Mathematical Programming: Modelling and Applications

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Mathematical formulation

Sets:
V, set of vertices of G
E, set of edges of G

Parameters:
c, weights of G
K, maximum number of clusters in the partition

Variables:
$\forall u \in V, k \leq K, \quad x_{uk}$, binary, indicates if the vertex u is contained into the cluster k:

$$x_{uk} = \begin{cases} 1 & \text{if } u \in k^{th} \text{ cluster} \\ 0 & \text{otherwise} \end{cases}$$

Mathematical formulation

Objective function:

what do we need to minimize?

we want to minimize the total weights of the edges between different clusters:

$$\min_x \frac{1}{2} \sum_{k \neq l \leq K} \sum_{(u,v) \in E} c_{uv} x_{uk} x_{vl}$$

Mathematical formulation

Constraints:

- each vertex must be assigned to only one cluster:

$$\forall u \in V \quad \sum_{k \leq K} x_{uk} = 1$$

- the trivial solution (all the vertices into one cluster) must be excluded:

$$\forall k \in K \quad \sum_{u \in V} x_{uk} \geq 1$$

Some observations

The objective function contains a product of binary terms.

- We introduce a new variable w_{ukvl} representing the product of the two binary variables.
- We substitute the products with the new variable w_{ukvl} everywhere, as for example in the objective function:

$$\min \frac{1}{2} \sum_{k \neq l \leq K} \sum_{(u,v) \in E} c_{uv} w_{ukvl}$$

- We add linearization constraints:

$$\forall u \in V, v \in V, l \in K, k \in K \;:\; (u,v) \in E \text{ or } (v,u) \in E$$

$$w_{ukvl} \leq x_{uk} \qquad w_{ukvl} \leq x_{vl} \qquad w_{ukvl} \geq x_{uk} + x_{vl} - 1$$