A Strategic Epistemic Logic for Bounded Memory Agents

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Introduction In [3] and [4] we presented an ATL-style epistemic logic for agents with arbitrary equivalence relations on histories, called euATL. In the current abstract, we review this logic and discuss its applications to problems concerning bounded memory agents. euATL is unique and relevant for problems in resource-bounded agents because it makes it possible to model and reason about systems where agents each have different, *arbitrary* equivalence relations on histories, and can be aware of their own past actions. While partial information strategic logics with memoryless agents, or with perfect recall agents, have already been studied in several papers, such as [1, 2] among others, in our systems we can model a situation, for example, where a subset of agents have bounded, finite memory (they only remember the last n states) and another subset of agents has unbounded memory. Since we allow arbitrary equivalence relations on histories including actions, we can also reason about situations where an agent loses all its information after entering a certain state, or loses some information from its memory when it chooses to take a certain action, or even remembers half of the previous states, etc. Thus, euATL is a practical logic for discussing situations where agents' memory is bounded, even in complex ways.

The logic euATL is defined on the models *epistemic concurrent game structures*, $\langle Q, \Pi, \Sigma, \mathcal{B}, \sim, \pi, Av, \delta\rangle$, where Q is a set of states, Π propositions, Σ agents with $|\Sigma| = n$, $\mathcal{B}$ actions, $\sim$ an equivalence relation for each agent, π a valuation function, $Av : Q \times \Sigma \rightarrow \mathcal{P}(\mathcal{B})$ defines the actions available to each agent in each state, and $\delta : Q \times \mathcal{B} \rightarrow \mathcal{P}(Q)$ is the transition function. So, the systems consist of agents and states, an equivalence relation on states for each agent, and at each state, every agent chooses an available action and the next state is chosen by the combination of these actions. We also require *determinacy*: when each agent chooses one available action, the combination gives exactly one possible next state. A *history* is a finite alternating sequence of states and n-tuples of actions $q_0.b_1^*.q_1 ... q_{k-1}.b_k^*.q_k$ such that in each n-tuple b_i^*, each agent takes an available action, and q_i is the outcome given by the δ function of taking actions b_i^* in state q_{i-1}.

We assume that besides the agents' equivalence relations on states, the systems are also equipped with equivalence relations on histories for each agent. If h_1 and h_2 are equivalent for agent i, we denote this as $h_1 \approx_i h_2$. By allowing agents to have *arbitrary* equivalence relations on the states, we make possible several interesting situations in our systems: we can model a perfect recall agent by allowing him to distinguish any pair of histories that have at least one pair of states that he can distinguish, or differ in the actions he took, and we can model memoryless agents by basing their equivalence relation only on the last state in the history, and similarly we can model finite memory agents, or even agents who always forget a certain state, etc. Also, by including actions in histories, we allow agents to remember, or forget, their own actions, rather than only remembering the past states. Finally, we define strategies for agents as usual: a strategy for agent i is a function $f_i : Hist \rightarrow \mathcal{B}$ that assigns the agent an available action for each possible history, and respects the agent's equivalence relation: if $h_1 \approx_i h_2$ then $f_i(h_1) = f_i(h_2)$.

Now we can discuss euATL. The syntax is

$$\phi ::= p \mid \neg\phi \mid \phi \vee \phi \mid K_i\phi \mid C_A\phi \mid \langle\langle A\rangle\rangle \bigcirc \phi \mid \langle\langle A\rangle\rangle \Box\phi \mid \langle\langle A\rangle\rangle \phi\mathcal{U}\phi$$

where $p \in \Pi$, $i \in \Sigma$, and $A \subseteq \Sigma$. Booleans, knowledge and common knowledge are interpreted as usual. The operator $\langle\langle A\rangle\rangle \bigcirc \phi$ means "the agents in group A have a strategy to make ϕ true at the next state, based on their knowledge." Thus, the strategy for each agent must succeed not only at the current history but at all other histories that the agent considers possible as well; otherwise the agent would not know that it would be effective to choose this strategy. So, the semantics for this operator is $L, h \models \langle\langle \Gamma\rangle\rangle_a \bigcirc \phi$ iff there exists a group strategy F_Γ for Γ such that $\forall h' \approx_\Gamma^* h$, $\forall \lambda \in out(h', F_\Gamma)$, $\lambda[0, |h'|+1] \models \phi$. Similarly, $\langle\langle A\rangle\rangle \Box\phi$ means that A has a strategy to make ϕ true always. $\langle\langle A\rangle\rangle \phi_1 \mathcal{U} \phi_2$ means A has a strategy to keep ϕ_1 true until ϕ_2 is true.

