Algebraic decompositions of corner triangulations

ongoing work with Schützenberger methodology

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MPS 2016, Bordeaux

based on joined work with C. Dervieux and D. Poulalhon
on the enumeration and realization of graphs of corner polyhedra

Enumeration and Schützenberger methodology for algebraic generating series

Schützenberger methodology (for algebraic gf)

Let A_n be a class of combinatorial objects enumerated by the integer a_n and suppose that the corresponding generating function $f(t) = \sum_{n\geq 0} a_n t^n$ is *algebraic*. An old idea, dear to M.P. Schützenberger, is to explain this algebraicity by expliciting a bijection between A_n and the words of a certain *algebraic* (*context-free*) *language* L defined on the alphabet X by a *non-ambiguous grammar*.

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Plane trees encoded by the Dyck words with grammar $D \to xD\bar{x}D \mid D \to \varepsilon$.

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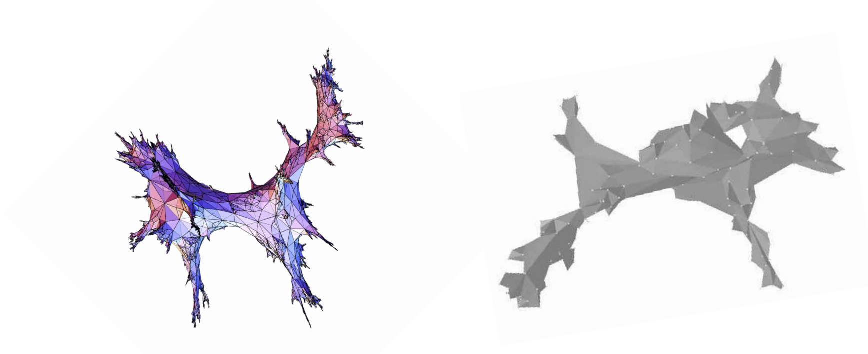
- At least two other great historical successes:

- Planar maps

70's Cori... Cori-Vauquelin'81



paper again quoted during last WCM...

- Convex polyominoes

80's Viennot, Delest...

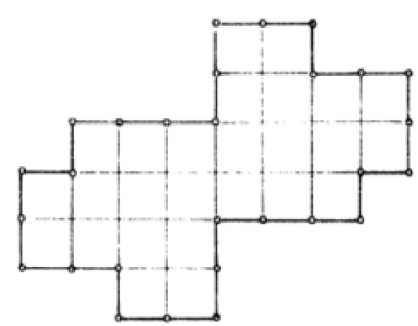


Fig. 1. A convex polyomino.

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ALGEBRAIC LANGUAGES AND POLYOMINOES ENUMERATION

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A notre 'bon maître' M.P. Schützenberger

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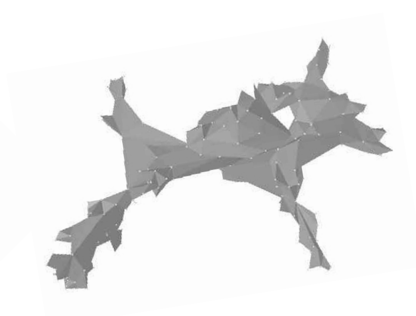

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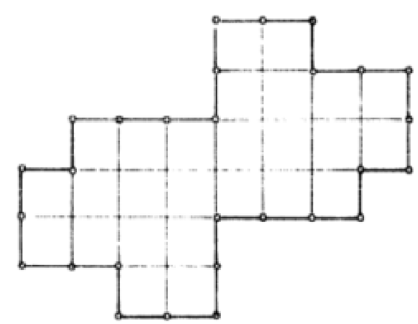

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The coding with words sheds more light upon the combinatorial comprehension of A_n. Each equation of the noncommutative algebraic system is in fact a combinatorial property of the objects of A_n.

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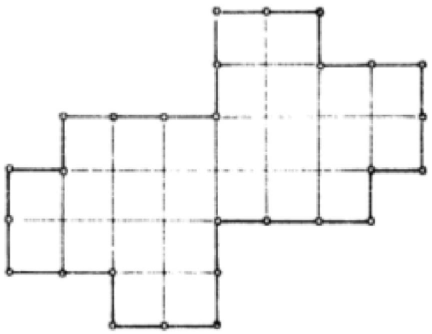

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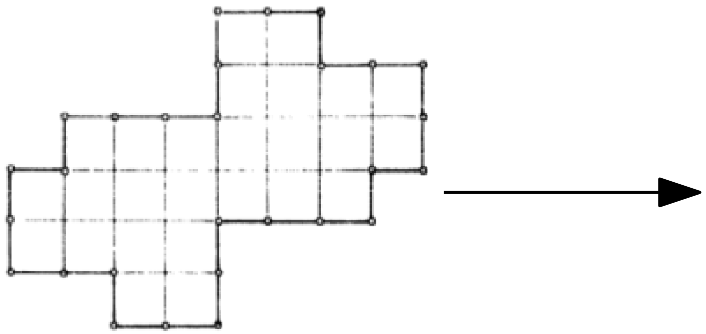

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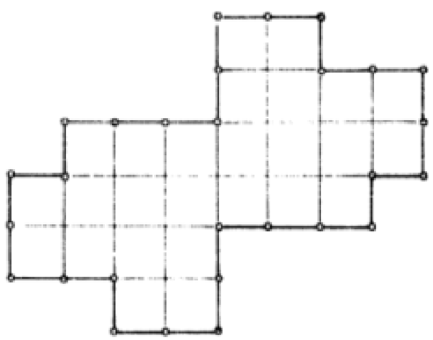

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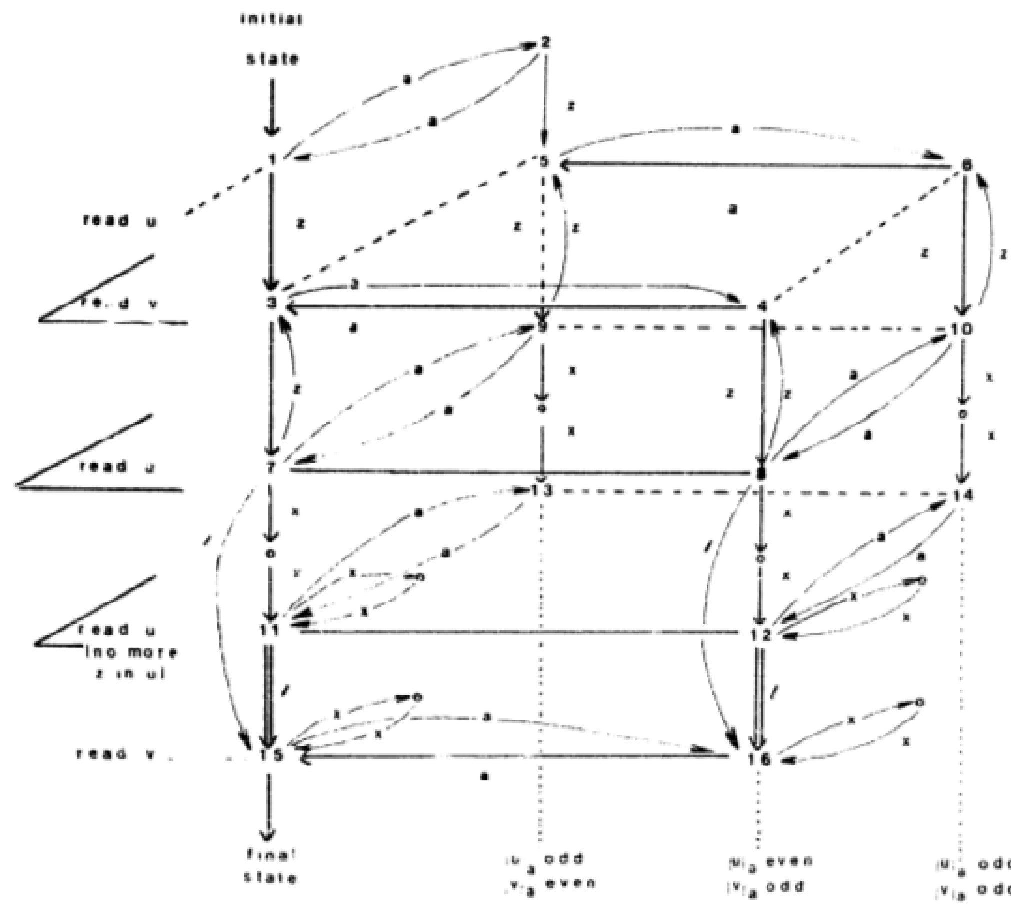

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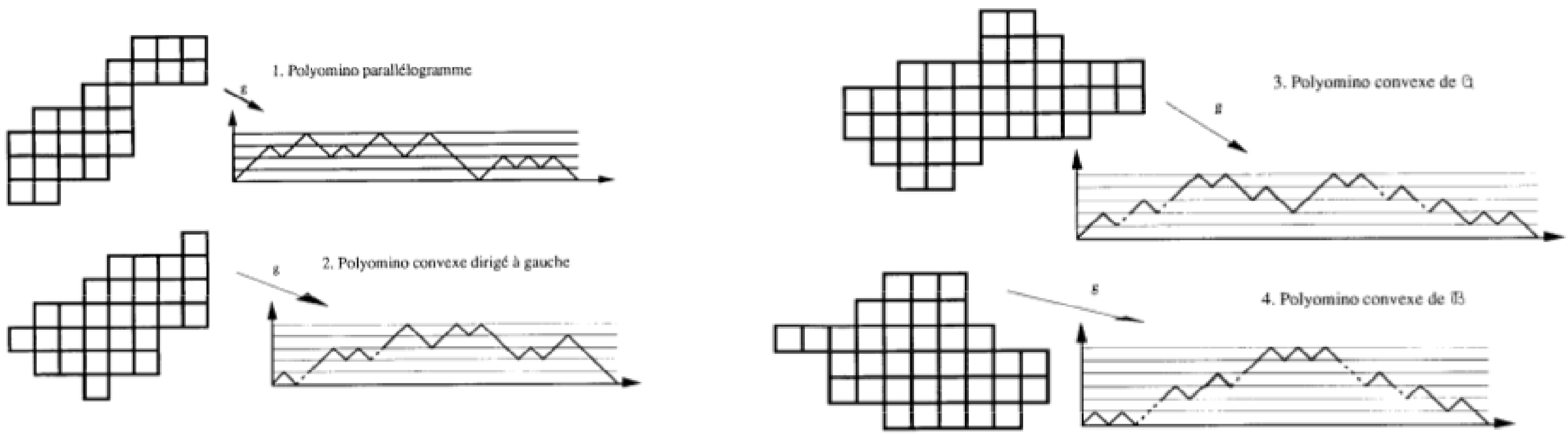

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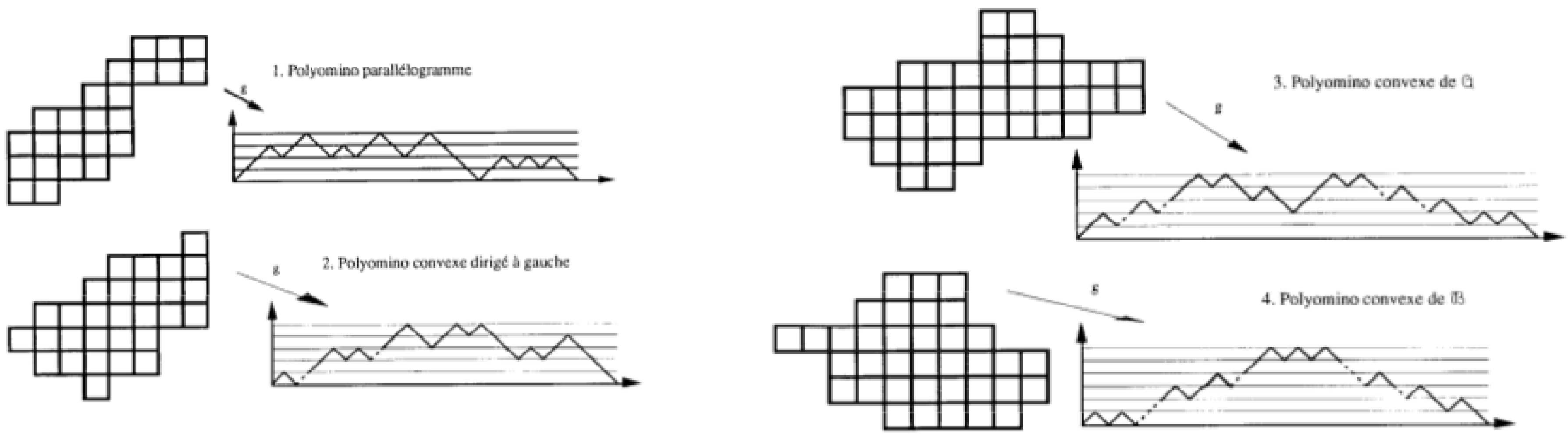


Fits in general trend from 60's, with many incarnations:
geometric interpretation of special functions (Foata-Schützenberger),
Viennotique (Viennot), Symbolic method (Flajolet),...
now "common knowledge" in enumerative combinatorics?

Schützenberger methodology (for algebraic gf)

Mireille has exposed monday some limitations of the methodology: algebraic gfs without (known/any) associated $\mathbb{N}$-algebraic structure, and she has shown to us other "natural sources" of algebraic gfs.

Instead in this talk, I will present some recent developments on the enumeration of planar maps that follow the original idea.

Apart from obvious spiritual satisfaction, one reason to insist on Schützenberger methodology is that among the "natural sources" of algebraic gfs, an algebraic decomposition is still the most likely to be algorithmically useful.

