Mathematical Programming: Modelling and Applications

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Sonia Cafieri

LIX, École Polytechnique

cafieri@lix.polytechnique.fr

Writing the mathematical model

<u>Preliminary observations:</u>

• There are some quantities (parameters) defined

for each product :

selling price, production cost, production quota, activation cost, minimum batch, storage cost

for each month:

number of production days

for each product and each month:

maximum demand for each product in each month

independently on product/month:

storage capacity.

• Furthermore:

For each product, there is a different quantity produced, sold, stocked during each month and the activation status of each production line is also dependent on the months.

Writing the mathematical model

Sets and indices:

• 3 products ➔ use an index $i \in I$

• 4 months ➔ use an index $j \in J$

define parameters / variables indexed on i, j

What decisions should we take?

product i , month j $\left\{ \begin{array}{l} \text{quantity produced} \\ \text{quantity sold} \\ \text{quantity stocked} \end{array} \right.$

also take into account the activation status

Mathematical model

- Parameters

P_j: number of production days in month j;

d_{ij}: maximum demand for product i in month j;

v_i : selling price for product i;

c_i: production cost of product i;

q_i : maximum production quota of product i;

a_i : activation cost for production i;

b_i : minimum batch for production i;

s_i : storage cost for product i;

C: storage capacity in number of units.

Mathematical model

- Variables

x_{ij} : quantity of product i produced during month j;
w_{ij} : quantity of product i sold during month j;
z_{ij} : quantity of product i stocked during month j;
y_{ij} : activation status for production i during month j;

All variables are non negative

y_{ij} binary

$$y_{ij} = \begin{cases} 1 & \text{if product } i \text{ is active during month } j \\ 0 & \text{otherwise} \end{cases}$$

- Objective function

maximize the total income

total income: 3 products – sum income obtained during each month

$$\max \sum_{i \in I} \left(v_i \sum_{j \in J} w_{ij} - c_i \sum_{j \in J} x_{ij} - s_i \sum_{j \in J} z_{ij} - a_i \sum_{j \in J} y_{ij} \right)$$

Mathematical model

- Constraints
- demand: $\forall i \in I, j \in J \quad w_{ij} \leq d_{ij}$
- production: $\forall j \in J \quad \sum_{i \in I} \frac{x_{ij}}{q_i} \leq P_j$
- balance: $\forall i \in I, j \in J \quad z_{i,j-1} + x_{ij} = z_{ij} + w_{ij}$
- capacity: $\forall j \in J \quad \sum_{i \in I} z_{ij} \leq C$
- activation: $\forall i \in I, j \in J \quad x_{ij} \leq P_j q_i y_{ij}$
- minimum batch: $\forall i \in I, j \in J \quad x_{ij} \geq b_i y_{ij}$
- december: $\forall i \in I \quad z_{i0} = 0$