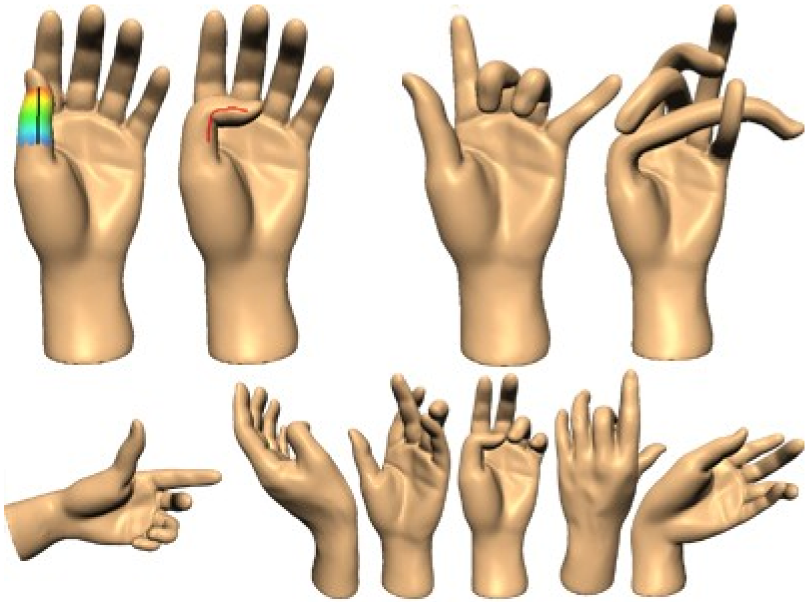
INF555

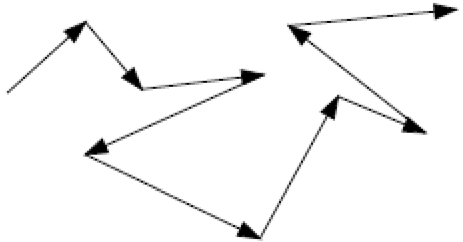


Fundamentals of 3D

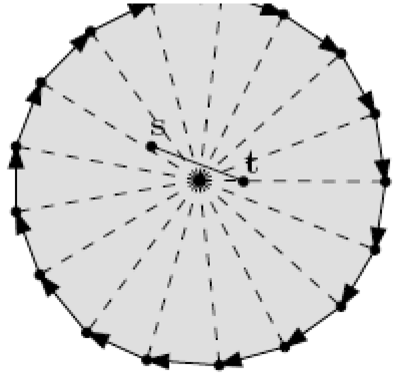
Lecture 8: Introduction to meshes

Frank Nielsen
nielsen@lix.polytechnique.fr

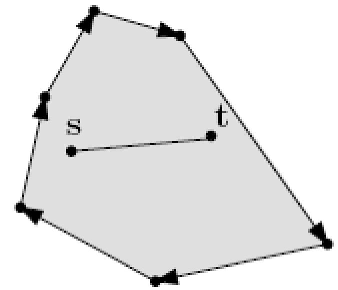
Polygons



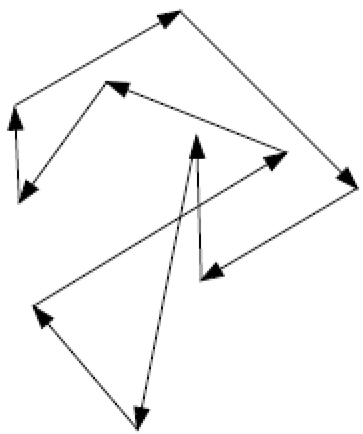
Polygonal chain