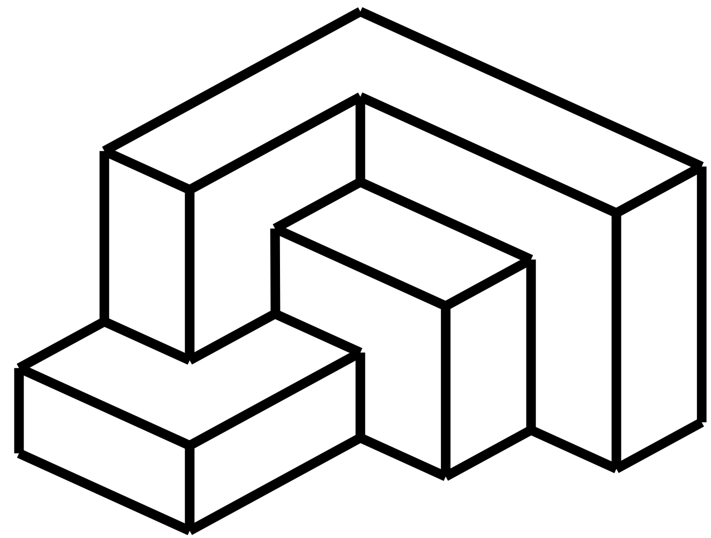
And now for something completely different

Corner polyhedra

Definition (Eppstein & Munford, 2014)

A polyhedron $\mathcal{P}$ in $\mathbb{R}^3$ is corner if

- $v_0 = (0,0,0) \in \mathcal{P}$
- edges are parallel to axes
- 3 edges meet at each vertex
- all vertices but v_0 are visible from infinity in direction $(1,1,1)$



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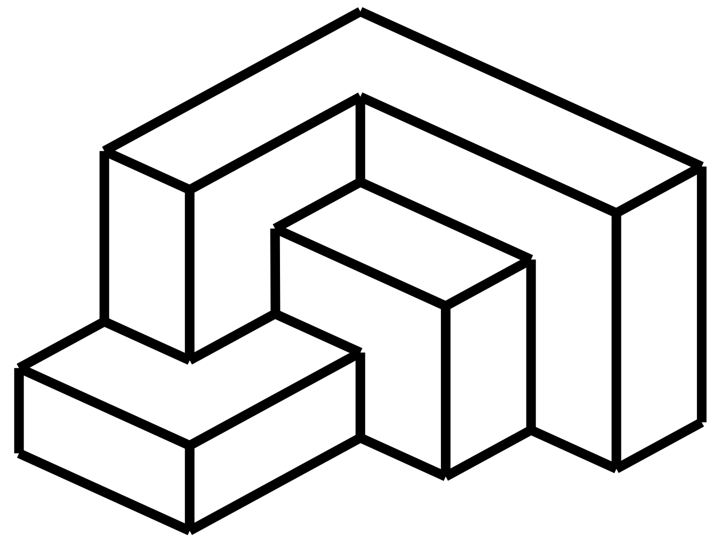

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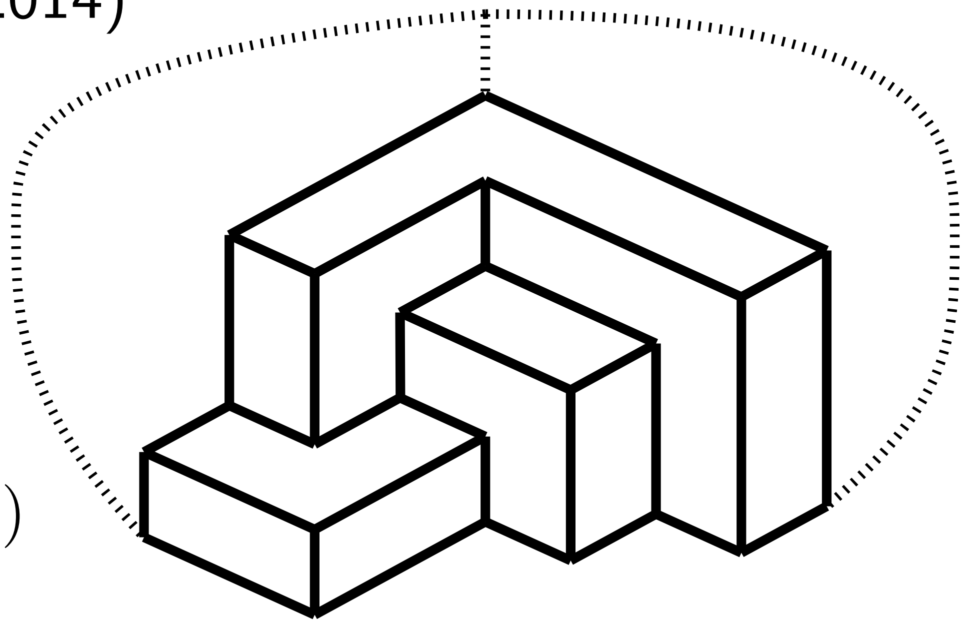

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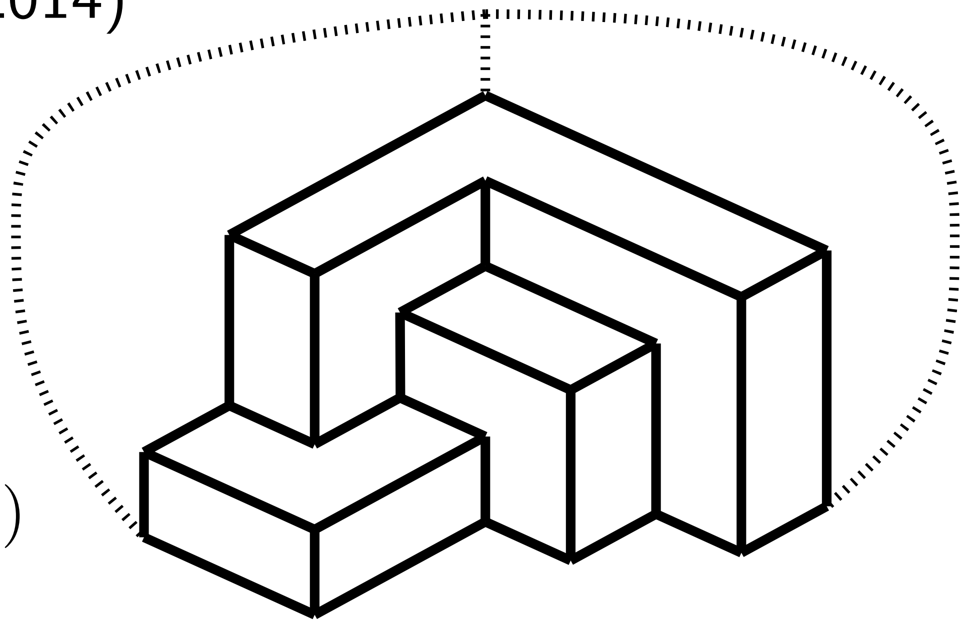

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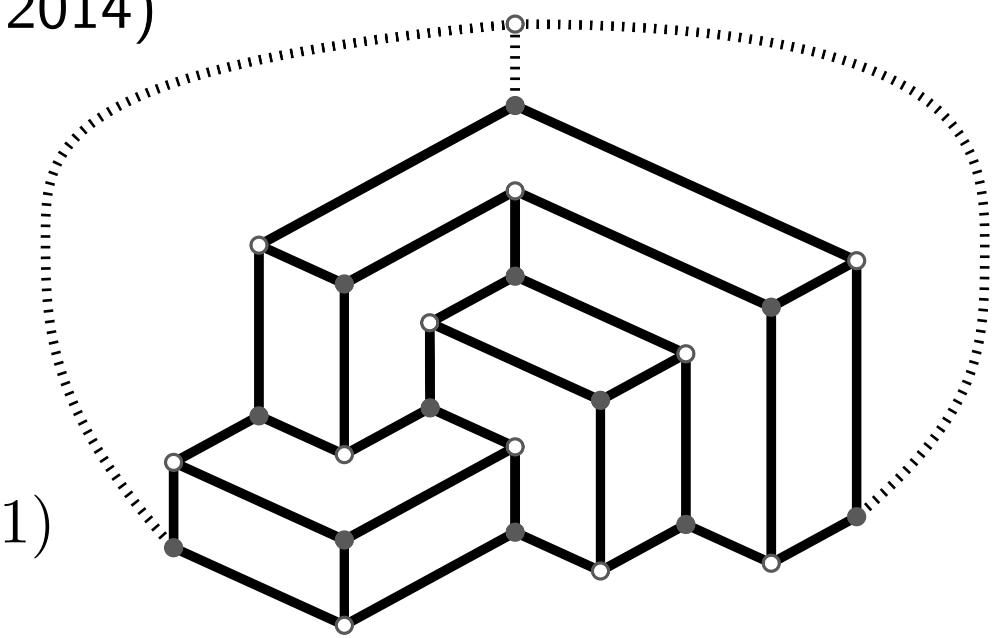

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bipartite, with black symmetric vertices, white asymmetric vertices.

Obstruction to being corner

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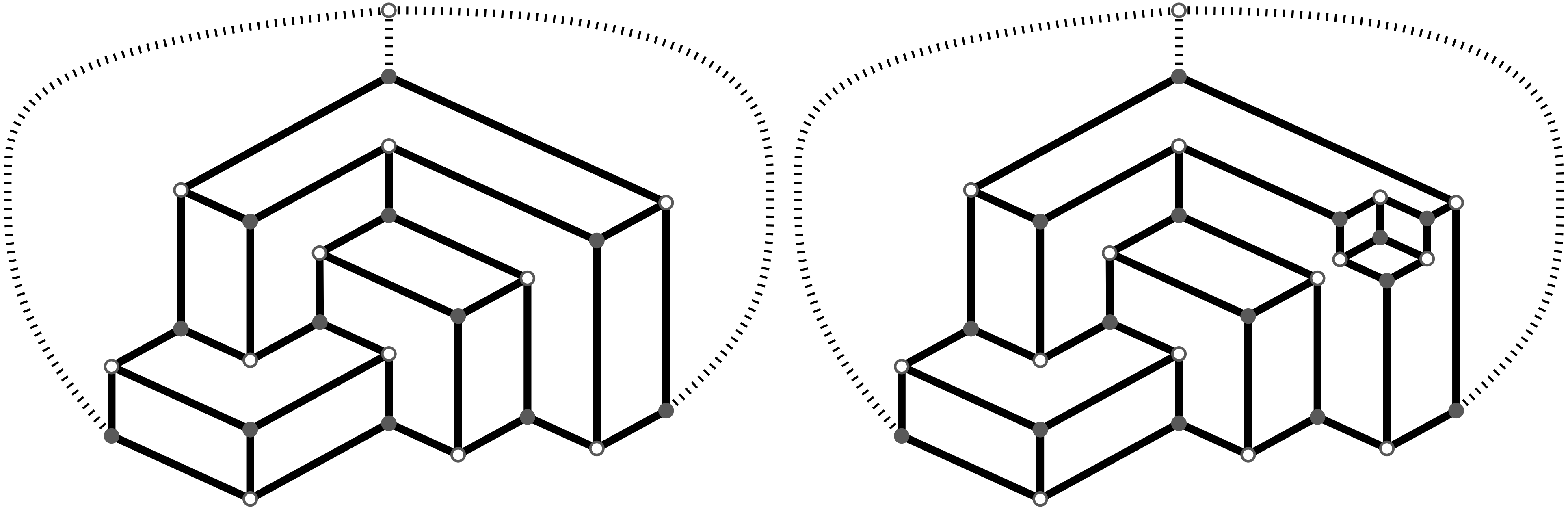
Are all such graphs realizable as skeletons of corner polyhedra?



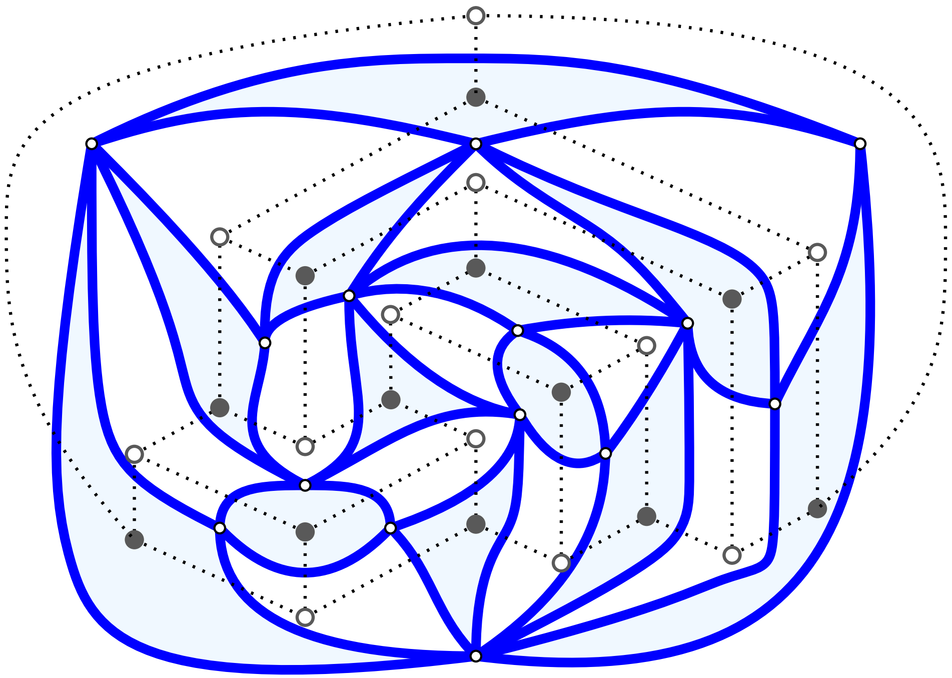
Corner triangulations can be composed at black vertices.

Similar insertions cannot be done at white vertices !

Corner triangulations

Consider the dual triangulation

Eulerian triangulation:
all faces are triangles
and they are bicolored.



