Harris-Stephens' combined corner/edge detector

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Abstract

We summarize Harris-Stephens[1] *combined* corner/thin edge detector that does not depend on rotation nor shift or affine change of intensity.

Let $I(x,y)$ denote the intensity pixels of Image I. Harris-Stephens extended the principle of Moravec's corner detector by considering the local auto-correlation energy $E(x,y) = \sum_{u,v} w_{u,v} \left(I(x+u,y+v) - I(u,v)\right)^2$, where (u,v) denote a neighborhood of (x,y). Traditionally, $w_{u,v} = 1$ if and only if $|x-u| \leq s$ and $|y-v| \leq s$, with $2s-1$ being the size of the square window patch. In order to be invariant to rotations, Harris-Stephens considered a smooth Gaussian circular window with $w_{u,v} = \exp(-\frac{u^2+v^2}{2\sigma^2})$ (and $u^2+v^2 \leq s^2$, with s denoting the radius size).

Observe when a window patch is:

• enclosed into an almost constant shaded region, shifts in *every* direction result in a small change of energy.

• straddling an edge, the shift along the edge yields a small variation of energy while moving perpendicular to that edge results in significant change.

• at a corner (including isolated points), then every direction yields a large change of energy.

Approximate the energy function at (x,y) by the bilinear form $E(u,v) = [u\ v]M_{x,y}[u\ v]^T$, where $M_{x,y}$ is a 2×2 positive definite matrix computed as

$$M_{x,y} = \sum_{u,v} w(u,v) \begin{bmatrix} (\frac{\partial I}{\partial x})^2 & \frac{\partial I}{\partial x}\frac{\partial I}{\partial y} \\ \frac{\partial I}{\partial y}\frac{\partial I}{\partial x} & (\frac{\partial I}{\partial y})^2 \end{bmatrix} = \sum_{u,v} w(u,v)\nabla I(u,v)(\nabla I(u,v))^T, \quad (1)$$

with is the discrete image gradient $\nabla I(u,v) = [I(u+1,v) - I(u-1,v) \quad I(u,v+1) - I(u,v-1)]^T$. Thus the energy is independent of a constant intensity shift $I(x,y) = I(x,y) + c$. The symmetric $M_{x,y}$ matrix represents the smoothened variance-covariance matrix of the intensity gradient at (x,y). Let α and β be the eigenvalues of M, geometrically interpreted as the elongations of the ellipsoid axes defined by M. For both α and β small, we are in a flat area (Trace(M) is small). For either α or β large, we are on an edge. For both α and β large, we are located on a corner ($\det M$ is large). In order to avoid computing a SVD to retrieve the eigenvalues, consider the detector response $R = \alpha\beta - k(\alpha+\beta)^2 = \det M - k(\mathrm{Trace}(M))^2$ (with $k \in [0.04, 0.06]$). According to R the pixels are classified as follows:

$R > 0$: corner pixel, $R \simeq 0$: pixel in flat region, $R < 0$: edge pixel.

Once R has been computed for each pixel, label a pixel a corner iff. it is a local maximum with respect to C_8 connectivity. Similarly, label a pixel an edgel iff. it is a local maximum in either the x or y direction. Finally, apply edge hysterisis (using low and high thresholds with non-maxima suppressions) and delete spurs and short edges: This gives a clean set of edges and corners that delimit visual putative surfaces. Observe that maxima of the response R is independent of an affine change of intensity. However, the main drawback of Harris-Stephens as well as its class of detectors[2] is that it is not scale-invariant.

References.

1. Chris Harris and Mike Stephens, *A Combined Corner and Edge Detector*, Proceedings of The Fourth Alvey Vision Conference (Manchester, UK), pp. 147–151, 1988.
2. M. Zuliani, C. Kenney and B. S. Manjunath, *A Mathematical Comparison of Point Detectors*, Workshop on Image and Video Registration (IVR/CVPR Workshop), Volume 27 , Issue 2, pp. 172–172, June 2004. (Provides a generalization of Harris-Stephens, Noble, and Shi and Tomasi corner detectors)