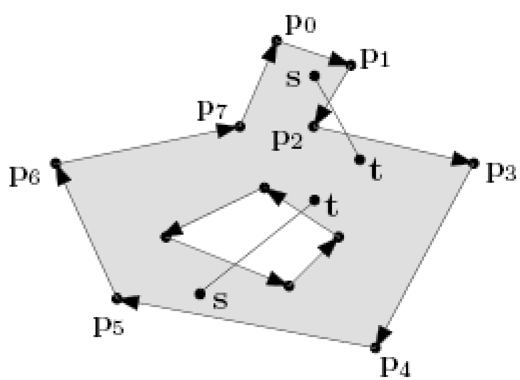
Regular (convex) polygon



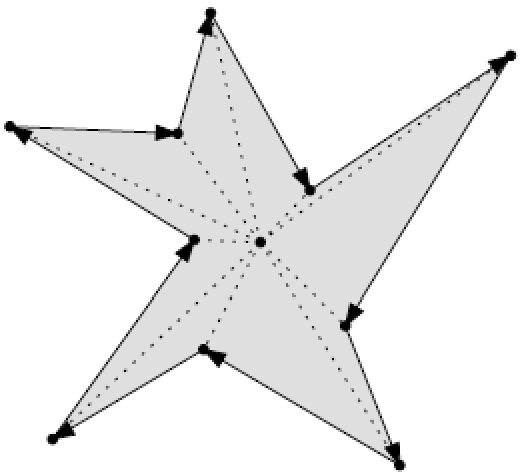
Convex polygon



Nonsimple polygon

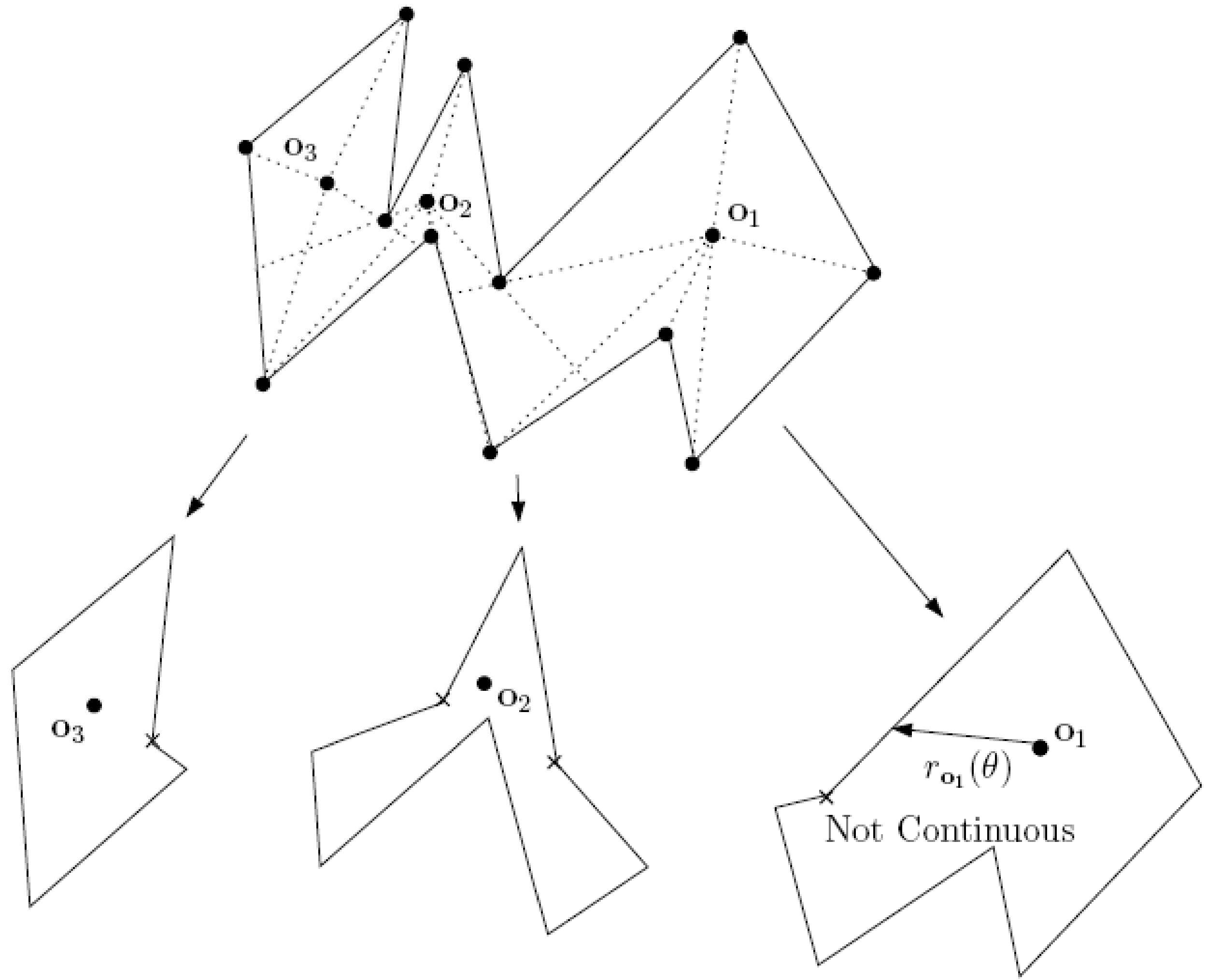


Nonconvex polygon with hole

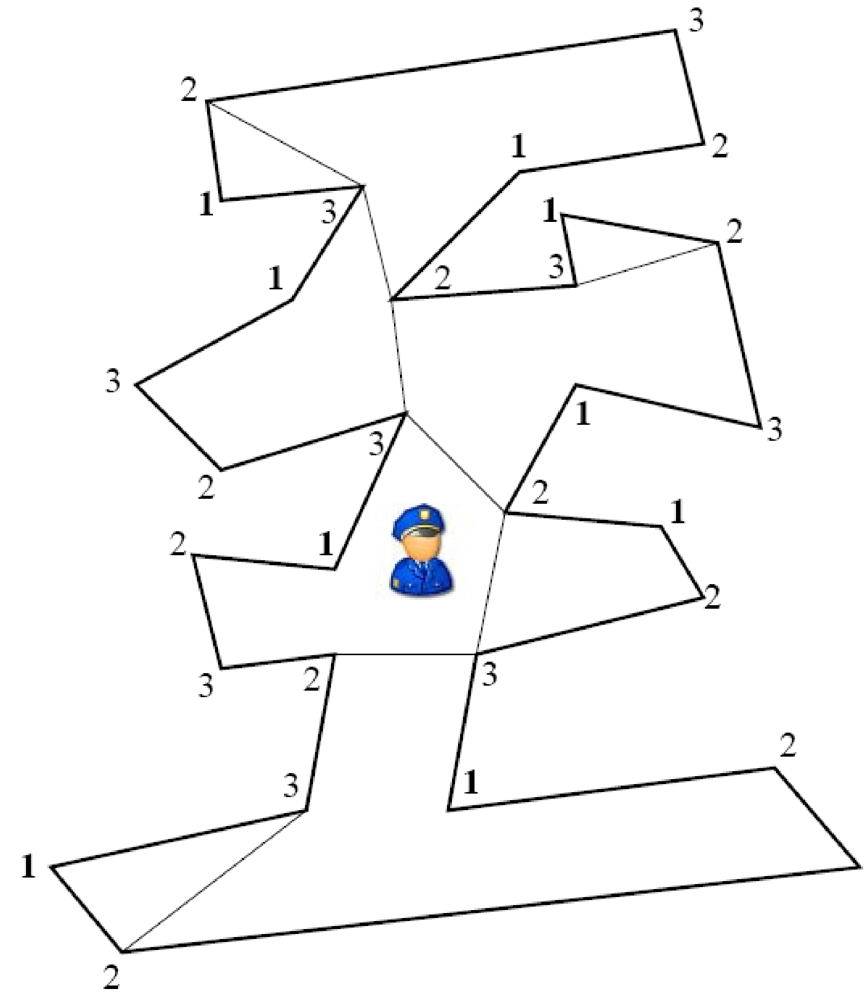


Star-shaped polygon

Polygons: Star-shaped decomposition

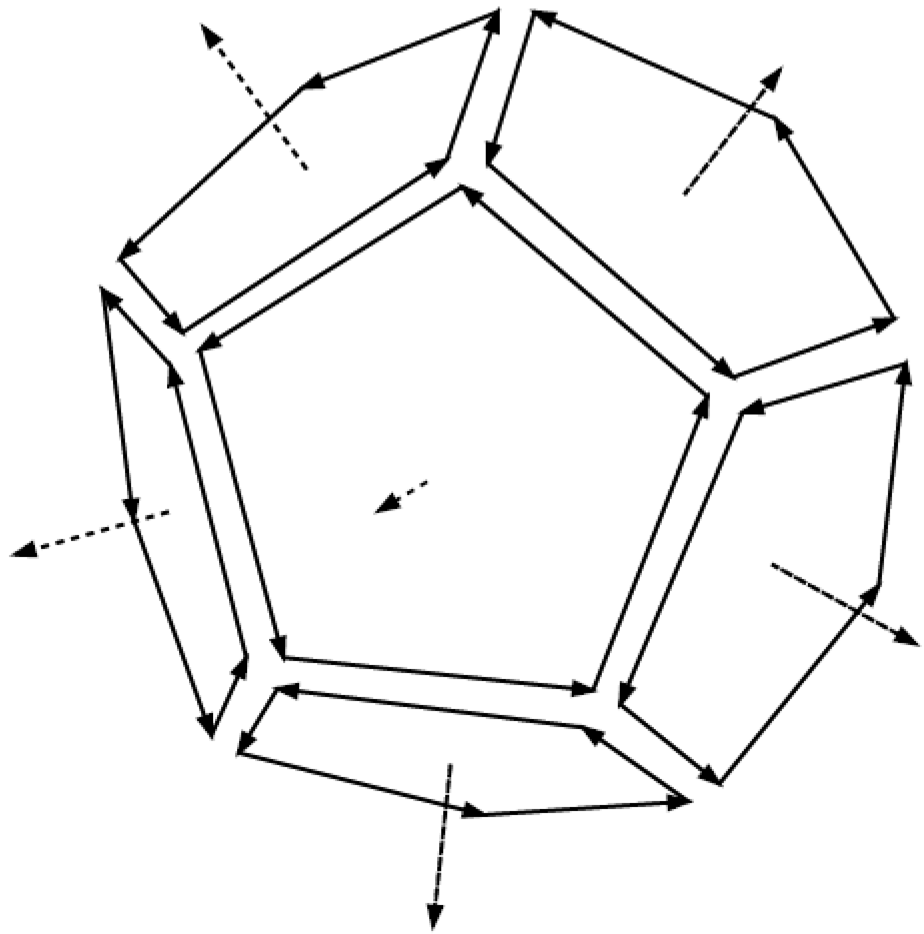