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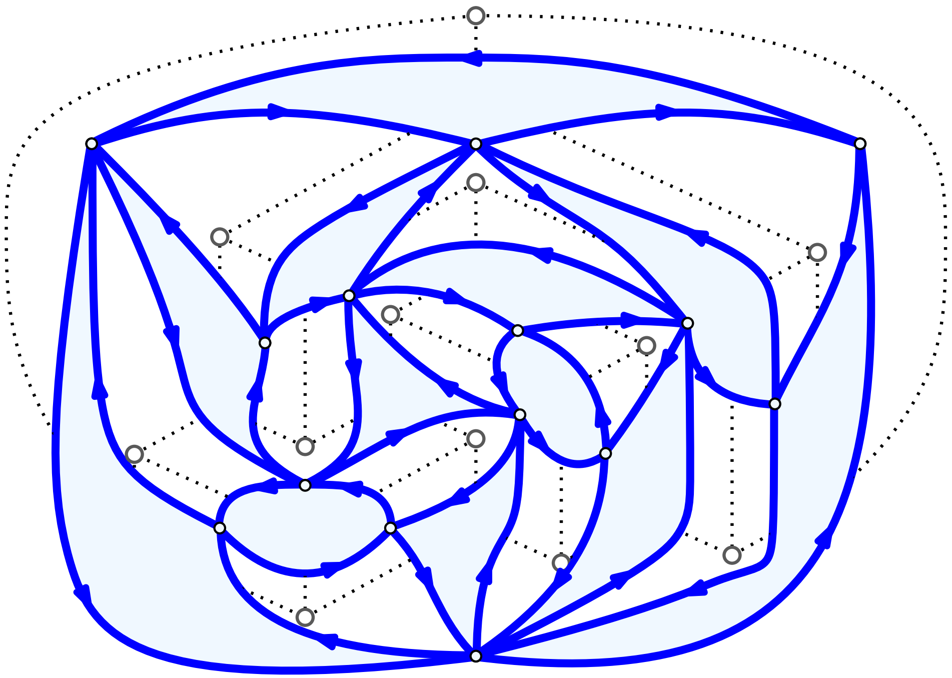
Canonical orientation: counterclockwise around black faces.



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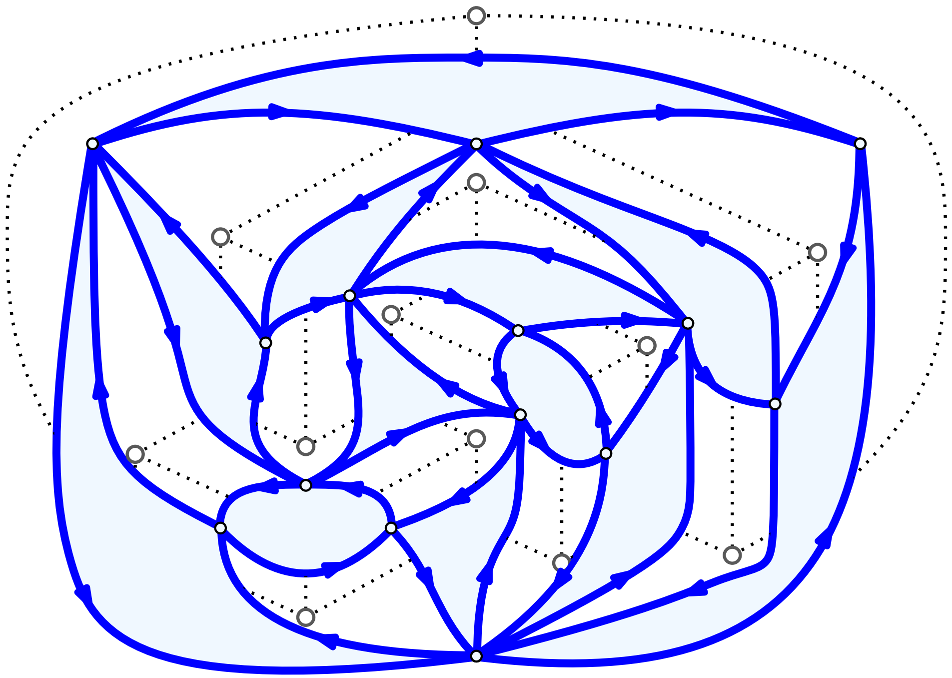

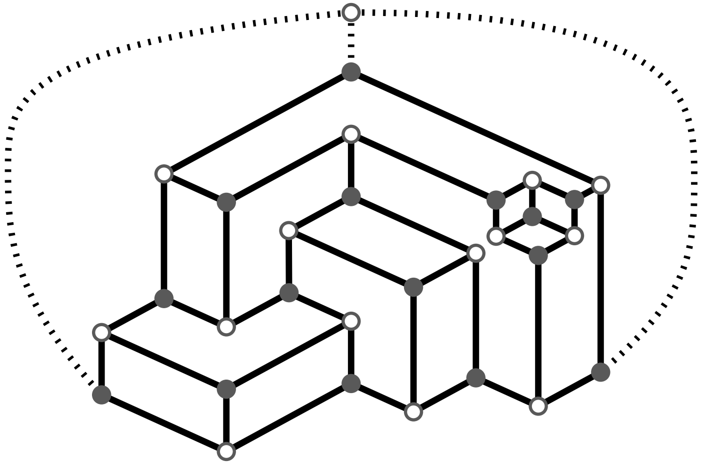

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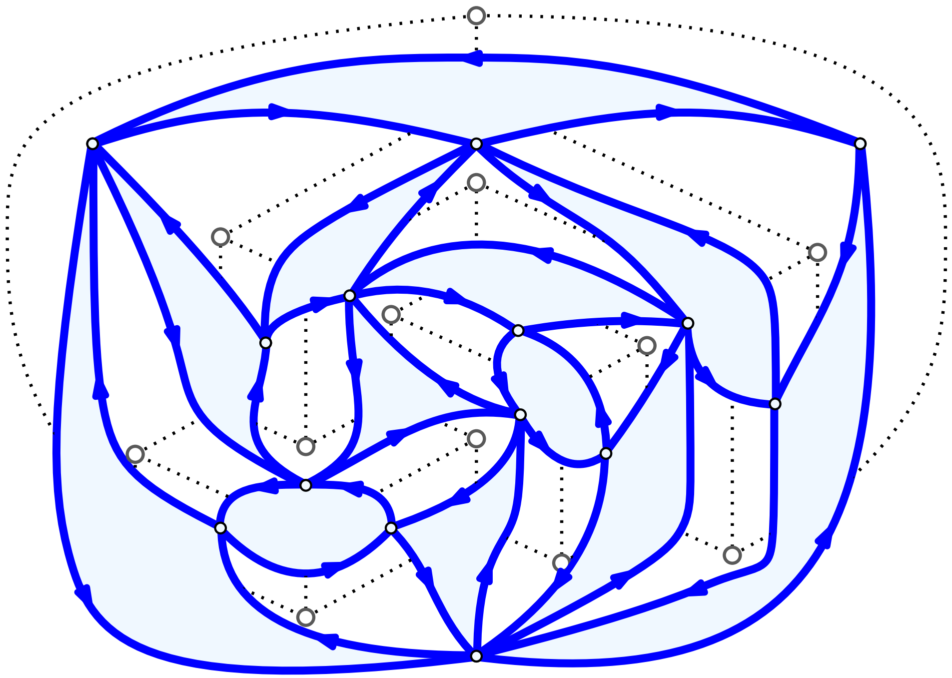

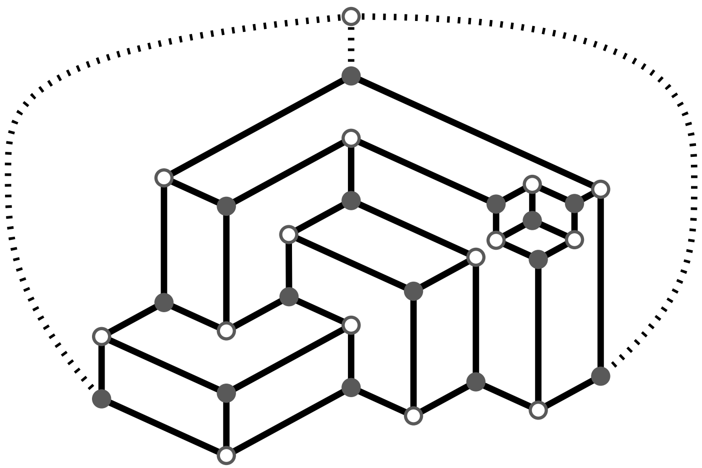

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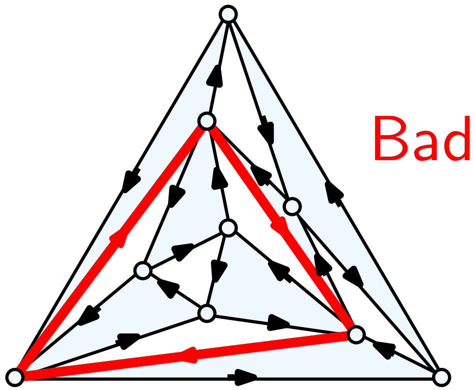

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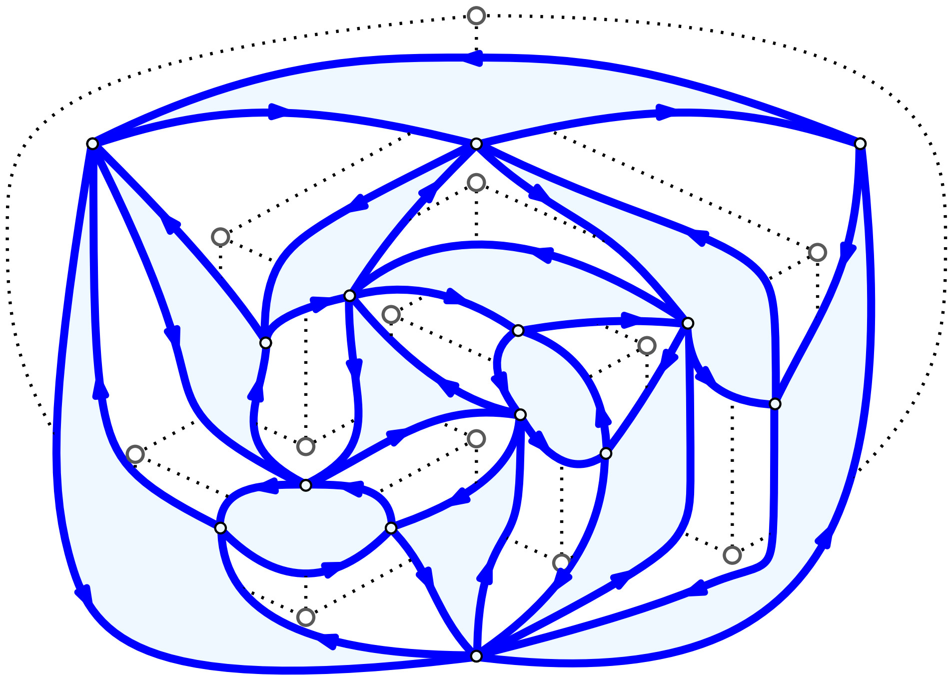

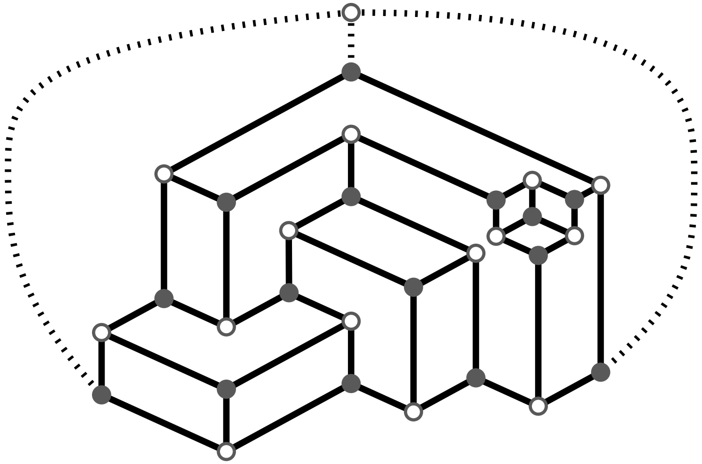

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Eppstein and Munford's theorem

Theorem (Eppstein-Munford)
A graph is the skeleton of a corner polyhedron if and only if it is planar and 3-connected, and its dual is a corner triangulation.

This result is a remarkable analog of the classic Steinitz theorem:

A graph is the skeleton of a convex 3d polyhedron if and only if it is planar and 3-connected.

Eppstein-Munford realization algorithm

Problem: Given a bicubic planar graph dual to a corner triangulation, draw it as a corner polyhedron.

