Mathematical Programming: Modelling and Applications

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Outline

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2. Densest subgraph problem
3. Graph partitioning problem
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Basic definitions

Definition

A **Graph** is an ordered pair $G = (V, E)$ comprising a set V of vertices or nodes together with a set E of edges or links, which are 2-element subsets of V.

- **Undirected graph**: a graph in which edges have no orientation.
- **Directed graph** or **Digraph**: a graph $G = (V, A)$, where A is a set of *ordered* pairs of vertices, even called arcs or directed edges.
- **Weighted graph**: a graph in which numbers (weights) are assigned to each edge.
  It can be *directed* and *undirected*.
  It is denoted by $G = (V, E, w)$ or $G = (V, A, w)$, where w represents the weights.

Basic definitions

Let $G = (V, A)$ be a directed graph. A function

$$\mu : A \longrightarrow \mathbb{N},$$

that associates an integer number to each arc of G, is called arc coloring of G.

Since we can associate a color to each integer number, the function μ actually associates a color to each arc of G.

Subgraphs of the graph G can be located by considering all its arcs having the same color:

$$H = (U, B) \quad : \quad U \subseteq V, B \subseteq A, \quad \forall e, f \in B \ \mu(e) = \mu(f).$$

Densest subgraph problem

Given

- a digraph $G = (V, A)$,
- an arc coloring μ of G,
- a color k,

find the densest uniformly coloured subgraph H of G,
i.e. the subgraph $H = (U, B)$ of G in which the arcs have the same color k and such that $|B| - |U|$ is maximun.

Formulate a mathematical model and solve it by AMPL/CPLEX.

Densest subgraph problem: data

15 vertices

color k: 2

Edges:

(1,15) ($\mu = 1$), (2,15) ($\mu = 1$), (2,3) ($\mu = 1$), (2,4) ($\mu = 1$),
(3,5) ($\mu = 1$), (4,5) ($\mu = 1$), (5,6) ($\mu = 1$), (5,11) ($\mu = 2$),
(5,12) ($\mu = 2$), (5,13) ($\mu = 2$), (5,14) ($\mu = 2$),
(6,9) ($\mu = 1$), (7,8) ($\mu = 1$), (7,11) ($\mu = 2$), (7,12) ($\mu = 2$),
(7,13) ($\mu = 2$), (7,14) ($\mu = 2$), (7,15) ($\mu = 1$), (8,10) ($\mu = 2$),
(8,14) ($\mu = 2$), (11,12) ($\mu = 2$), (11,13) ($\mu = 2$), (12,13) ($\mu = 2$)