Mathematical Programming: Modelling and Applications

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Outline

Inheritance problem

A rich aristocrat passes away, leaving a large legacy which must be shared between the two sons.

The value of each item of the legacy is known.

Find the partition that minimizes the difference between the values of the two parts. Formulate a mathematical program and solve it with AMPL.

Inheritance problem: data

13 items

1. 25000$
2. 5000$
3. 20000$
4. 40000$
5. 12000$
6. 12000$
7. 12000$
8. 3000$
9. 6000$
10. 10000$
11. 15000$
12. 10000$
13. 13000$

Mathematical formulation

The problem is known as the `Subset-Sum` problem:

given a set A of n elements each with an evaluation function $v : A \to \mathbb{R}$, we want to find a partition of A in A_1, A_2

(i.e. to find $A_1 \neq \emptyset, A_2 \neq \emptyset : A_1 \cap A_2 \neq \emptyset, A_1 \cup A_2 = A$)

such that

$$|v(A_1) - v(A_2)| = |\sum_{a \in A_1} v(a) - \sum_{a \in A_2} v(a)|$$

is minimum.

Mathematical programming nonlinear formulation

- Set:

  A: set of objects of to be divided

- Parameters:

  all the values of the objects in A

- Variables:

  $\forall a \in A \quad x_a, y_a$ binary variables such that:

  if object a is assigned to brother x: $x_a = 1$ and $y_a = 0$

  if object a is assigned to brother y: $x_a = 0$ and $y_a = 1$

Mathematical programming nonlinear formulation

- Objective function:

$$\min |\sum_{a \in A_1} v(a) - \sum_{a \in A_2} v(a)|$$

  ensures that the inheritance is split between the two brothers as fairly as possible

- Constraint:

$$\forall a \in A \qquad x_a + y_a = 1$$

Because of the absolute value, this formulation is nonlinear:
Mixed-Integer NonLinear problem.

Mathematical programming nonlinear formulation: using AMPL

- use the AMPL function `abs()` to compute the absolute value;
- use the solver `boncouenne` for MINLP problems;
- display the assignment of each item to one of two brothers and also the total value of the legacy received by each one of the two brothers: you can define parameters `value1,value2` in the run file and use `let` to assign values to these parameters.

Mathematical formulation, Linear

The `Subset-Sum` problem can also be formulated as follows:

let V be the total value of inheritance: $V = \sum_{a \in A} v(a)$,

minimize the inheritance assigned to one of the brothers with the constraint that it should not be less than $V/2$

(equivalently maximize the inheritance assigned to one of the brothers with the constraint that it should not exceed $V/2$)

$$\min \quad \sum_{a \in A} v(a) x_a$$
$$\text{s.t.} \quad \sum_{a \in A} v(a) x_a \geq \frac{V}{2}$$
$$\forall a \in A, \quad x_a \in \{0, 1\}$$

This is an integer linear programming problem.