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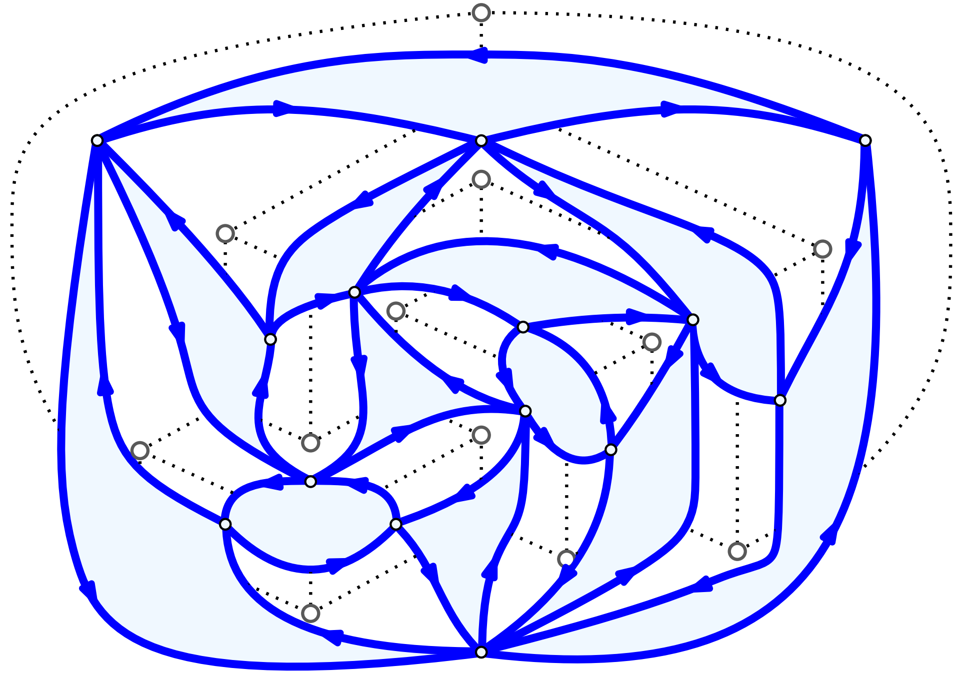

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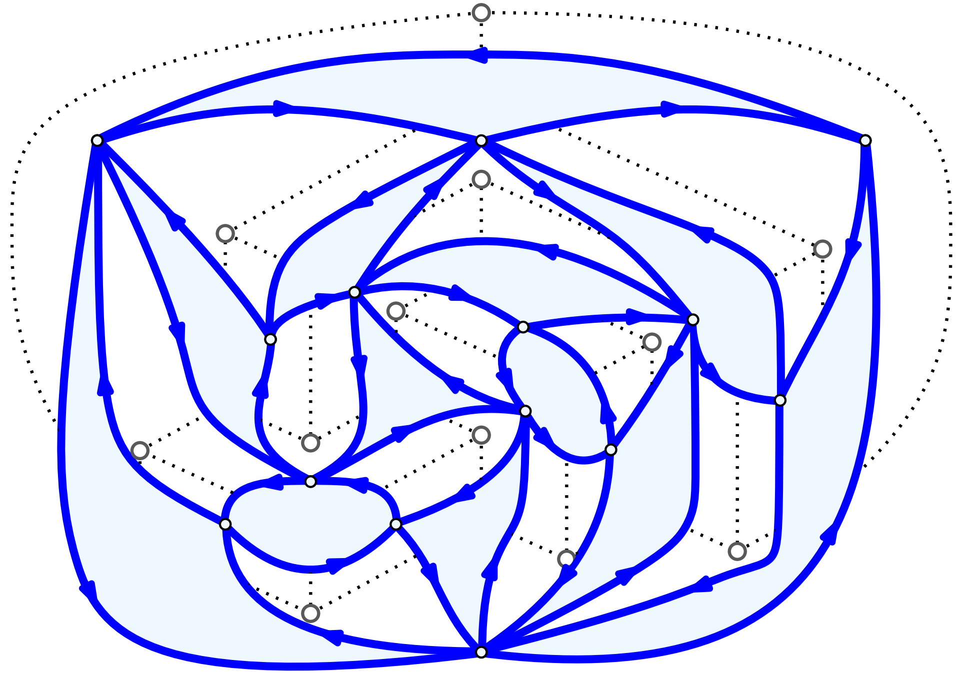

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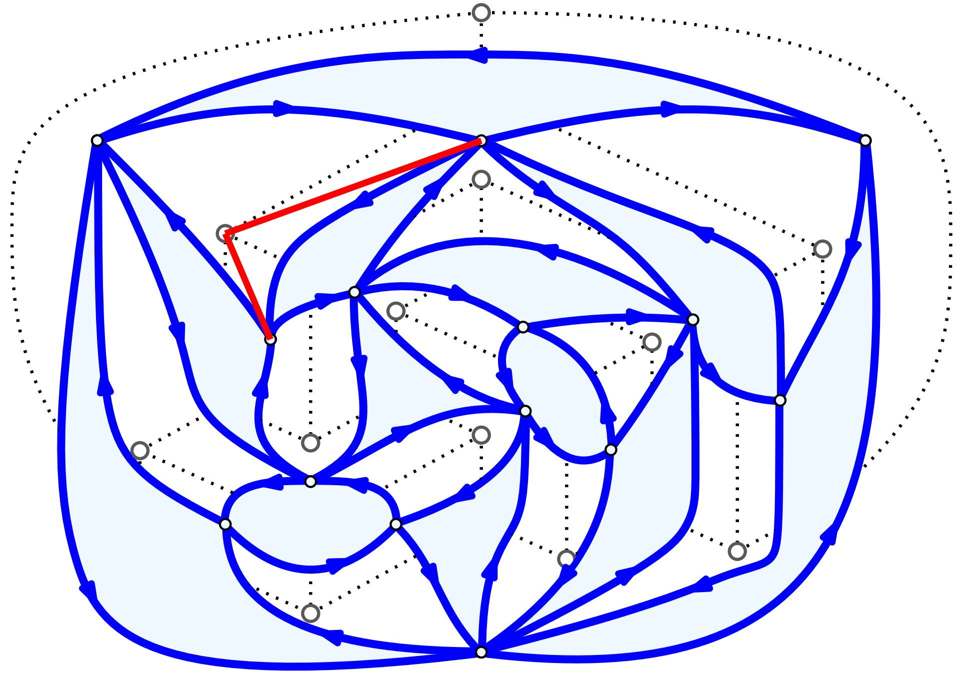

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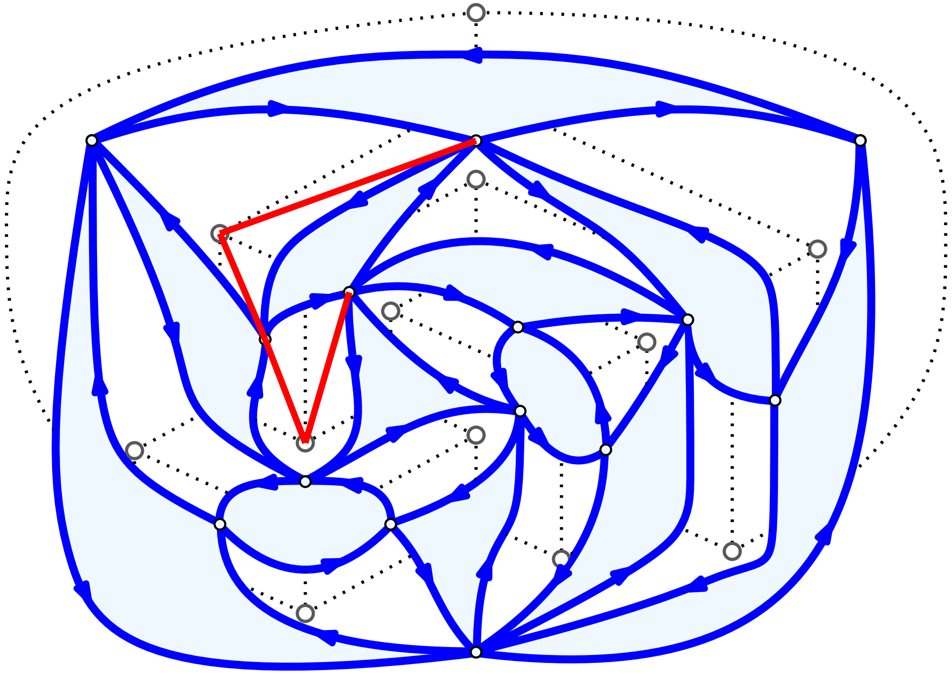

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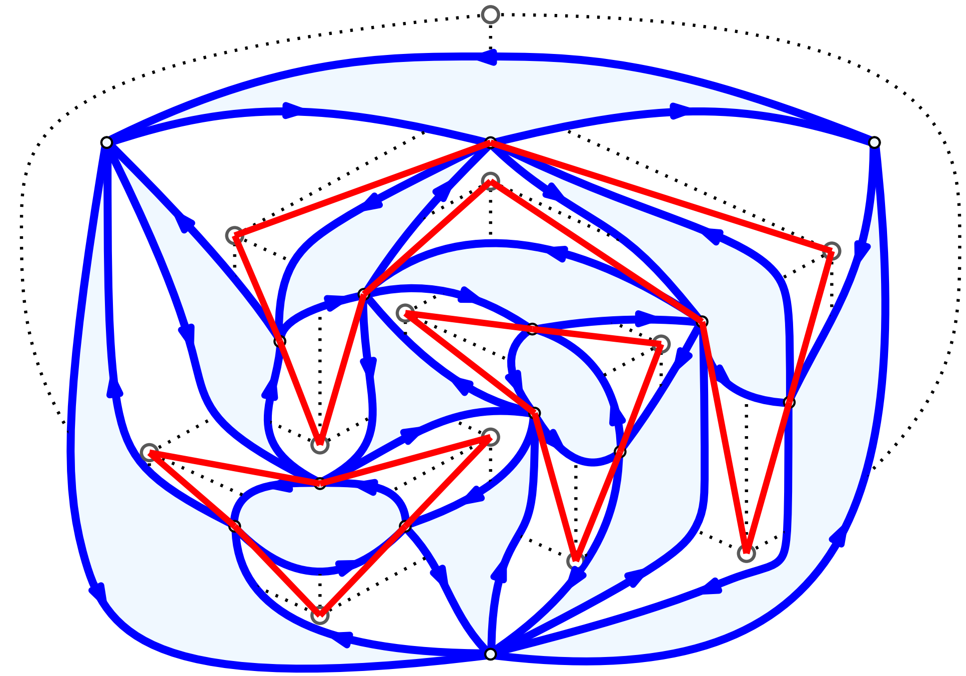

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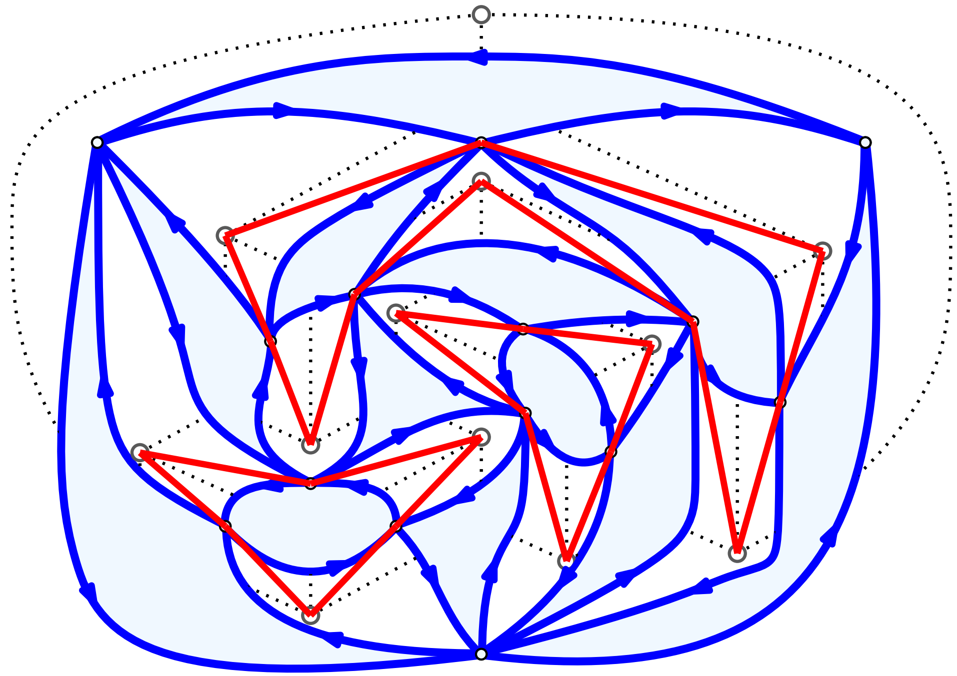

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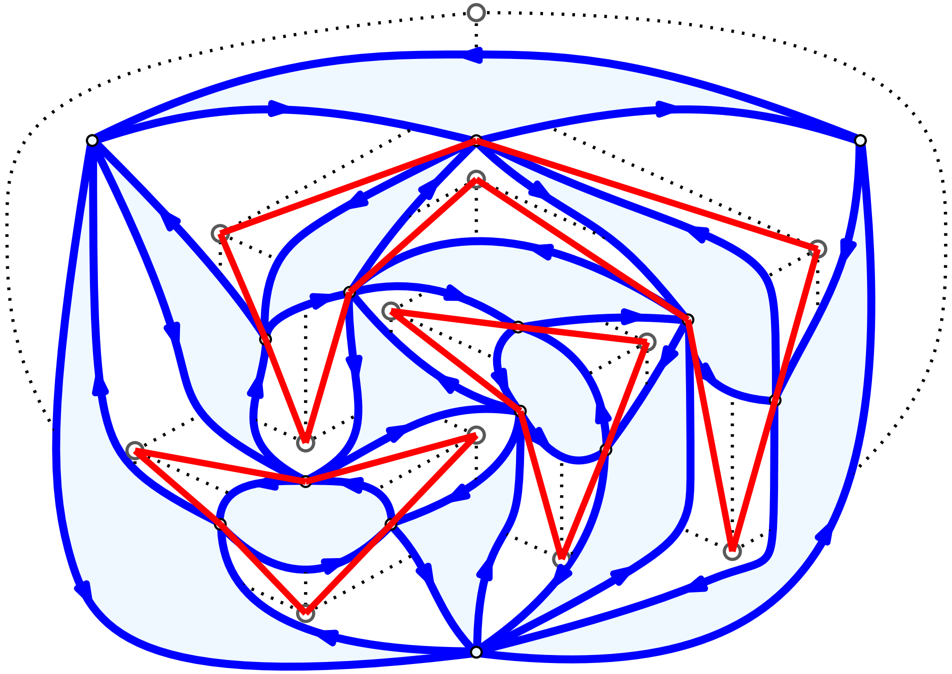

