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Results and conjectures on the Sandpile Identity on a lattice

Arnaud Dartois [1] and Clémence Magnien [1]

[1] *LIX, École Polytechnique, 91128 Palaiseau Cedex, France*

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In this paper we study the identity of the Abelian Sandpile Model on a rectangular lattice. This configuration can be computed with the burning algorithm, which, starting from the empty lattice, computes a sequence of configurations, the last of which is the identity. We extend this algorithm to an infinite lattice, which allows us to prove that the first steps of the algorithm on a finite lattice are the same whatever its size. Finally we introduce a new configuration, which shares the intriguing properties of the identity, but is easier to study.

Keywords: Abelian sandpile, Identity, Burning algorithm, Infinite lattice, Toppling

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1 Preliminaries

The abelian Sandpile Model was first introduced in (BTW87), and has been widely studied as one of the simplest models for Self-organized criticality (SOC) (KLGP00; Bak97).

This model is defined on a rectangular lattice, in the squares of which are disposed a certain number of *grains* of sand. The evolution rule, called the *toppling* rule, is the following: if a square contains at least four grains, it topples, losing four grains and giving one to each of its neighboring squares. If the toppled square is on the edge of the lattice, grains fall out of the lattice and are lost.