Polygons: Star-shaped decomposition



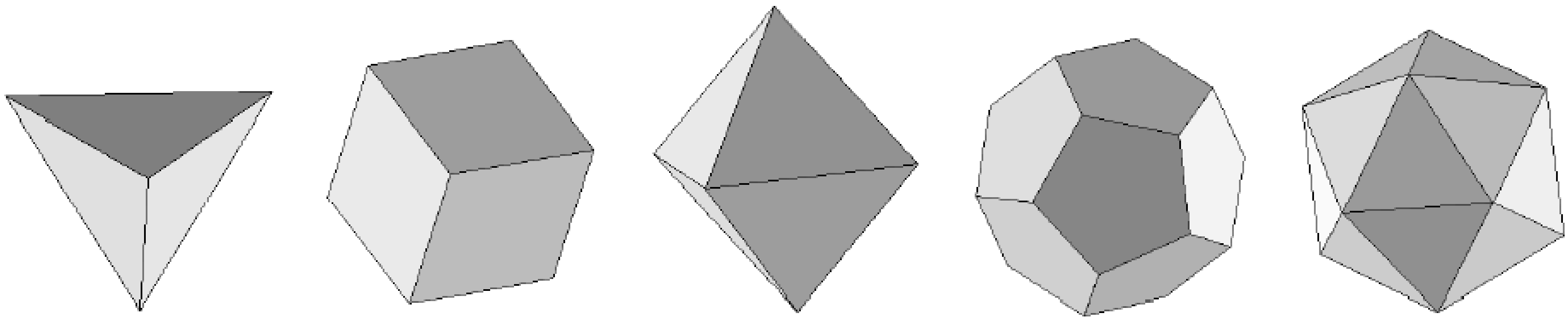
Art gallery, illumination problems, robots'race, etc.
Place guards...

Orienting the fact edges for outer normals



Polyhedron, convex polyhedra

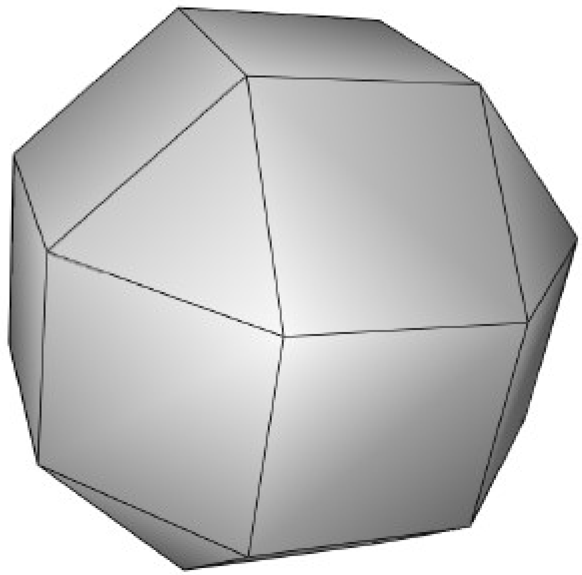
Platonic solids: 5 convex polyhedra


(a) (b) (c) (d) (e)

