Mathematical Programming: Modelling and Applications

October 2009

Sonia Cafieri

LIX, École Polytechnique

cafieri@lix.polytechnique.fr

Writing the mathematical model

- Parameters and Sets

Consider a set containing the products.

What are the parameters?

- Variables

What do we have to decide?

Note : we have a certain number of goods and, for each of them, an amount of produced and exported units.

- Objective function

What do we have to maximize?

Note: the profits can be expressed in terms of exported quantities and produced quantities.

Writing the mathematical model

- Constraints

1. The amount of work needed to produce all the products (man-months) cannot exceed the total available workforce per year.
2. There is a maximum possible production for some of the products.

   Note: this limit is not imposed on all products. How can this condition be expressed?
3. For each product, the produced quantity must balance the exported quantity and the quantity needed to produce the other products.

   Suggestion: start writing, for each product, the relation among the total produced quantity, the exported quantity, the quantity needed for each other product. Then, try to write these constraints in a more compact form.

Mathematical model

- Parameters and Sets

$P = \{a,m,e,p\}$: set of products (a = steel, m = engines, e = electronic, p = plastic)

H: total available amount of work (man-years)

M_i : maximum possible production for product $i \in P$

h_i : amount of work required to manufacture a unit of product $i \in P$

m_i : price of raw materials necessary to manufacture a unit of product $i \in P$

p_i : market price for product $i \in P$

- Variables

x_a , x_m , x_e , x_p : produced units of steel, engines, electronics and plastics

y_a , y_m , y_e , y_p : exported units of steel, engines, electronics and plastics

All variables are non negative

Mathematical model

- Objective function

maximize the profits

profits: exported quantities *minus* produced quantities

$$\max \sum_{i\in P} p_i y_i - \sum_{i\in P} m_i x_i$$

- Constraints

- amount of work: $\sum_{i\in P} h_i x_i \le H$
- maximum production: $\forall i \in P \quad x_i \le M_i$
- balance:

$$x_a = y_a + 0.8x_m + 0.01x_e + 0.2x_p$$
$$x_m = y_m + 0.02x_a + 0.01x_e + 0.03x_p$$
$$x_e = y_e + 0.15x_m + 0.05x_p$$
$$x_p = y_p + 0.01x_a + 0.11x_m + 0.05x_e$$

Mathematical model

Note that the following constraints

$$x_a = y_a + 0.8x_m + 0.01x_e + 0.2x_p$$
$$x_m = y_m + 0.02x_a + 0.01x_e + 0.03x_p$$
$$x_e = y_e + 0.15x_m + 0.05x_p$$
$$x_p = y_p + 0.01x_a + 0.11x_m + 0.05x_e$$

can be written as:

$$\forall i \in P \quad x_i = y_i + \sum_{j \in P} a_{ji} x_j$$

where:

$$\forall i, j \in P \quad a_{ij} = \begin{cases} 0 & \text{if } \; i = j \\ \text{units of product } i \\ \text{needed to produce} & \text{otherwise} \\ 1 \text{ unit of product } j \end{cases}$$