A New Algorithm for Interconnect Capacitance Extraction Based on a Fictitious Domain Method

S. Putot[a], F. Charlet[a], P. Witomski[b]

[a]LETI(CEA-Technologies Avancées) BP85X,
17, rue des Martyrs, 38054 Grenoble Cedex 9, France
[b]LMC-IMAG, BP 53, 38041 Grenoble Cedex 9, France

Abstract

We propose a new approach for fast and accurate extraction of capacitance in multiconductor cells embedded in multiple dielectric media. We use the so-called fictitious domain method with Lagrange multipliers for the problem formulation. It leads to a coupled linear system which unknowns are the potential on a regular 3D grid of a simple-shaped domain, imbedding the dielectric media, and the charge on a mesh of the conductor surfaces. Thanks to the regular grid, the storage of information related to the volumic mesh is not necessary and we can use fast solvers. Numerical results on 3D complex structures show that the method is more efficient, both in time and memory, than a finite elements or a boundary elements method.

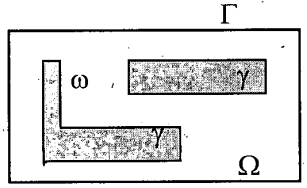
1. INTRODUCTION

An accurate computation of the parasitic capacitance requires the surfacic charge got by the normal derivative of the potential on the surface of conductors. The potential in the dielectric media is the solution of the Laplace equation with boundary conditions on the surfaces of conductors. For these simulations on a complex 3D domain, large computing resources, both in cpu time and memory, are needed. The finite elements (as in Clever from Silvaco) can take into account complicated geometries and inhomogeneous dielectrics, but with large cpu times and memory needs. The other methods (boundary integrals as in [1] or Monte-Carlo methods as in [2]) are more efficient for some problems, but less robust when the complexity increases, for instance in the case of multiple dielectric media. We propose to use the fictitious domain method, which combines advantages of volumic finite elements and boundary integral methods. In particular, we can easily take into account multiple dielectric media, while avoiding the meshing of a complex three-dimensional domain.

The approach described in this paper is based on [3] : the potential calculation is extended inside the conductors, and the boundary condition is taken into account via the introduction of a Lagrange multiplier that can be interpreted as the surfacic charge. The main idea of fictitious domain methods is to replace the problem on a domain of complex geometry by another one on a simple shape domain imbedding the initial domain. That allows the use of a regular grid on a simple shape domain, and therefore the use of fast solvers.

The method was developed for complex 3D structures with multiple dielectric media, as it is shown in part 4.

2. THE FICTITIOUS DOMAIN FORMULATION OF THE PROBLEM



Let ω the dielectric space between the conductors, of permittivity ε, and γ the union of the conductor surfaces. We denote by Ω the set constituted by the dielectric regions and the conductors, and by Γ the boundary of Ω.

The potential u in ω is the solution of

$$(P)\quad \begin{cases} \nabla(\varepsilon\,\nabla u) = 0 \quad in\ \omega & (1)\\ u = g \quad on\ \gamma & (2)\\ + boundary\ conditions\ on\ \Gamma & (3) \end{cases}$$

Considering the condition $u = g$ on γ as an equality constraint (potential set to a constant value), the solution u of problem (P) can also be seen as the restriction to ω of the solution $\tilde{u}$ of the extended minimisation problem

$$\min_{v_{|\gamma}=g} \frac{1}{2}\int_\Omega \varepsilon |\nabla v|^2\, dx\ ,$$

where v is in the space $X \doteq \left\{ v \in H^1(\Omega);\ verifying\ (3) \right\}$.

We introduce the Lagrangian associated to that minimisation problem, defined by

$$L(v,\mu) = \frac{1}{2}\int_\Omega \varepsilon |\nabla v|^2 dx - \int_\gamma \mu (v-g)\, d\gamma\ ,$$

where μ is the Lagrange multiplier associated to the constraint $v_{|\gamma} = g$. We denote by M the space of the Lagrange multiplier. Then we look for the saddle-point $(\tilde{u},\lambda) \in X \times M$ solution of

$$\forall v \in X, \forall \mu \in M,\quad L(\tilde{u},\mu) \le L(\tilde{u},\lambda) \le L(v,\lambda)\ .$$

Noting that the derivative of this Lagrangian is equal to zero at the optimum $(\tilde{u},\lambda)$, the extended potential $\tilde{u}$ is the first term of $(\tilde{u},\lambda) \in X \times M$ solution of

$$(Q)\quad \begin{cases} \int_\Omega \varepsilon \nabla\tilde{u}\,\nabla v\, dx = \int_\gamma \lambda\, v\, d\gamma \quad \forall v \in X & (1)\\ \int_\gamma \mu\left(\tilde{u} - g\right) d\gamma = 0 \quad \forall \mu \in M & (2) \end{cases}$$

The potential is the restriction of $\tilde{u}$ to the dielectric region ω, and the surface charge q is equal to the multiplier λ.