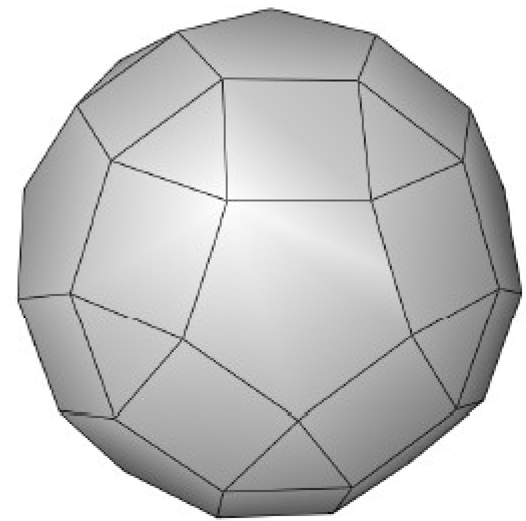
Platonic solid	Schläfli symbol	# Vertices	# Faces	# Edges
Tetrahedron (a)	$(3, 3)$	4	4	6
Hexahedron (b)	$(4, 3)$	8	6	12
Octahedron (c)	$(3, 4)$	6	8	12
Dodecahedron (d)	$(5, 3)$	20	12	30
Icosahedron (e)	$(3, 5)$	12	20	30

Identical faces

Uniform polyhedra



rhombicuboctahedron

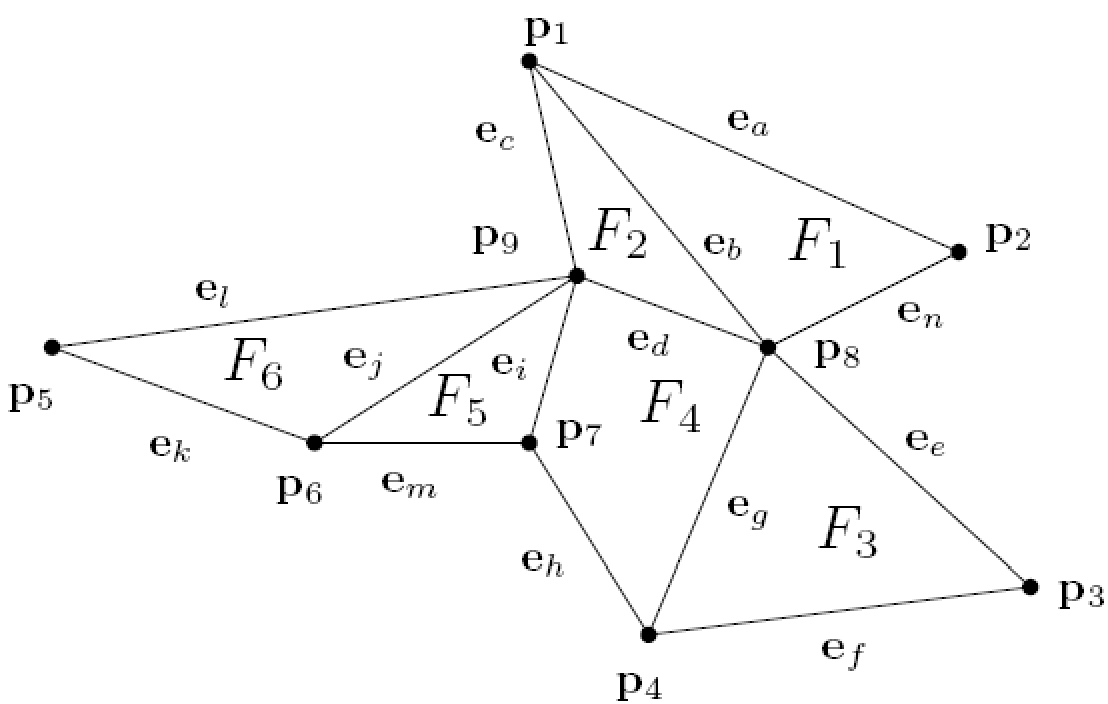


rhombicosidodecahedron

faces=regular polygons (not necessarily the same),
isometry mapping of its vertices (=symmetry)

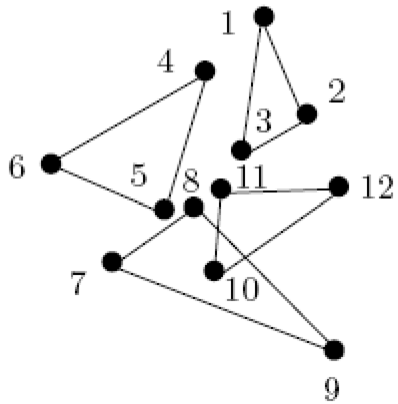
Meshes: Notations

$$\mathcal{M} \qquad \begin{array}{ll} |\mathcal{V}(\mathcal{M})| = 9 & \mathcal{V}(\mathcal{M}) = \{\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3, \mathbf{p}_4, \mathbf{p}_5, \mathbf{p}_6, \mathbf{p}_7, \mathbf{p}_8, \mathbf{p}_9\} \\ |\mathcal{E}(\mathcal{M})| = 14 & \mathcal{E}(\mathcal{M}) = \{\mathbf{e}_a, \mathbf{e}_b, \mathbf{e}_c, \mathbf{e}_d, \mathbf{e}_e, \mathbf{e}_f, \mathbf{e}_g, \mathbf{e}_h, \mathbf{e}_i, \mathbf{e}_j, \mathbf{e}_k, \mathbf{e}_l, \mathbf{e}_m, \mathbf{e}_n\} \\ |\mathcal{F}(\mathcal{M})| = 6 & \mathcal{F}(\mathcal{M}) = \{F_1, F_2, F_3, F_4, F_5, F_6\} \end{array}$$