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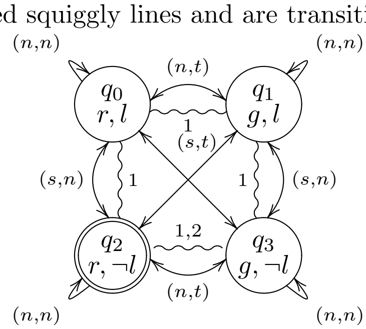
Applications First we consider a very simple example of a system with one memoryless agent and one perfect recall agent. a_1 is memoryless, makes no observations at all and at each time-step only decides whether to flip a switch or not. This switch controls the lights. a_2 has perfect recall and has a card which is red on one side and green on the other. At each time-step he can turn over the card or not. If the lights are on, he can see which side of the card is up, and if they are off he cannot see the color of the card. The propositions are l for lights on, r for card red and g for card green. The actions are s for a_1 flipping the switch, t for a_2 turning over the card, and n for do nothing. In the diagram, the actions are shown as pairs with the first action for a_1 and the second for a_2. The agents' equivalence relations are shown by the labelled squiggly lines and are transitive.



Suppose the system starts in q_2, with the lights off and the card red. Neither of the agents knows what state the system is in: a_1 does not observe anything and a_2 does not know whether the card is red or green. We would like to check whether the following formulas are true at q_2. $\langle\langle\{a_2\}\rangle\rangle \bigcirc g$: can a_2 make g true at the next state? $\langle\langle\{a_2\}\rangle\rangle \bigcirc \langle\langle\{a_2\}\rangle\rangle \bigcirc g$: can a_2 make g true after two steps? $\langle\langle\{a_1, a_2\}\rangle\rangle \bigcirc g$: can a_1 and a_2 together make g true at the next state? And $\langle\langle\{a_1, a_2\}\rangle\rangle \bigcirc \langle\langle\{a_1, a_2\}\rangle\rangle \bigcirc g$: can both agents together bring about g after two steps? $\langle\langle\{a_2\}\rangle\rangle \bigcirc g$ is not true because a_2 does not know whether g is true now so he knows no strategy to make it true at the next step. Similarly, $\langle\langle\{a_2\}\rangle\rangle \bigcirc \langle\langle\{a_2\}\rangle\rangle \bigcirc g$ is not true. Also, $\langle\langle\{a_1, a_2\}\rangle\rangle \bigcirc g$ is not true because neither agent knows the current color of the card so together they cannot make it green at the next step. Finally, $\langle\langle\{a_1, a_2\}\rangle\rangle \bigcirc \langle\langle\{a_1, a_2\}\rangle\rangle \bigcirc g$ is true. a_1 can use the strategy of flipping the lightswitch no matter what the current history is (since he is memoryless he must behave the same in all histories) and then at the next step, a_2 will learn that the card is red and do the strategy of turning it over. Note that these strategies succeed from all states either agent considers possible, as required. This example demonstrates that it is interesting to be able to combine agents with different levels of memory ability in one system. Even though a_2 has perfect recall, he is able to accomplish more with cooperation of the memoryless agent a_1.

Although this example is extremely simple, it is easy to imagine other scenarios where it would be practical to model agents with different memory abilities in one system. For example, in a system where there are some friendly agents with bounded memory, and other adversarial agents with unknown memory abilities, we could model the friendly agents as limited memory agents, and the adversarial agents as perfect recall agents in order to represent the worst case scenario, which would be practical for verifying security properties in a system. Also, allowing arbitrary equivalence relations on histories gives a great deal of flexibility in modelling agents with their memory bounded in interesting ways: e.g. we can model an agent whose memory fills up after he has seen n states, so he only remembers the first n states and gains no new information after this, or we could model a system where agents' actions affect their memory, such as an agent who forgets everything after performing a certain action, or even an agent who forgets the first or last state after performing some action. Thus, our systems allow us a great deal of flexibility in modelling agents with bounded abilities.

Future Work In the future, we hope to add expressions about agents' memory abilities into the logic, rather than having them be fixed properties of the underlying systems. Ideally, we would be able to have formulas expressing complex statements such as "if at least one of a_1 or a_2 have perfect recall, then the coalition $\{a_1, a_2\}$ can achieve the goal, but if both a_1 and a_2 are memoryless, then the coalition cannot achieve the goal."

References

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