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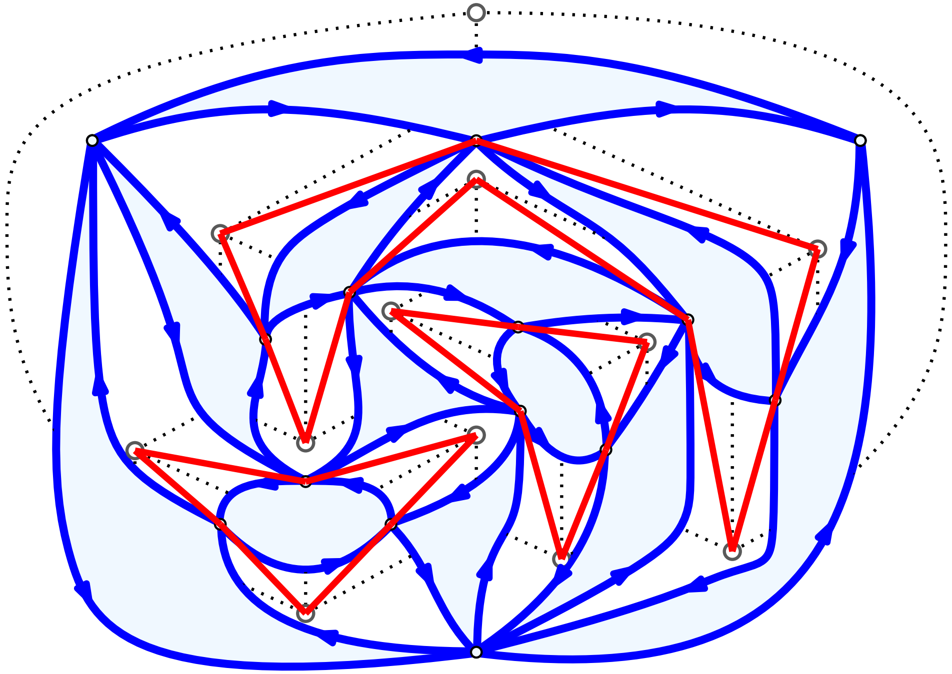

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Instead the first step requires a decomposition of the triangulation into 4-connected components, and reductions rules.

Algebraic decomposition of maps

Decompositions for triangulations

The family of corner triangulations is a "natural" family of planar maps

For bicolored triangulations (or bicubic maps) several decompositions have been found since the 60's.

Indeed:

Theorem (Tutte, 62) The number of rooted planar bicolored triangulations with n black triangles is $E_n = \frac{3}{n+2}\frac{2^n}{n+1}\binom{2n}{n}$.
The gf $E(z) = \sum_{n\geq 1} E_n z^n$ satisfies $E(z) = B(z) - B(z)^2$ where $B(z)$ is the unique power series solution of $B(z) = z(1+2B(z))^2$.

These are bipartite maps in disguise, for which a variant of Cori-Vauquelin bijection was given by D. Arquès (85).

An algebraic generating function

Applying Tutte's composition approach, one can extract the gf of corner triangulations from the gf of bicolored triangulations:

Let E^c_n denote the number of corner triangulations with n black faces, and let $E^c(z)$ be their generating function:

$$E^c(z) = \sum_{n \geq 1} E^c_n z^n = z + z^4 + 3z^6 + 4z^7 + 15z^8 + \dots$$

Theorem (Dervieux, Poulalhon, S. 2015)

$$E^c(z) = \frac{z}{1+z}\left(1 + \frac{zA(z) + z^2A(z)^2}{1+z} - \frac{z^2A(z)^2 + 2z^3A(z)^3}{(1+z)^2}\right)$$

where $A(z) = \frac{1-\sqrt{1-4z}}{2z}$ is the Catalan gf, solution of $A(z) = 1 + zA(z)^2$.

According to Schützenberger methodology...

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where $A(z) = \frac{1-\sqrt{1-4z}}{2z}$ is the Catalan gf, solution of $A(z) = 1 + zA(z)^2$.

There should be a bijection between corner triangulations and words of (the difference of two) algebraic languages generated by some explicit non ambiguous grammar.

Equivalently one can look directly for an "algebraic" decomposition of the combinatorial structures.

Playing with pieces of triangulations

To obtain an algebraic structure, one needs a natural interpretation of the cartesian product.

For words: concatenation of words in codes $w_1 w_2$

For tree: joining root of independant subtrees.

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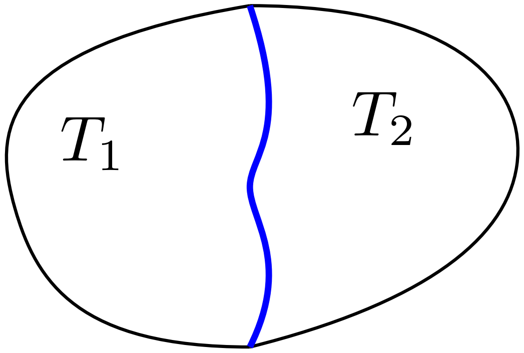

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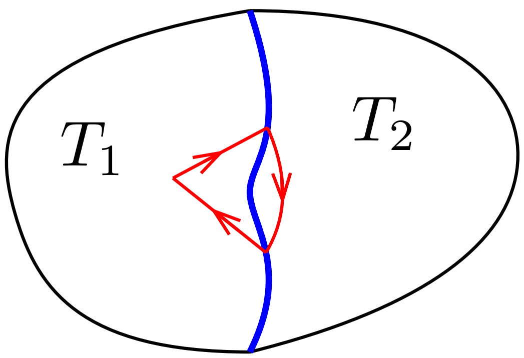

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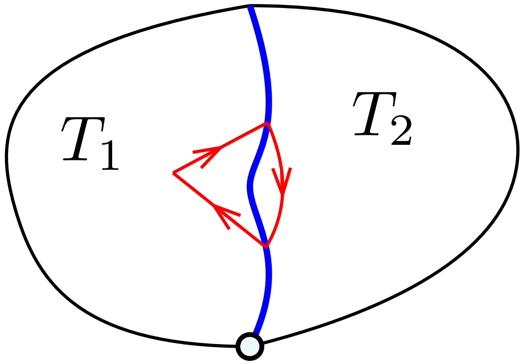
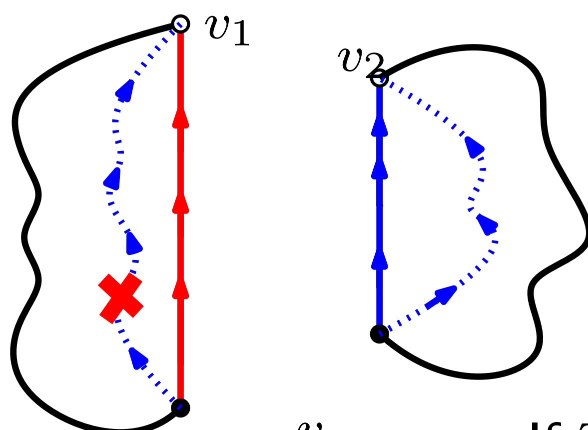
Knowing the root is not enough to cut back into pieces: we need a cut path!



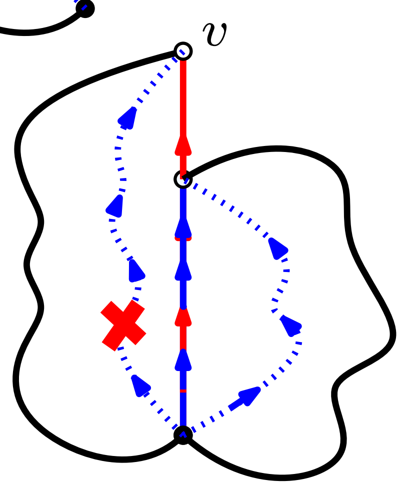
Playing with pieces of triangulations

A cure to this problem is to use geodesics (shortest paths to a fixed point) and in particular leftmost geodesics

T_1 has a leftmost geodesic as right boundary



T_2 has a geodesic as left boundary



If T_1 and T_2 are pieces of corner triangulations then

$T_1 \bullet T_2$ has no double edges.

$T_1 \bullet T_2$ has no ccw triangles.

$\Rightarrow$ it is a piece of corner triangulation!

Finally T_1 and T_2 can be recovered from $T_1 \bullet T_2$ because the cut path can be characterized as the leftmost geodesic from the root to the basepoint v...

Almond triangulations

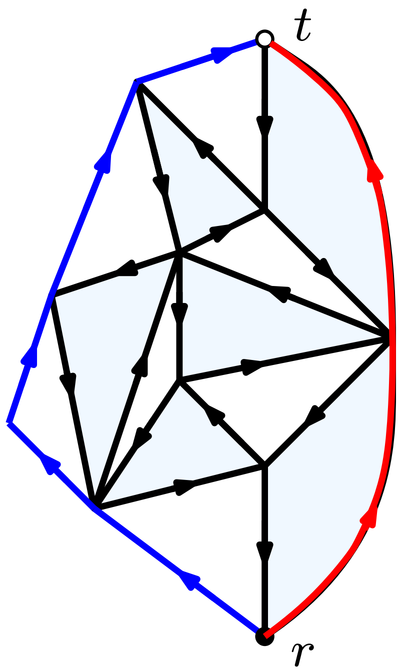
So what should be our intepretation of Catalan equation? $\mathcal{A} = 1 + z \times \mathcal{A} \times \mathcal{A}$

In order to be able to glue on left and right hand side, need "geodesic" boundaries

Definition: an almond triangulation is a simple bicolored triangulation of a polygon without non facial clockwise triangles and with an apex vertex t such that:

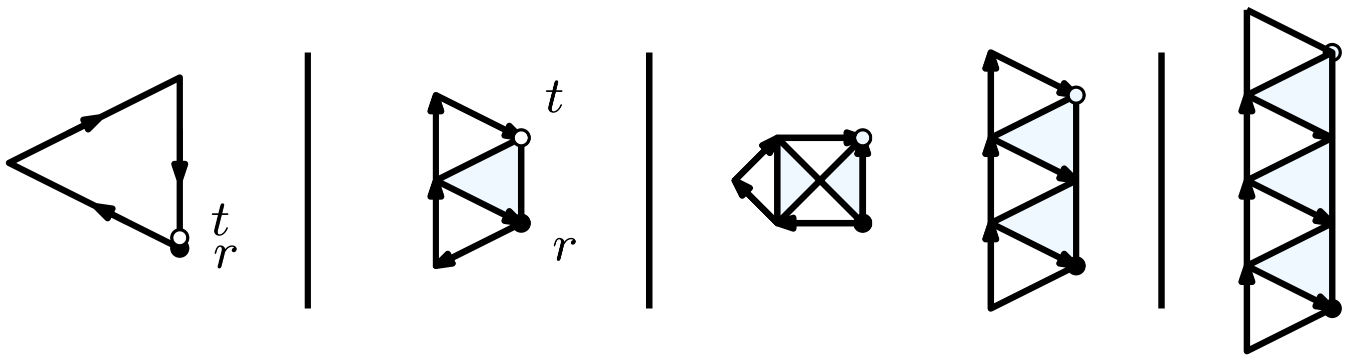
The right boundary is the unique geodesic from r to t. Let ℓ denote its length.

The left boundary is an almost geodesic path from r to t, of length $\ell + 3$.



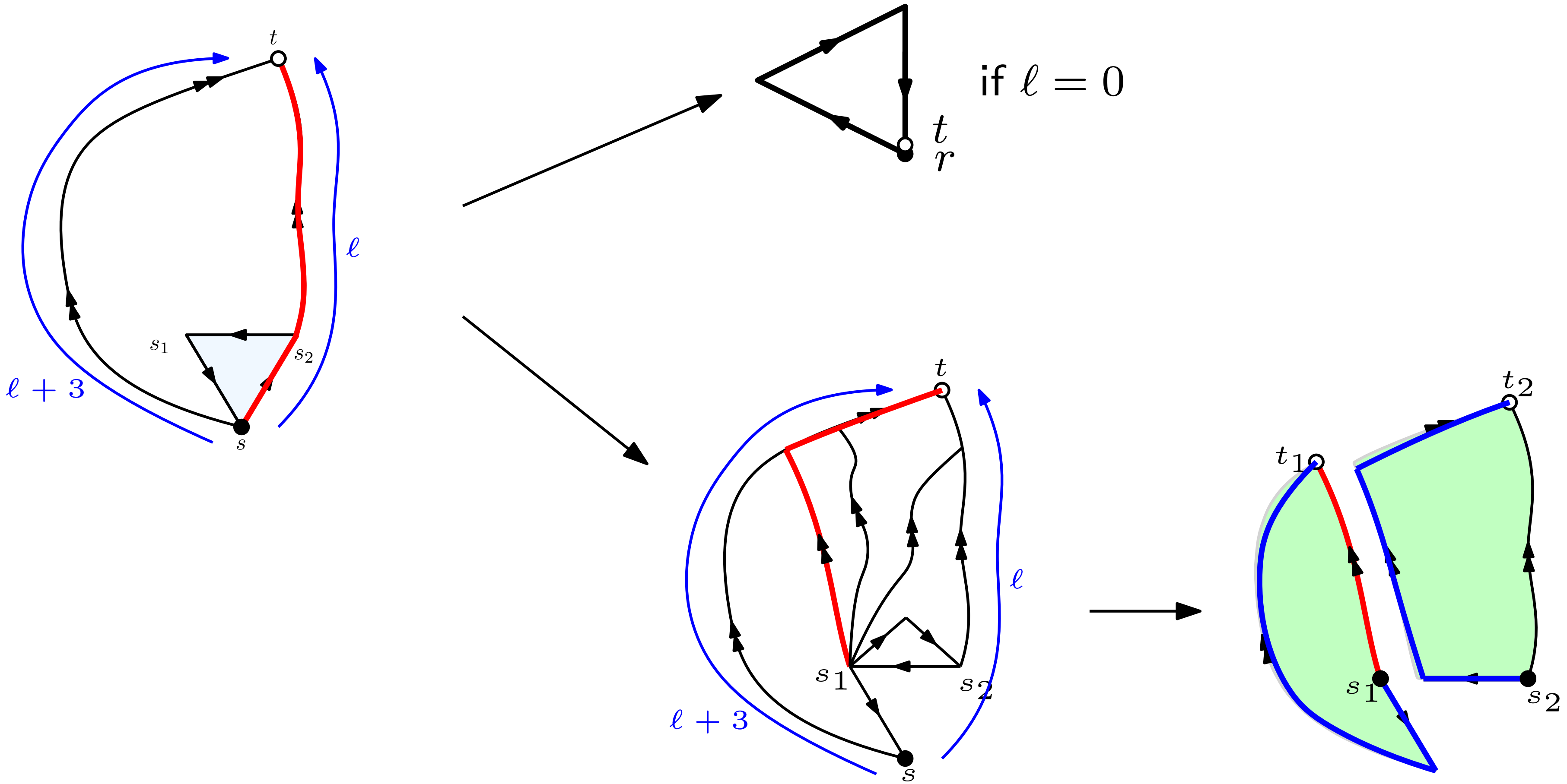
Some almonds:

The smallest almond is the clockwise triangle, which is also the unique with $\ell = 0$.



