TD #1

Advanced Mathematical Programming

Leo Liberti, CNRS LIX Ecole Polytechnique
liberti@lix.polytechnique.fr

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Section 2

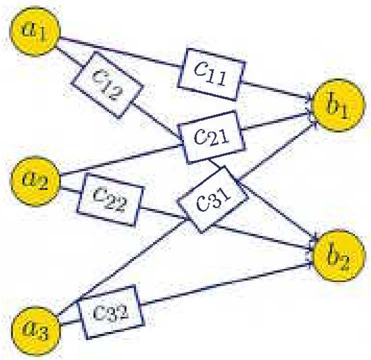
Modelling

The transportation problem

Given a set P of production facilities with production capacities a_i for $i \in P$, a set Q of customer sites with demands b_j for $j \in Q$, and knowing that the unit transportation cost from facility $i \in P$ to customer $j \in Q$ is c_{ij}, find the optimal transportation plan

The art of modelling!

- *Use drawings — they help to think*



First fundamental question

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    2. costs — unit, refers to quantities
    3. capacities and demand based on quantities
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    5. (pitfall: the question "quantity *of what*?" is irrelevant — and you don't know in advance which questions are irrelevant)

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- *As you go on with the model, you might find your initial choices were poor — you might have to go back and change them*

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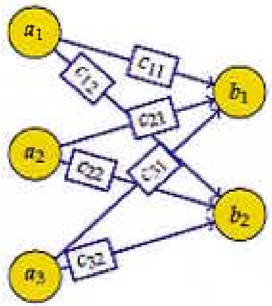

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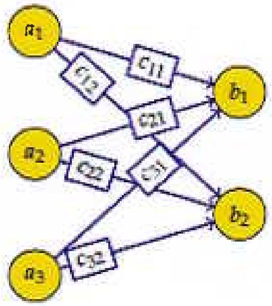

- How about:
  z_i = qty. produced at i
  y_j = qty. demanded at j

Let's try this choice

1. *Sets and indices*
   a. $i \in P \subset \mathbb{N}$
   b. $j \in Q \subset \mathbb{N}$
2. *Parameters*
   a. $\forall i \in P \quad a_i \in \mathbb{R}_+$
   b. $\forall j \in Q \quad b_j \in \mathbb{R}_+$
   c. $\forall i \in P, j \in Q \quad c_{ij} \in \mathbb{R}_+$
3. *Decision variables*
   a. $\forall i \in P \quad z_i \in [0, a_i]$
   b. $\forall j \in Q \quad y_j \in [b_j, \infty]$
4. *Constraints*
   a. All that is produced must be delivered: $\sum_{i \in P} z_i = \sum_{j \in Q} y_j$

      necessary condition, but is it sufficient?
5. *Objective function*: ???

   no way of knowing what fraction of the production out of i went to j, so how do we consider transportation costs?

Bummer! Let's go back

- Failure to express "*fraction of i going to j*" must inspire us!
  Let's try x_{ij} = qty. transported from i to j

1. *Sets*: as before
2. *Parameters*: as before
3. *Decision variables*
   a. $\forall i \in P, j \in Q \quad x_{ij} \in \mathbb{R}_+$
4. *Objective function*

$$\min \sum_{i \in P} \sum_{j \in Q} c_{ij} x_{ij}$$

5. *Constraints*
   a. No facility can produce more than the maximum:
      $\forall i \in P \quad \sum_{j \in Q} x_{ij} \leq a_i$
   b. No customer must receive less than its demand:
      $\forall j \in Q \quad \sum_{i \in P} x_{ij} \geq b_j$

Much better!