Mathematical Programming: Modelling and Applications

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SDP with YALMIP/2

vector $v \in \{\pm 1\}^n$, $n = |V|$, represents cut in graph G
$S = \{i : v_i = +1\} \qquad V \backslash S = \{i : v_i = -1\}$.

$$(MC) \qquad \begin{array}{rll} \mu^* = & \max & \sum\limits_{1 \le i < j \le n} w_{ij} \left(\frac{1 - v_i v_j}{2} \right) \\ & \text{s.t.} & v \in \{\pm 1\}^n \end{array}$$

Equivalently,

$$(\text{MC1}) \qquad \begin{array}{rll} \mu^* = & \max & v^T Q\, v \\ & \text{s.t.} & v_i^2 = 1, \qquad i = 1, \ldots, n, \end{array}$$

where $Q = \frac{1}{4}(\text{Diag}\,(A\,e) - A)$ is $\frac{1}{4}$ Laplacian matrix, L,
$A = (w_{ij})$ is weighted adjacency matrix of G.

SDP with YALMIP/2

With $Q := \frac{1}{4}L$, $X := vv^T, v \in \{\pm 1\}^n$,
then $v^T Qv = \operatorname{trace} QX$ and equivalent formulation is:

$$\text{(MC1)} \qquad \begin{array}{rll} \mu^* = & \max & \operatorname{trace} QX \\ & \text{s.t.} & \operatorname{diag}(X) = e \\ & & \operatorname{rank}(X) = 1 \\ & & X \succeq 0, X \in \mathcal{S}^n, \end{array}$$

relax by deleting the ***hard*** constraint $\operatorname{rank}(X) = 1$

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Code it in yalmip.