$+$ 4 more

The decomposition of almond triangulations



leftmost shortest path fro s_1 to t

$\ell - 2$ ✗

$\ell + 1$ ✓

$$\mathcal{A} \quad = \quad 1 \quad + \quad z \times \mathcal{A} \times \mathcal{A}$$

Almond triangulations and Catalan numbers

Theorem (Dervieux, Poulalhon, S. 2015)

The generating function of almond triangulations according to the number of black triangles is the Catalan gf, that is, the unique fps satisfying

$$A(z) = 1 + z \cdot A(z)^2$$

and the number of almond triangulations with n black faces is the nth Catalan number.

Corner triangulations are not almonds... but almost

Corner triangulations are essentially slices of height 1...

$$(1+z)E^c(z) = z + zS^1(z)$$

Slices of arbitrary height are essentially pairs of almonds...

$$(1+z)S(z) = zA(z) + z^2A(z)^2$$

Slices of height at least 2 are essentially triples of almonds...

$$(1+z)^2S^+ = z^2A(z)^2(1+2A(z))$$

Hence the difference !

Theorem (Dervieux, Poulalhon, S. 2015)

$$(1+z)E^c = z + z\left(\frac{S(z)}{1+z} - \frac{S^+(z)}{(1+z)^2}\right)$$

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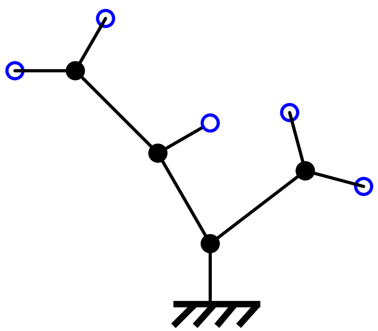
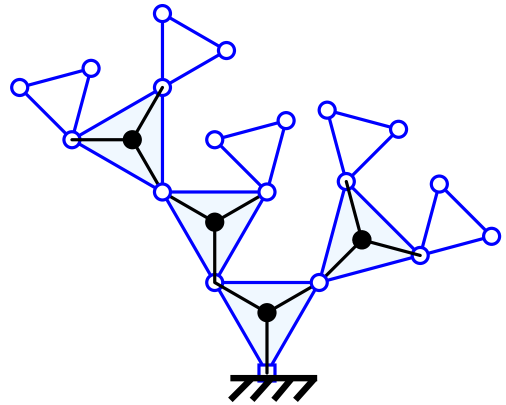
Build on ideas of Cori-Vauquelin, Fusy, Chapuy, Bouttier, Guitter, Albenque...

Reformulation in terms of trees

Binary tree



Triangular cactus



n nodes

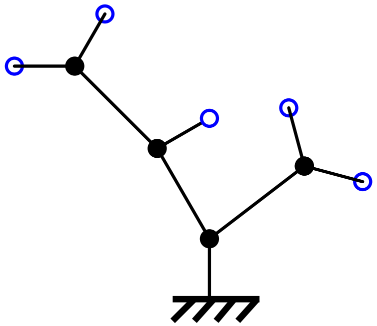
$n+1$ leaves

n black triangles

$n+1$ white triangles

Reformulation in terms of trees

Binary tree



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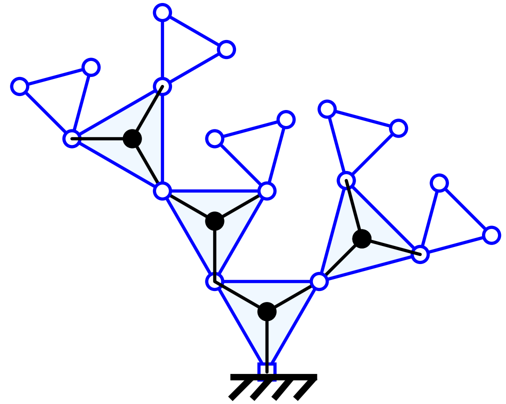
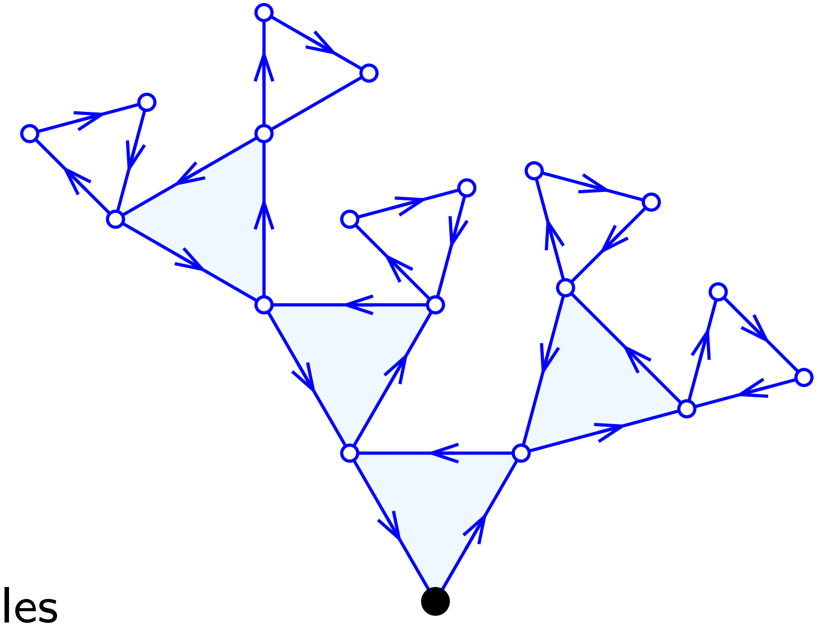


n black triangles

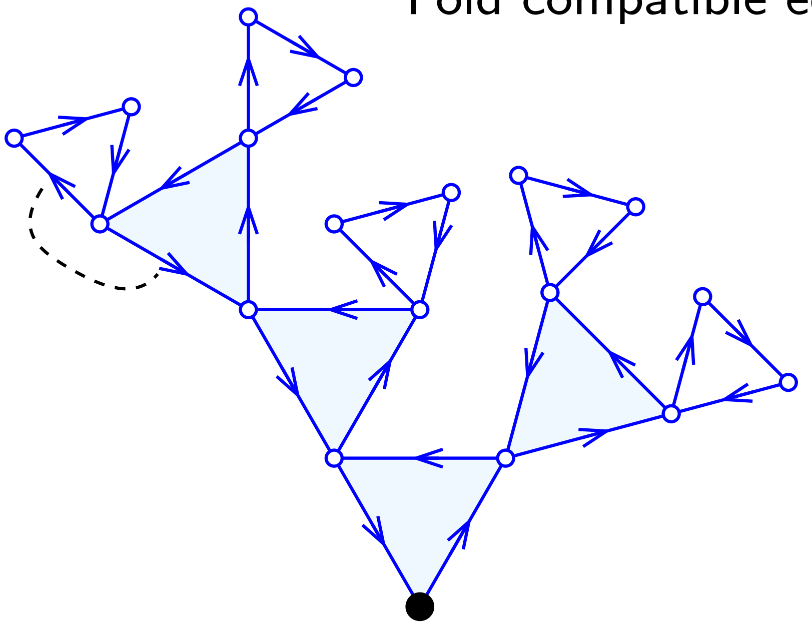
$n+1$ white triangles

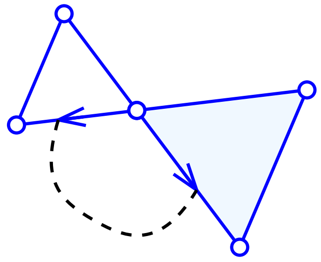


The number of such cacti is the nth Catalan number.

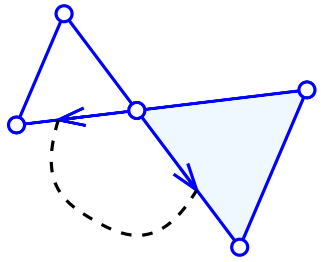
Reformulation in terms of trees

Fold compatible edge around the cactus





Reformulation in terms of trees



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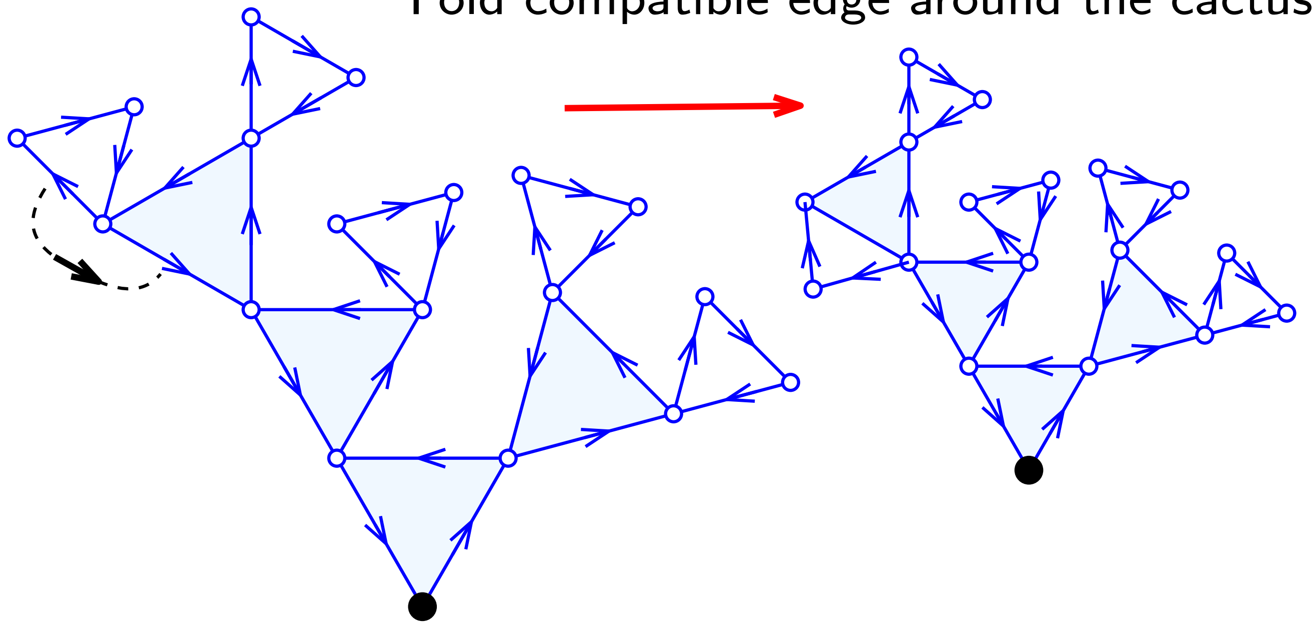

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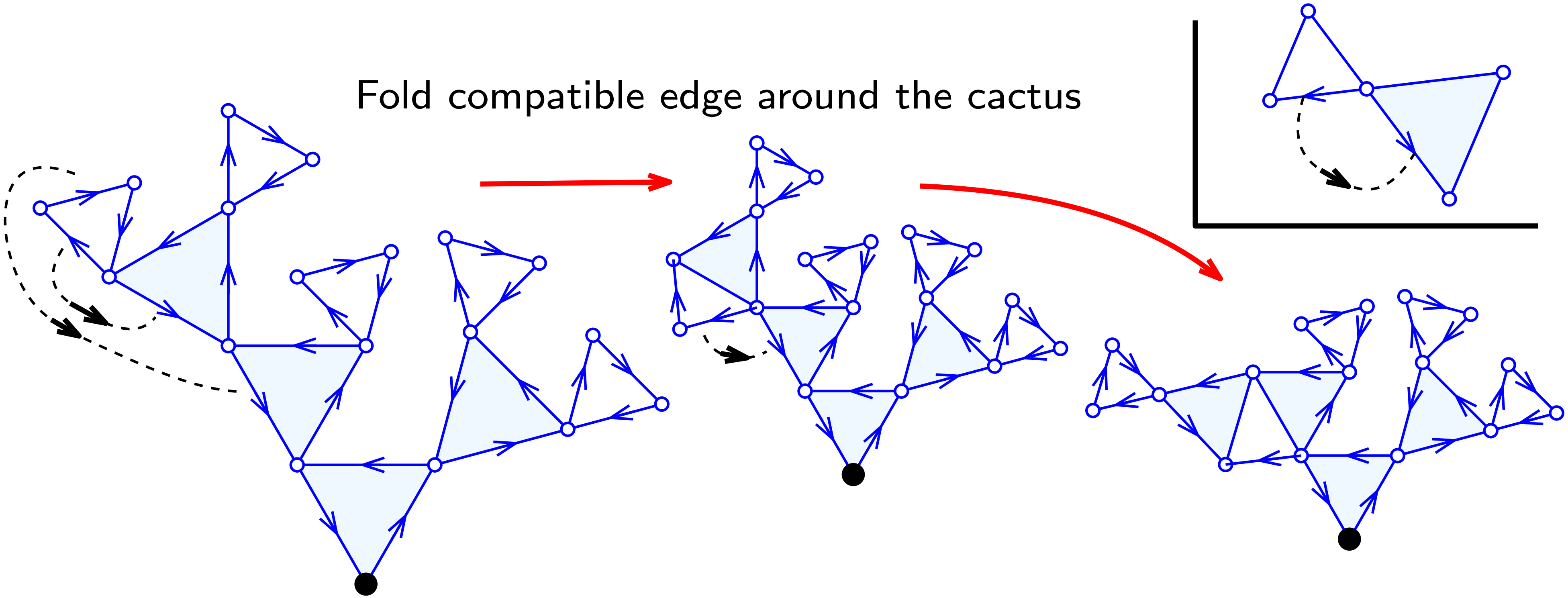