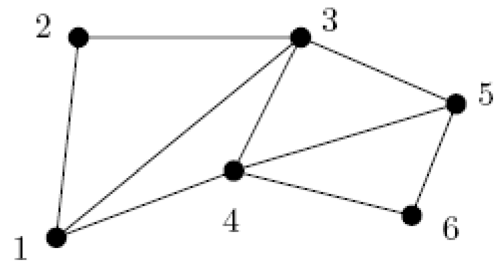


$$E(\mathbf{p}_8) = \{\mathbf{e}_b, \mathbf{e}_d.\mathbf{e}_g, \mathbf{e}_e, \mathbf{e}_n\} \qquad N(\mathbf{p}_8) = \{\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3, \mathbf{p}_4, \mathbf{p}_9\}$$

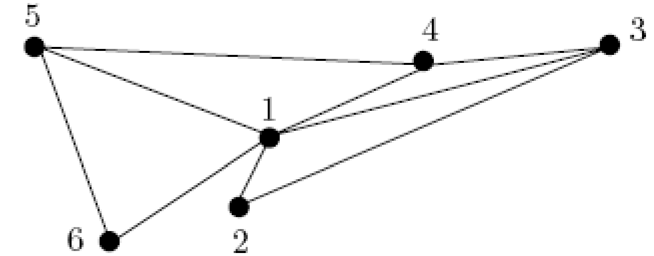
Meshes: Connectivity



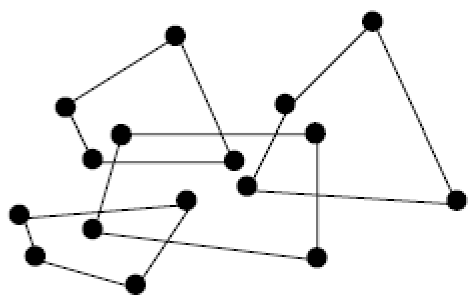
Triangle Soup



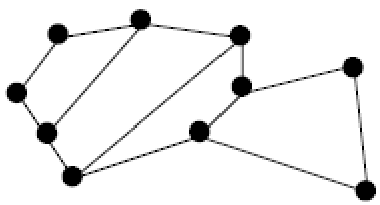
Triangle Strip



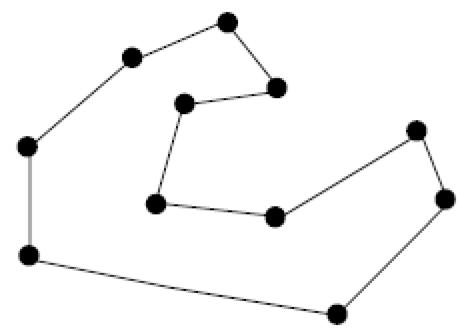
Triangle Fan (Umbrella)



