

Mathematical Programming: Modelling and Applications

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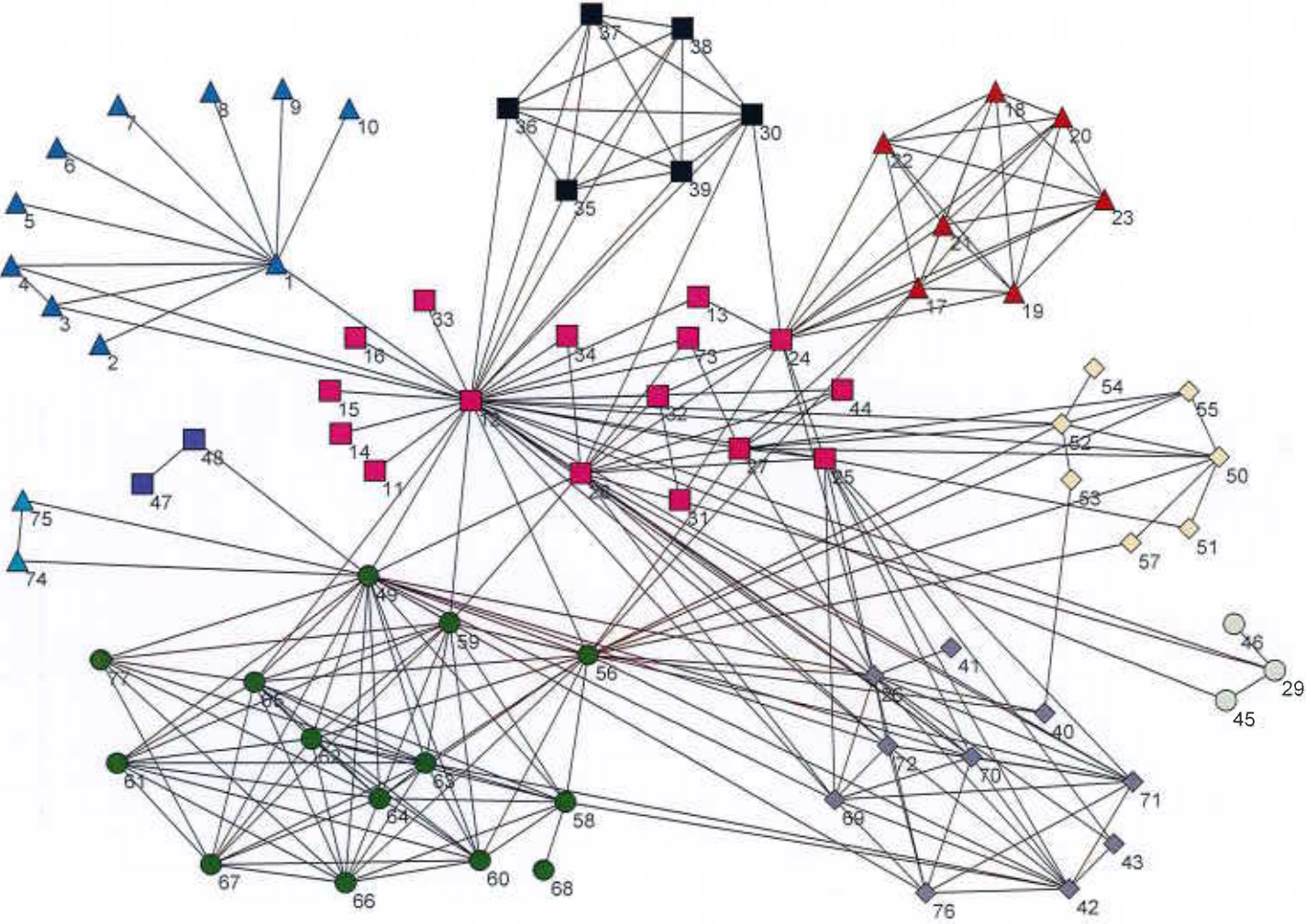
Graph partitioning

Definition

Graph partitioning is the problem of finding a suitable partition of a set of data represented through a graph G .

- Graph partitioning is a **clustering problem**.
- Each cluster is a **subgraph** of the graph G .
- Intuitively, the best partition is the one that separates **sparsely connected dense** subgraphs from each other.
- **sparsely connected**: the number of edges between vertices belonging to *different* clusters is minimal.
- **dense**: the number of edges between vertices belonging to *the same* cluster is maximum.

Graph partitioning: example



Graph partitioning problem

Given a weighted undirected graph $G = (V, E, c)$, where

- V is the set of vertices of G ,
- E is the set of edges of G ,
- c is the set of weights eventually assigned to the edges,

and an integer $K \leq |V|$,

find a partition of $k \leq K$ subsets (clusters) of V minimizing the total weights of edges between different clusters.

Write the mathematical formulation and solve the problem using AMPL.

Graph partitioning problem: data

16 vertices

maximum number of clusters: 4

Edges:

(1,15) (2,15) (2,3) (2,4)

(3,5) (4,5) (5,6) (5,16)

(6,9) (7,8) (7,16) (8,10)

(8,16) (11,16) (12,16) (13,16)

(14,16)

Edge Weights: 1 for each edge