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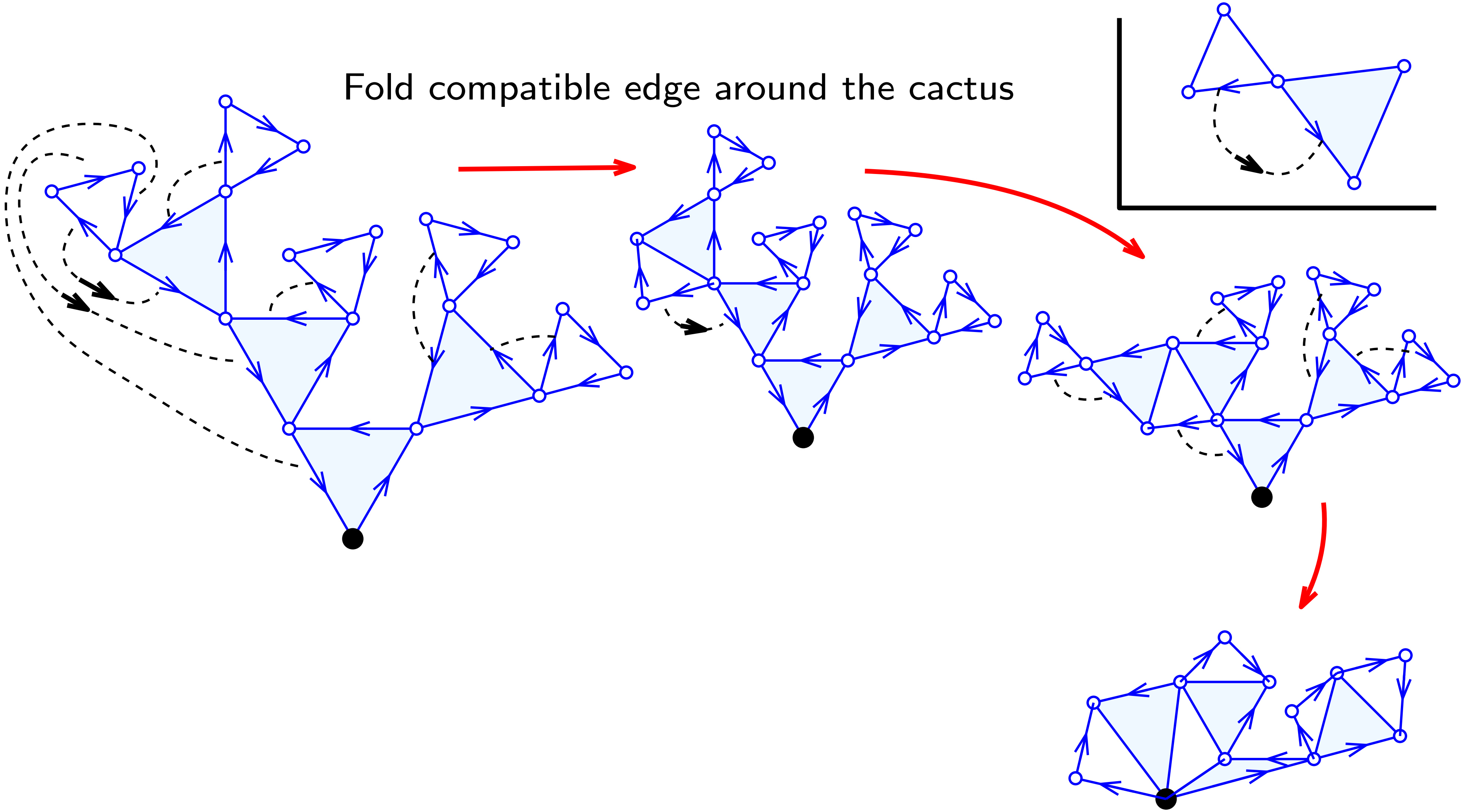

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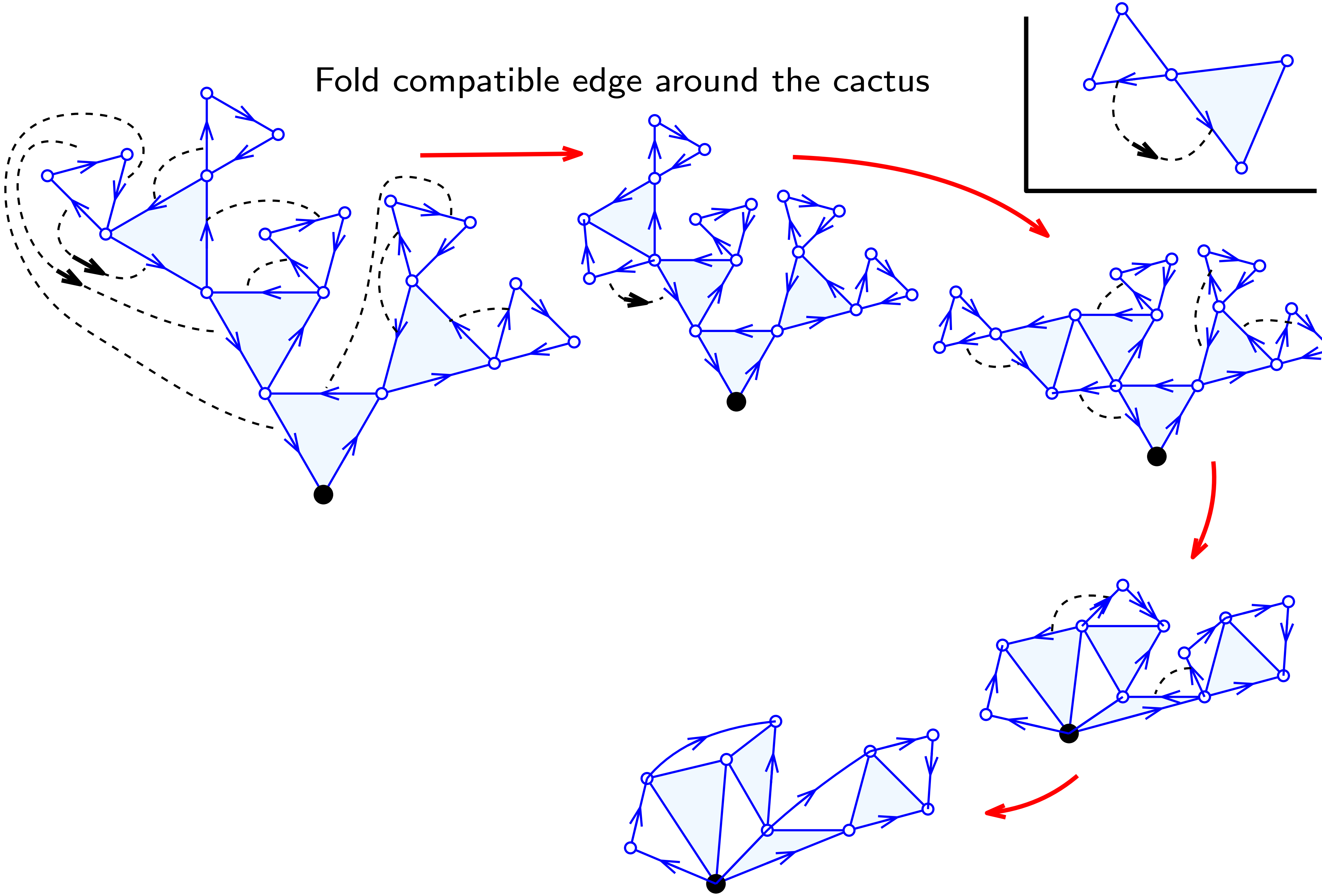

Reformulation in terms of trees

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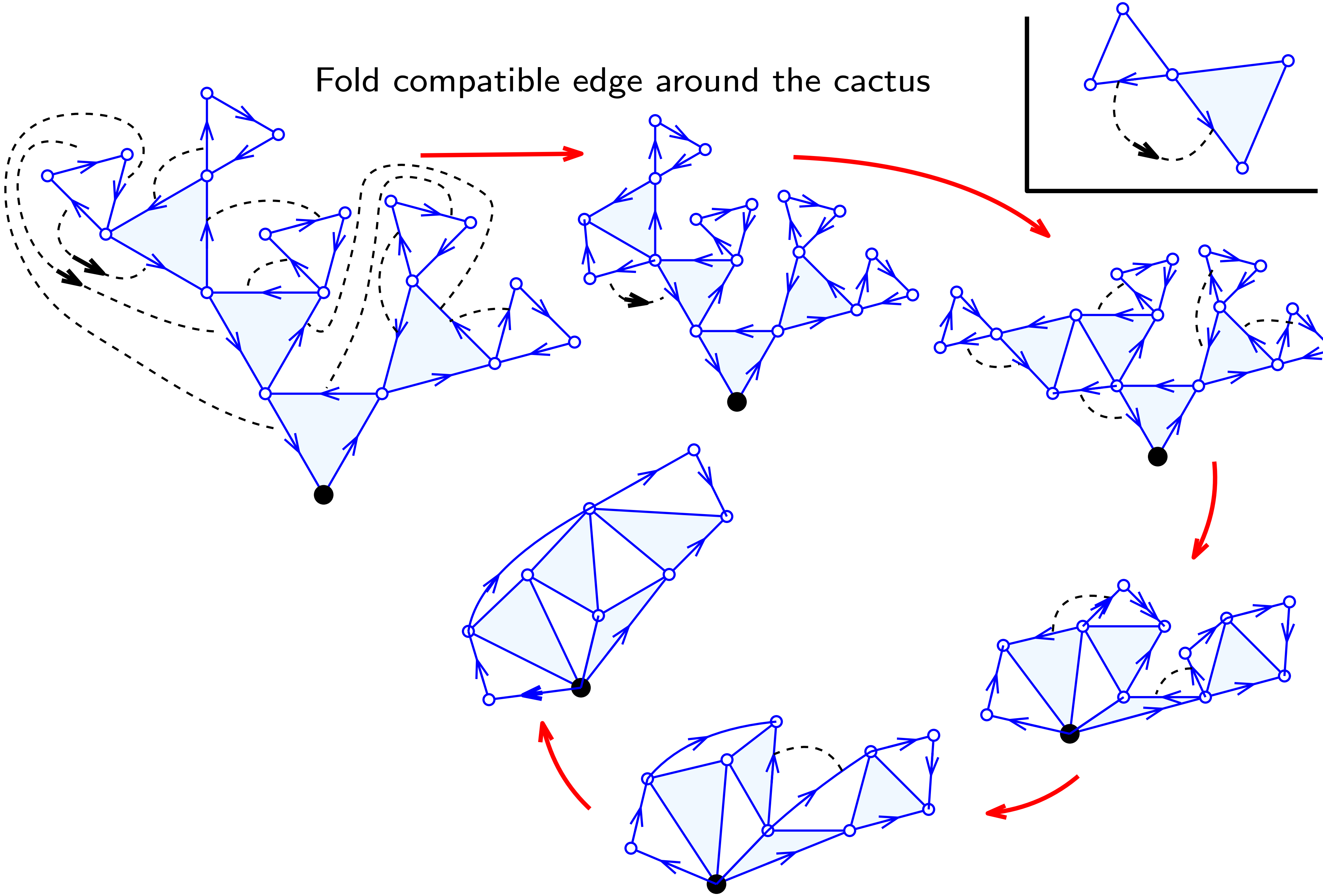

