



# Mixed production problem

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A firm is planning the production of 3 products A1, A2, A3 .

In a month production can be active for 22 days.

The following are given:

- maximum demands (units=100Kg)
- selling price (\$/100Kg)
- production costs (per 100Kg of product)
- production quotas (maximum amount of 100Kg units of product that would be produced in a day if all production lines were dedicated to the product).



# Mixed production problem

Product	A1	A2	A3
Maximum demand	5300	4500	5400
Selling price	\$124	\$109	\$115
Production cost	\$73.30	\$52.90	\$65.40
Production quota	500	450	550

Formulate an AMPL model to determine the production plan to maximize the total income



# Mathematical model

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What is to be identified to write the mathematical formulation?

- Decision variables
  - Objective function
  - Constraints
  - Parameters
- 
- What are the decision variables?

$x_i \quad i \in \{1,2,3\}$  : quantity of product  $i$  to produce

any bound?

$$\forall i \in \{1,2,3\} \quad x_i \geq 0$$

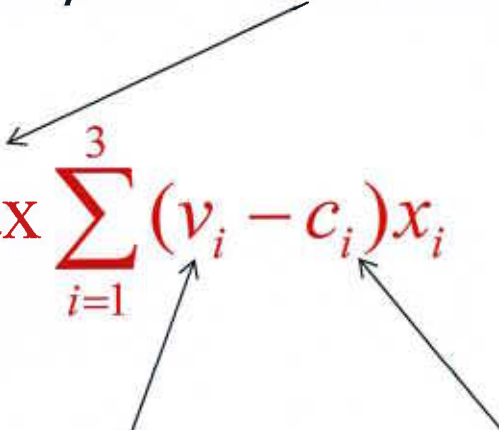


# Mathematical model

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- What is the objective function?

*determine the production plan to **maximize the total income***


$$\max \sum_{i=1}^3 (v_i - c_i)x_i$$

*each  $x_i$  has a **selling price** and a **production cost***



# Mathematical model

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- What are the constraints?

demand:  $\forall i \in \{1,2,3\} \quad x_i \leq d_i$

production:  $\sum_{i=1}^3 \frac{x_i}{q_i} \leq P$

*P = number of production days in a month*



# Mathematical model

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- What are the parameters?

$P$  = number of production days in a month

$d_i$  = maximum market demand for product  $i$

$v_i$  = selling price for product  $i$

$c_i$  = production cost for product  $i$

$q_i$  = maximum production quota for product  $i$



# Modelling and solving the problem using AMPL

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Remember that it is necessary to write:

1. a **model** file (extension `.mod`)

contains the mathematical formulation of the problem

- logical structure of the problem -

2. a **data** file (extension `.dat`)

contains the numerical values of the problem parameters

- more data files may correspond to the same model -

3. (possibly) a **run** file (extension `.run`)

specifies the solution algorithm



# AMPL model/data

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## Model file

Logical structure:

1.Parameters \_\_\_\_\_ `param name_parameter;`

2.Variables \_\_\_\_\_ `var name_variable;`

3.Objective function \_ `maximize (minimize) name_objective:...`

4.Constraint(s)\_\_\_\_\_ `subject to name_constraint: ...`

## Data file

```
param name_parameter1 := ...;
```

```
param name_parameter2 := ...;
```

```
.....
```





## AMPL model – mixed production

Starting from the mathematical formulation, try to code the AMPL model

➤ You already know the parameters:

```
days  
demand  
price  
cost  
quota
```

All of them are non negative:  $\geq 0$

demand, price, cost, quota are indexed on a set containing the products:

```
set PRODUCTS;  
  
param days  $\geq 0$ ;  
param demand { PRODUCTS }  $\geq 0$ ;  
param price { PRODUCTS }  $\geq 0$ ;  
param cost { PRODUCTS }  $\geq 0$ ;  
param quota { PRODUCTS }  $\geq 0$ ;
```



# AMPL model – mixed production

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➤ Decision variables:  $x$

All of them are non negative:  $\geq 0$

Are indexed on a set containing the products (previously declared).

➤ Objective function:

In order to code in ampl the objective function of the mathematical formulation, you just need to know how to write a sum in ampl:

```
sum {i in PRODUCTS} ...
```

➤ Constraints:

1. Each  $x_i$  must be less than or equal to its demand: `subject to requirement {i in PRODUCTS} .....`
2. The sum of  $x_i/q_i$  must be less than or equal to the number of the days of production: `subject to production: sum {i in PRODUCTS} .....`



## Exercise

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One step more:

- Change the mathematical program and the AMPL model to cater for a fixed activation cost on the production line, as follows:

Product	A1	A2	A3
Activation cost	\$170000	\$150000	\$100000

- Change the mathematical program and the AMPL model to cater for both the fixed activation cost and for a minimum production batch:

Product	A1	A2	A3
Minimum batch	20	20	16



# Mathematical model updated

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The basic model is unchanged. But something has to be added.

Parameters. We have 2 parameters more:

$a_i$  = activation cost for the plant producing  $i$

$b_i$  = minimum batch of product  $i$

Variables. For each product  $i$ , the production line can be activated or not

$y_i$  = activation status of the product  $i$

Binary variable:

$$\forall i \in \{1,2,3\} \quad y_i = \begin{cases} 1 & \text{if product } i \text{ is active} \\ 0 & \text{otherwise} \end{cases}$$



# Mathematical model updated

Objective function. Takes into account the possible activation for each product:

$$\max \sum_{i=1}^3 ((v_i - c_i)x_i - a_i y_i)$$

Constraints. Two constraints more:

original constraints +

activation:  $\forall i \in \{1,2,3\} \quad x_i \leq Pq_i y_i$

minimum batch:  $\forall i \in \{1,2,3\} \quad x_i \geq b_i y_i$