Quad Soup

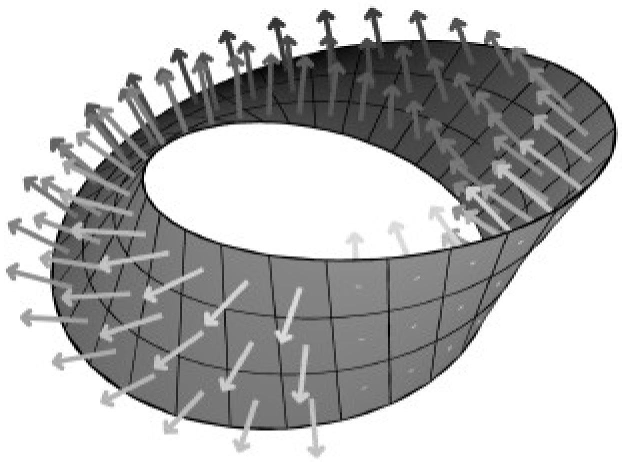


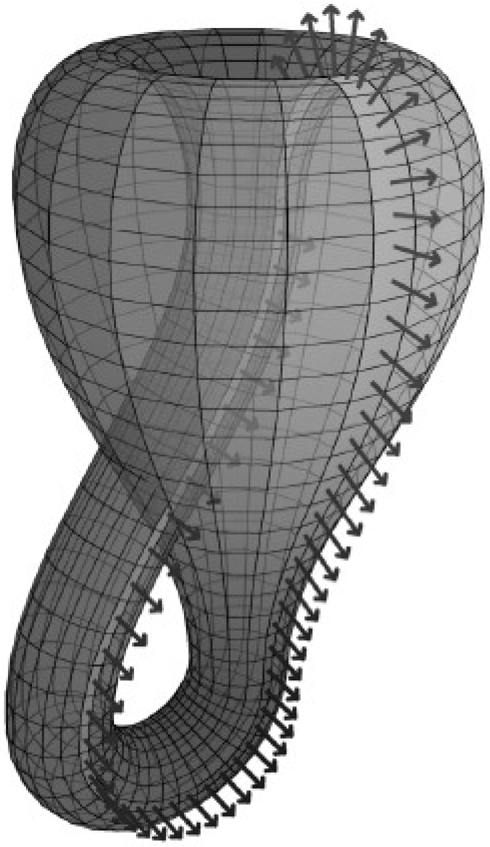
Quad Strip



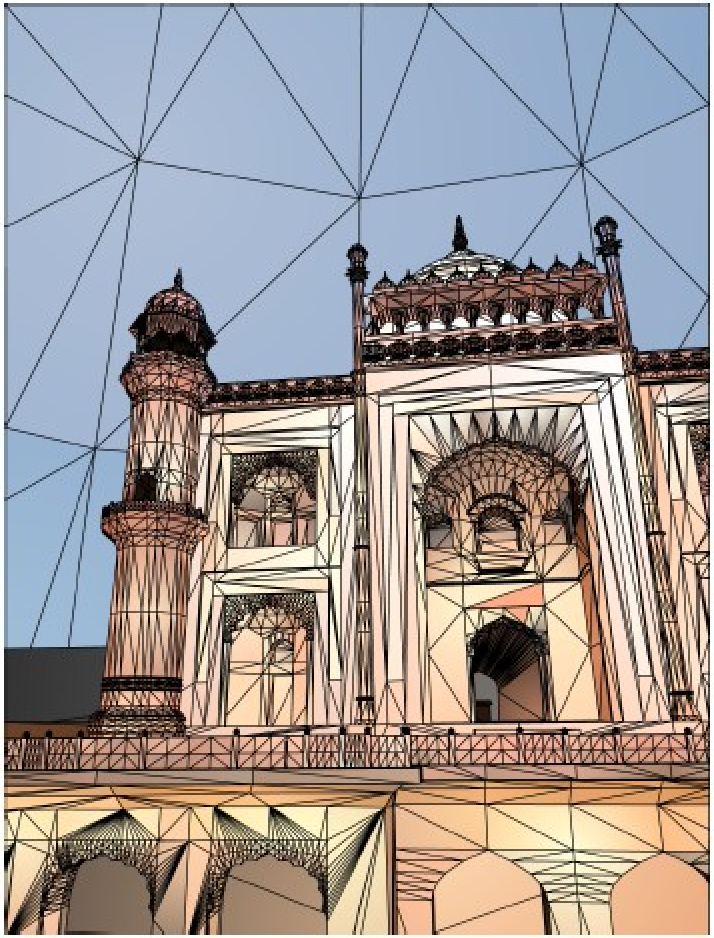
Polygon

Meshes: Non-orientable surfaces

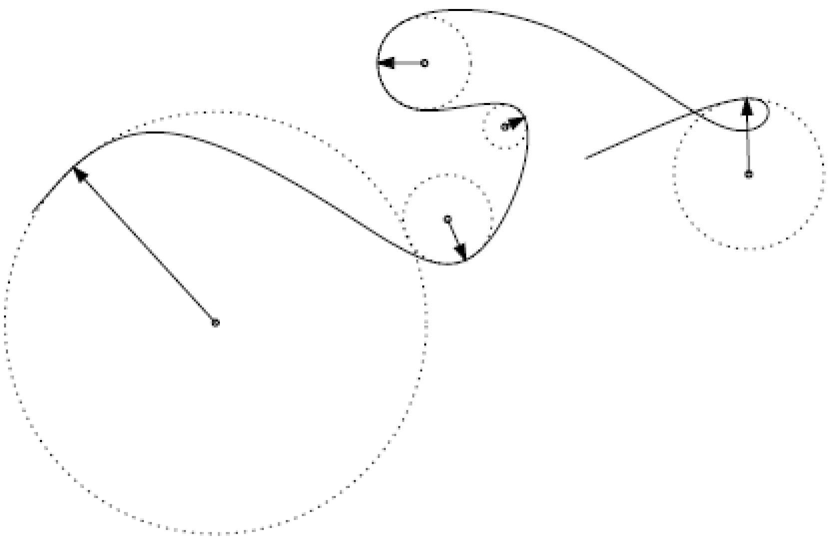




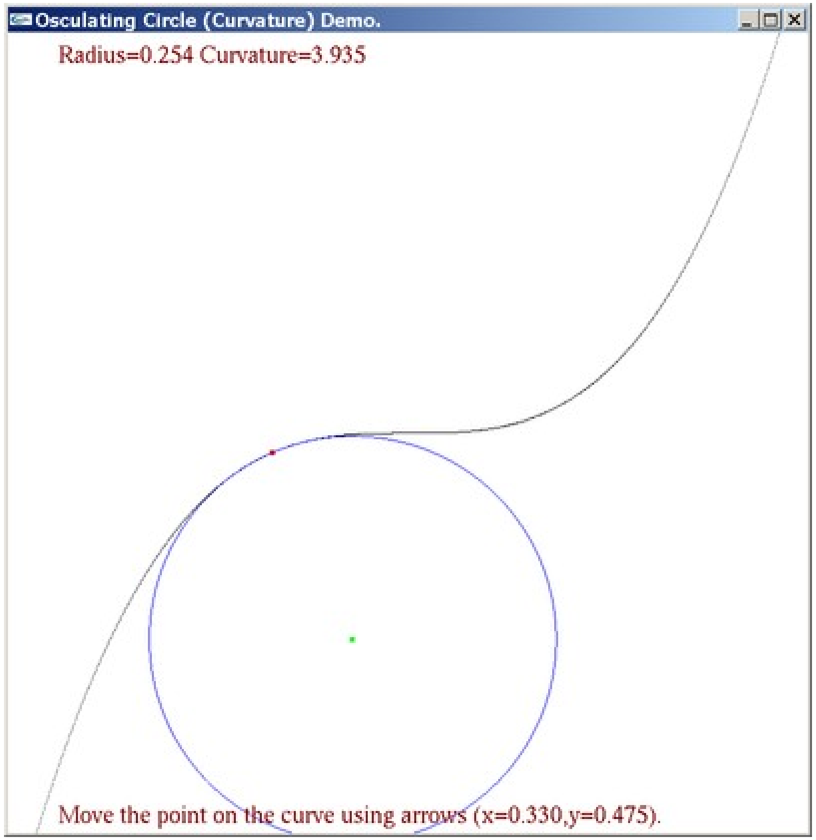
Textured meshes



Osculating circles and curvature



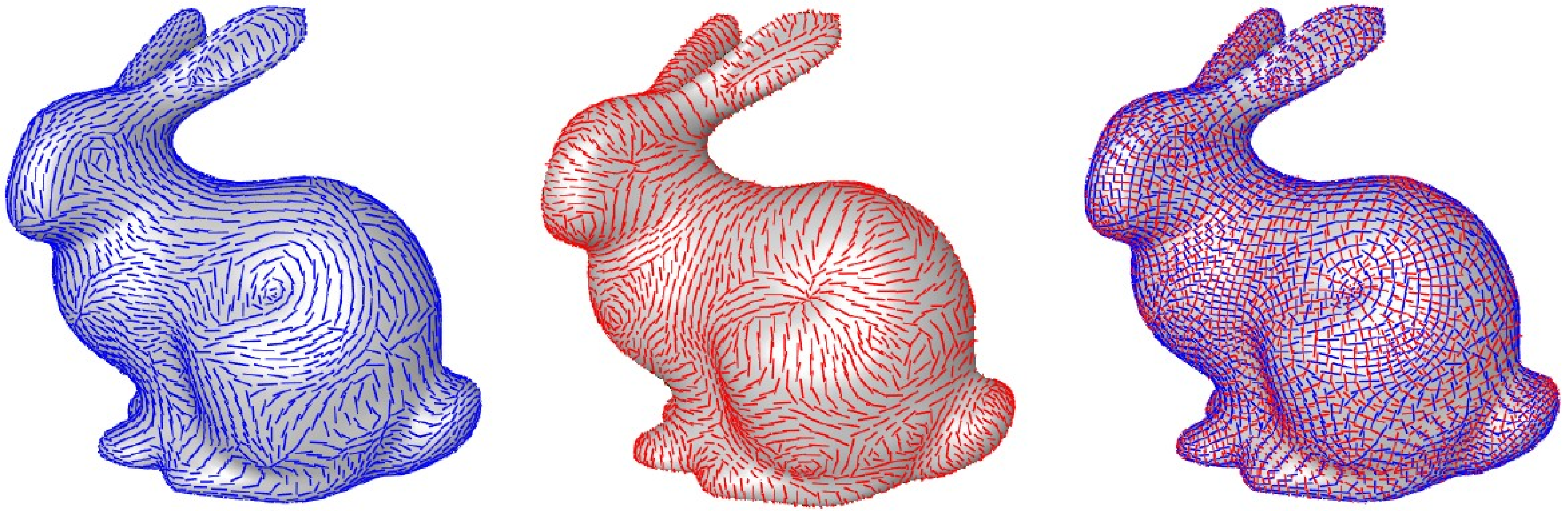
$$\rho(\mathbf{p}) = \frac{1}{R(\mathbf{p})}$$



Curvature is the inverse of the radius of the osculating circle.

Mesh: Sectional curvatures and principal directions

Directions are perpendicular to each other



Minimum curvature Maximum curvature