3. DISCRETIZATION AND SOLUTION

We first present the classical approximation as proposed in [3]. We introduce the finite dimensional spaces $X_h \subset X$ and $M_\eta \subset M$ such that a volume variable $v_h \in X_h$ is approached by Q_1 finite elements on a regular grid Ω_h of parallelepipeds of the fictitious domain Ω, and a surface variable $\mu_\eta \in M_\eta$ is approached by P_0 finite elements on a set of triangles γ_η meshing the conductor surfaces γ. We approximate the variational problem (Q) by

$$(Q_{h,\eta})\quad \begin{cases} Find\ (u_h,\lambda_\eta) \in X_h \times M_\eta\ \ such\ that\\ \int_\Omega \varepsilon \nabla u_h\, \nabla v_h\, dx = \int_\gamma \lambda_\eta\, v_h\, d\gamma \quad \forall v_h \in X_h\\ \int_\gamma \mu_\eta \left(u_h - g\right) d\gamma = 0 \quad \forall \mu_\eta \in M_\eta \end{cases}$$

Denoting U_h and Λ_η the vectors representing the decomposition of u_h and λ_η on the finite element bases defining X_h and M_η, we can rewrite the discrete formulation of system (Q) as

$$(Q)_{h,\eta}\quad \begin{cases} Find\ U_h\ and\ \Lambda_\eta\ \ such\ that\\ A_h U_h - B_{h,\eta}\Lambda_\eta = 0\\ B_{h,\eta}^T U_h = g_h\ . \end{cases}$$

This can be reformulated as a symmetric and positive system in Λ_η,

$$B_{h,\eta}^T A_h^{-1} B_{h,\eta} \Lambda_\eta = g_h .$$

This approximation gives accurate results for the potential, but not for the charge. Consequently we improved the method by introducing non regular functions in the space of discretisation X_h. This leads to a system

$$\begin{cases} Find\ U_h\ and\ \Lambda_\eta\ such\ that \\ A_h U_h - B_{h,\eta}\Lambda_\eta = 0 \\ B_{h,\eta}^T U_h + C_{h,\eta}\Lambda_\eta = g_h . \end{cases}$$

We still haven't proved theoretically the convergence of this approximation, but practically the results for the charge are much more accurate, in particular for coarse volume grids.

Classically, the existence of (u_h, λ_η), and the convergence of the method when h and η tend to zero are linked to the uniform discrete inf-sup condition

$$\text{There exists } C \text{ independent of } h \text{ such that } \inf_{\mu \in M_\eta} \sup_{v \in X_h} \frac{\int_\gamma \mu v \, d\gamma}{\|v\|_X \|\mu\|_M} = C > 0 .$$

In our case, this condition implies a compatibility relation between the dimensions of the two spaces X_h et M_η, that is between the discretization steps h and η of the two meshes : the surface mesh must not be too refined compared to the volume mesh. In the 2D case for example, it is proved theoretically (see [6]) that this condition holds if the volume grid step is less than three times smaller than the surface mesh step. However, numerical experiments show that for our improved approximation, this condition can be relaxed.

The system in Λ_η can then be solved efficiently by the conjugate gradient algorithm. The matrix A_h represents Poisson's equation on a regular 3D grid, and therefore doesn't need to be stored. Moreover, at each iteration of the gradient algorithm, the product $A_h^{-1} x$ can be computed efficiently with a fast Poisson solver using FFT ([4]).

The matrix $B_{h,\eta}$ is sparse ($\approx 20 N_S$ elements are stored, where N_S is the number of nodes on the surface mesh). Indeed, it can be interpreted as a discrete trace operator, and represents the coupling between the surface and the volume meshes. Therefore, its nonzero terms are, for every element of the surface mesh, those corresponding to the summits of the parallelepipeds having a nonzero intersection with this element.

Treatment of dielectric layers :

The dielectric permittivity is artificially extended inside the conductors by its value in the dielectric media in the horizontal layer. Introducing plane dielectric layers of different permittivities breaks the regularity of the matrix A_h in the vertical direction. However we remark that the fast Poisson solver takes advantage of the regularity of the grid and equation only in two directions. This propriety allows us to adapt the fast solver to the case of dielectric layers : we can in particular adapt the height of the parallelepipeds so that dielectric interfaces coincide with the grid, and that the dielectricpermittivity is constant on any parallelepiped of the grid. Thus, dielectric layers are taken into account at no extra cost or degree of approximation.

4. RESULTS

In the next table, we compare the cpu time and the memory (on a Sun Ultra 60) needed by the fictitious domain method with those needed by a finite element method (Clever), and a boundary integral method (Fastcap). For each of the 3 structures, we give its dimensions, the number of levels of metallic lines and the number of dielectric layers. We made sure the results were obtained with same

precision of 5% for all programs. We used the same surface meshes for Fastcap and Fictitious Domains simulations. Note also that in the Fastcap simulations, we used an homogeneous dielectric medium because the treatment of dielectric layers is very penalising with this method, therefore the results given here for Fastcap are optimistic.

		Clever	Fastcap	Fictitious Domain
35μm × 30μm	**Time (sec)**	**1389**	**500**	**43**
5 levels ; 3 dielectrics	**Memory (MB)**	**339**	**443**	**30**
20μm × 20μm	**Time (sec)**	**311**	**58**	**12**
3 levels ; 2 dielectrics	**Memory (MB)**	**62**	**27**	**7**
125μm ×150μm	**Time (sec)**	**1507**	**17**	**13**
2 levels ; 4 dielectrics	**Memory (MB)**	**387**	**18**	**6**

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