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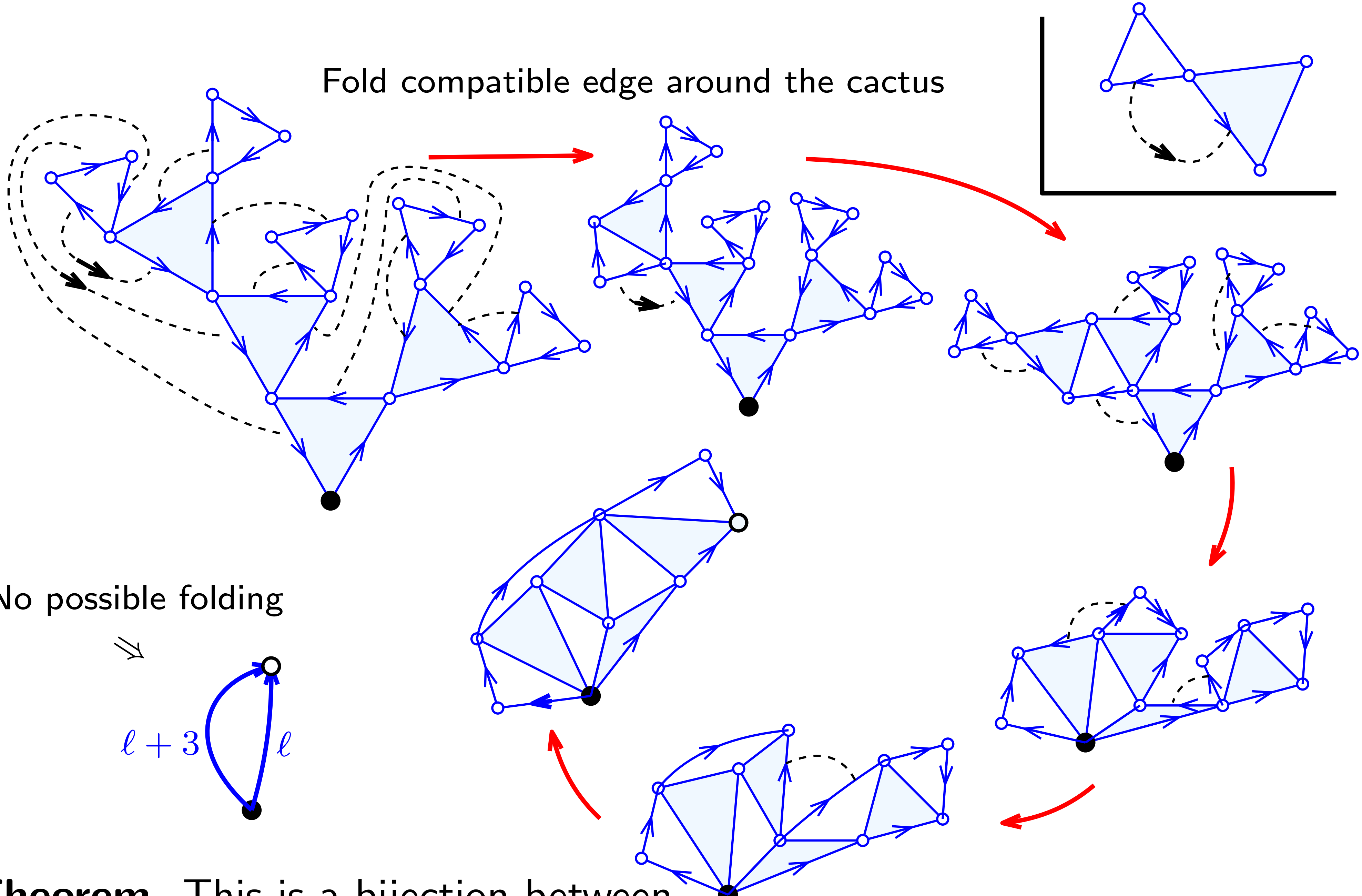

Theorem. This is a bijection between binary trees with n nodes and almond triangulations with n black faces.

Back to the realization of corner polyhedra graphs

Returning to corner polyhedra

Can we use the algebraic decomposition to endow a corner triangulation with a EM-structure ?

Returning to corner polyhedra

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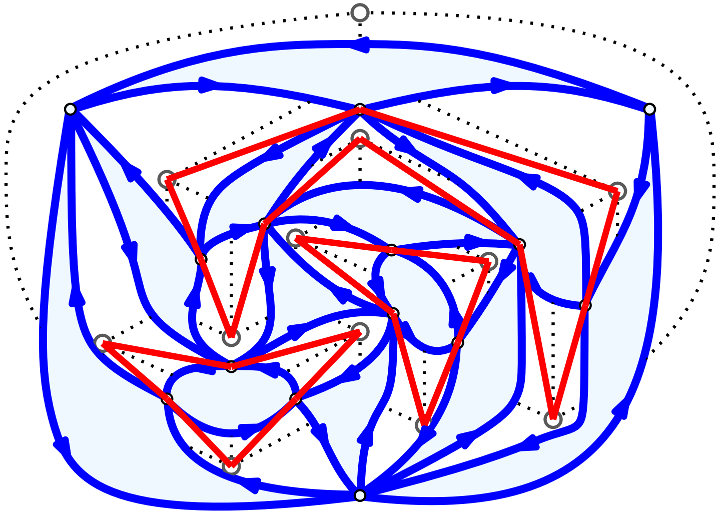
Recall:

Starting from
the corner triangulation:

1. First endow it with a EM-structure.
2. Then use the EM-structure to compute coordinates.



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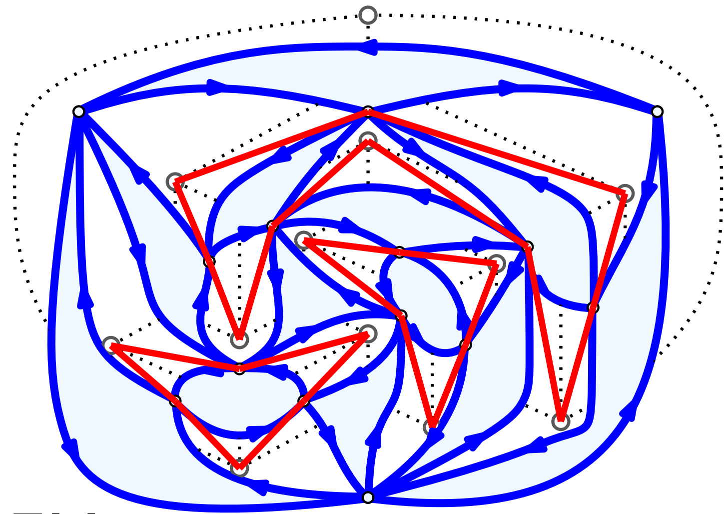
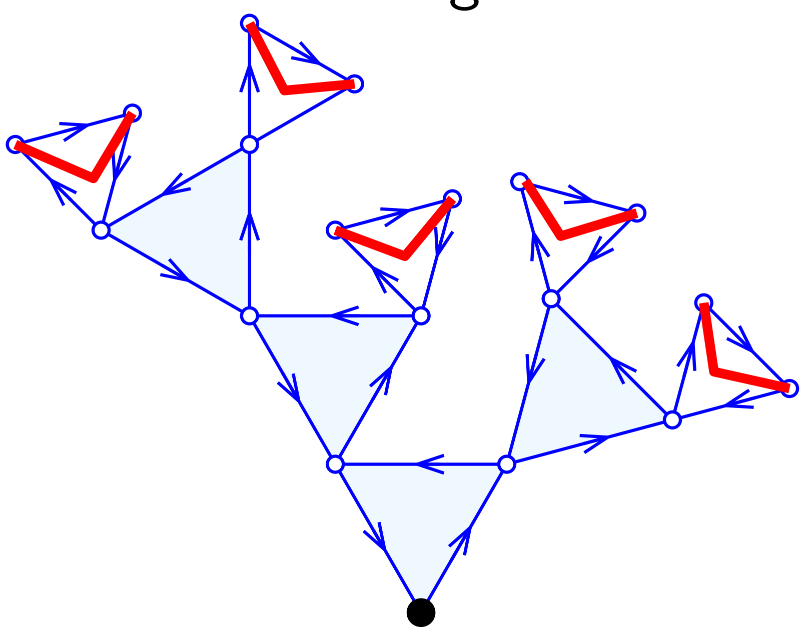


Endow the triangles the cactus with a local EM-structure:



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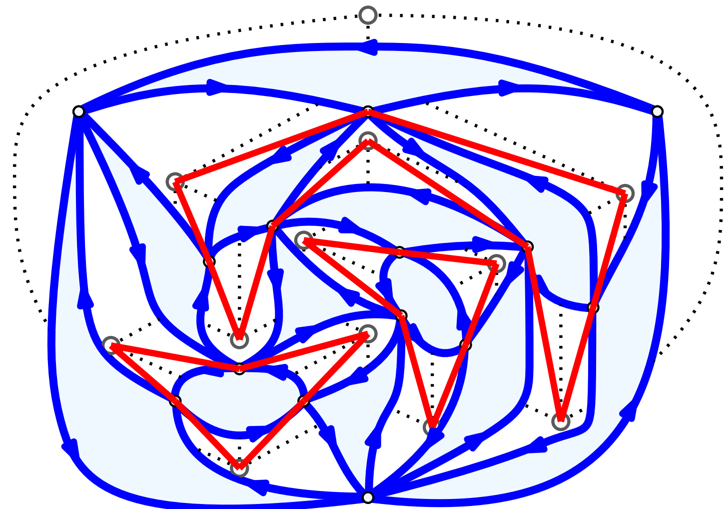

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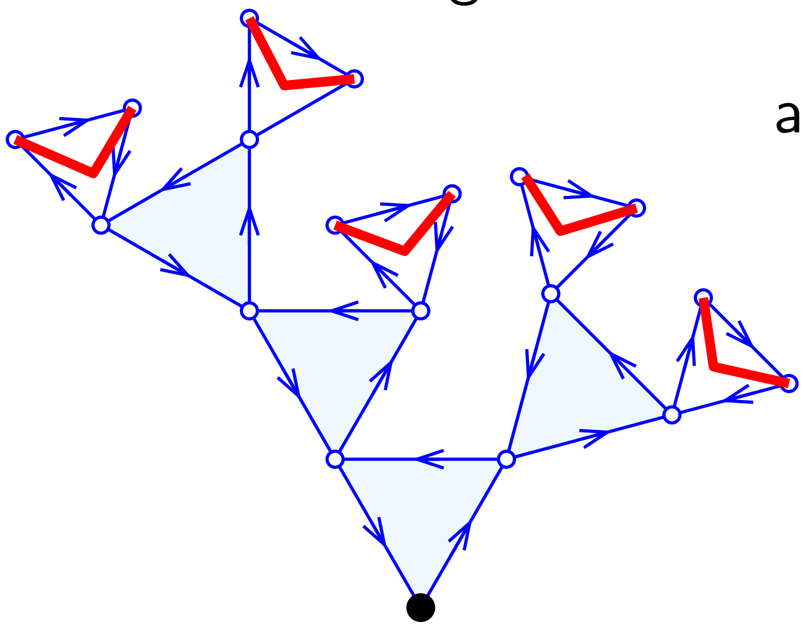
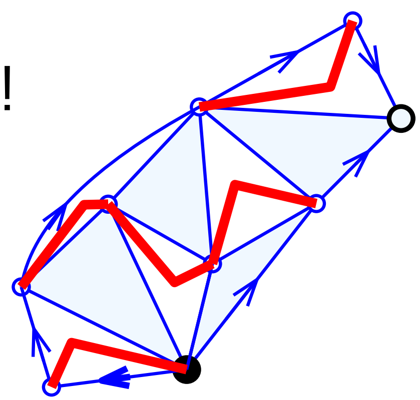


and fold !



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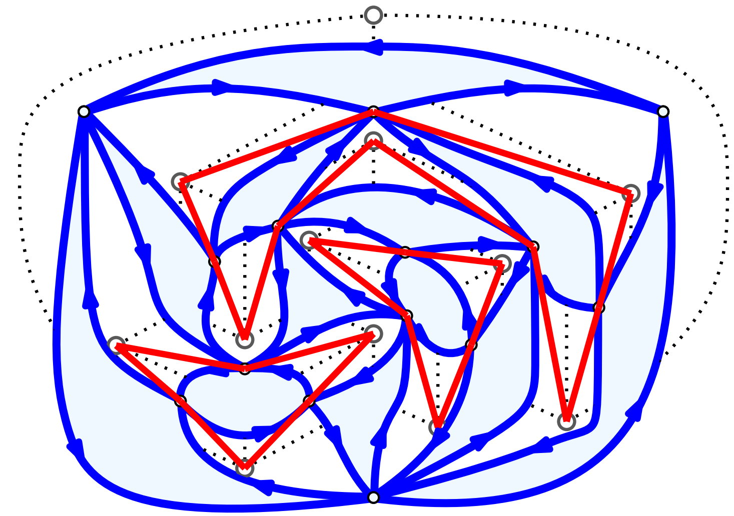

Alternative algorithm for Step 1

Decomposition the triangulation in two almonds and get two cactus from their algebraic decompositions.

Endow the two cacti with local EM-structure and refold them.

Glue the two almond back into a corner triangulation, now with a EM-structure !

Random corner polyhedra

Outline of a linear time random generation algorithm:

Pick your favorite random binary tree
(I get mine with Rémy's algo but it's admittingly a waste of random bits).

Make it into a cactus (pretending is enough).

Fold the cactus into a corner triangulation with EM-structure.

Use EM-structure to compute vertex coordinates

As coordinates are given by rank in topological sorting, they are all integers smaller than $2n$ for a corner polyhedra with $2n$ vertices.

Conclusion

In this talk we have

- Counted graphs of corner polyhedra: gf is rational in Catalan gf.
- Identified a nice subfamily of Almond triangulations, counted by C_n
- Obtained direct algebraic decompositions and bijections.
- Used the decomposition to give an alternative realization algorithm.

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In this talk we have not

- Discussed how Cori-Vauquelin's bijection underlies recent furious developments around the Brownian Random Planar Map.

Distant echoes of Schützenberger 's suggestion to Robert Cori to study Tutte work on maps!

COLLÈGE DE FRANCE 1530

COURS PECCOT

M. Nicolas CURIEN
Professeur à l'Université Paris-sud Orsay

invité par l'Assemblée des professeurs, donnera une série de leçons sur le sujet suivant :

ÉPLUCHAGE DES CARTES PLANAIRES ALÉATOIRES

Ces leçons auront lieu au Collège de France (11, place Marcelin-Berthelot, Paris 5e), les mardis 3, 10, 17 et 24 mai 2016, de 10 h à 12 h, salle 2.

L'Administrateur du Collège de France
Alain Prochiantz

Thank you !