sectional curvatures.

Intersection of a surface S with a plane containing point $\mathbf{p}$ and its normal:
2D curve that can be analyzed using the osculating circles.

Mesh: Gaussian and mean curvatures

Gaussian curvature:

$$\rho_G = \rho_{\max} \times \rho_{\min}$$

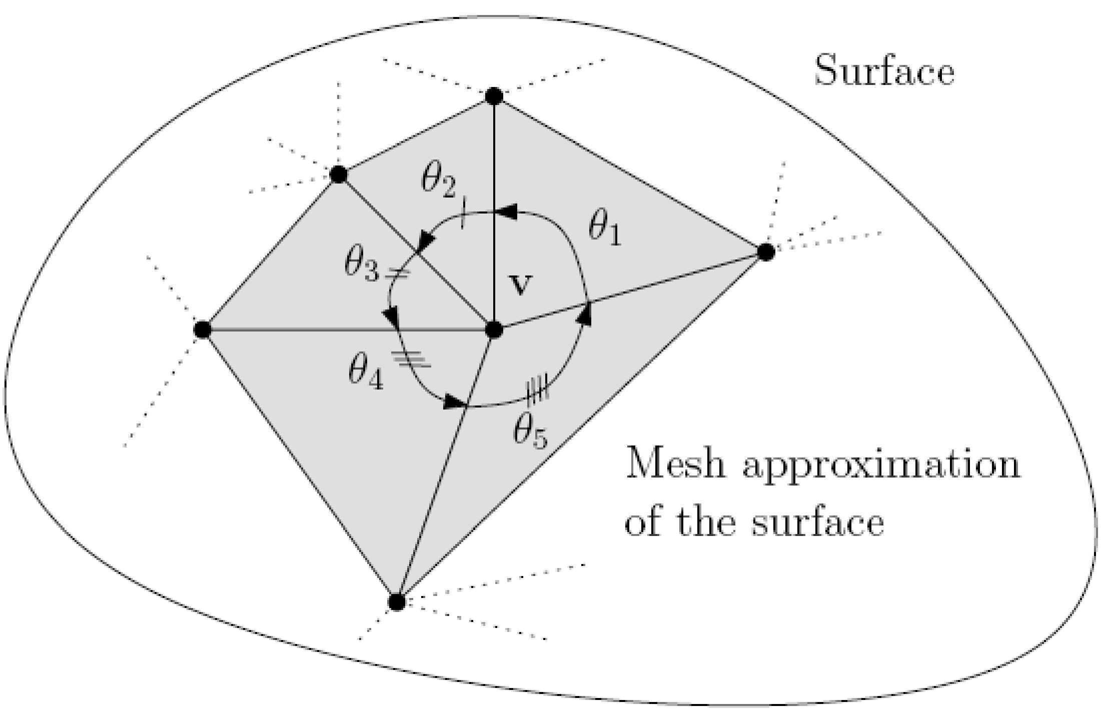
Mean curvature:

$$\frac{\rho_{\max} + \rho_{\min}}{2}$$

Mesh: Integral Gaussian curvature/angle excess

$$\int\int_{A \in T(\mathbf{v})} \rho_G(A)\,dA \simeq -\theta(\mathbf{v}).$$

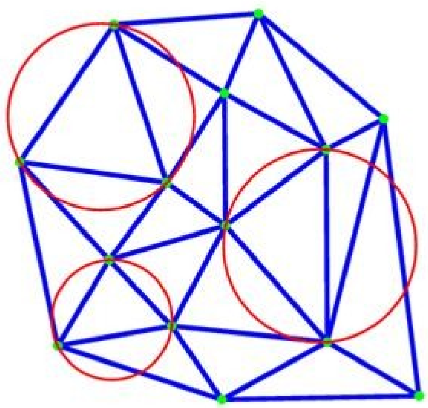
$$\rho(\mathbf{v}) = 2\pi - \sum_{i=1}^{|T(\mathbf{v})|} \theta_i.$$



Mesh: Ingredients of topology

Euler's formula is a topological invariant:

$$\#\text{Vertices} - \#\text{Edges} + \#\text{Faces} = 2.$$



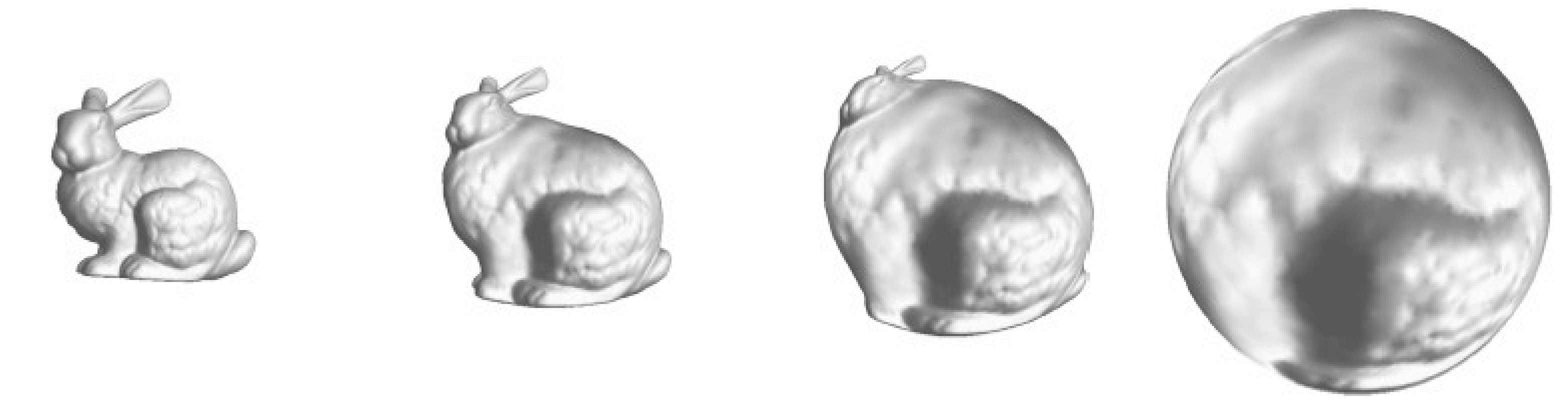
Closed triangulated manifold:

$$\#\text{Vertices} \le \frac{2}{3}\#\text{Edges}, \qquad \text{and} \qquad \#\text{Vertices} \le 2\#\text{Faces} - 4.$$

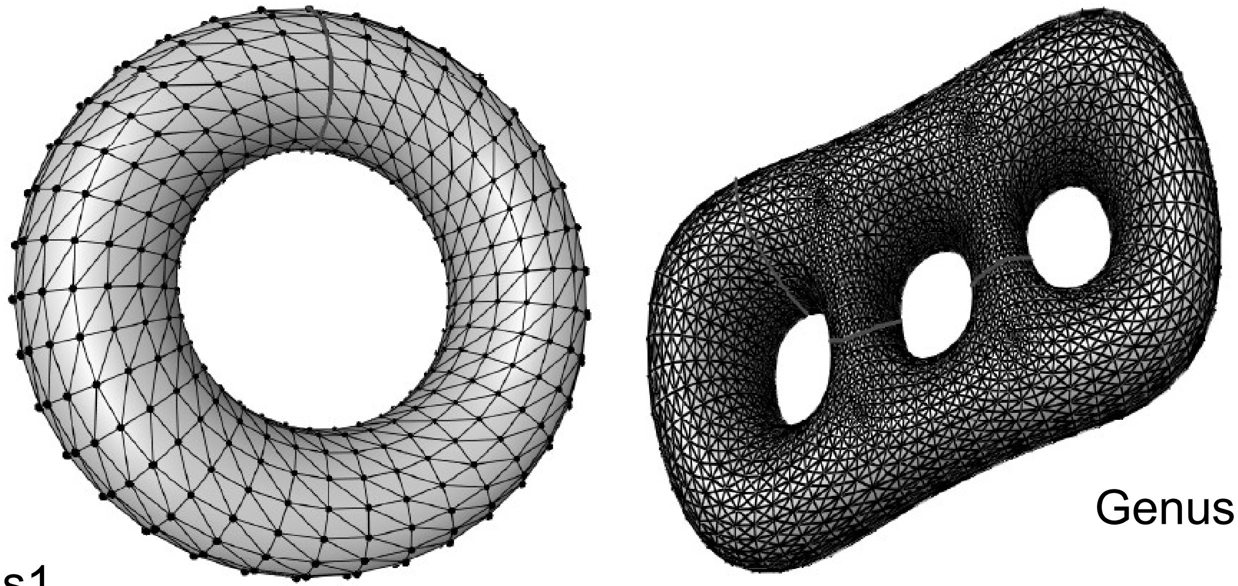
$$\#\text{Edges} \le 3\#\text{Vertices} - 6, \qquad \text{and} \qquad \#\text{Edges} \le 3\#\text{Faces} - 6.$$

$$\#\text{Faces} \le \frac{2}{3}\#\text{Edges}, \qquad \text{and} \qquad \#\text{Faces} \le 2\#\text{Vertices} - 4.$$

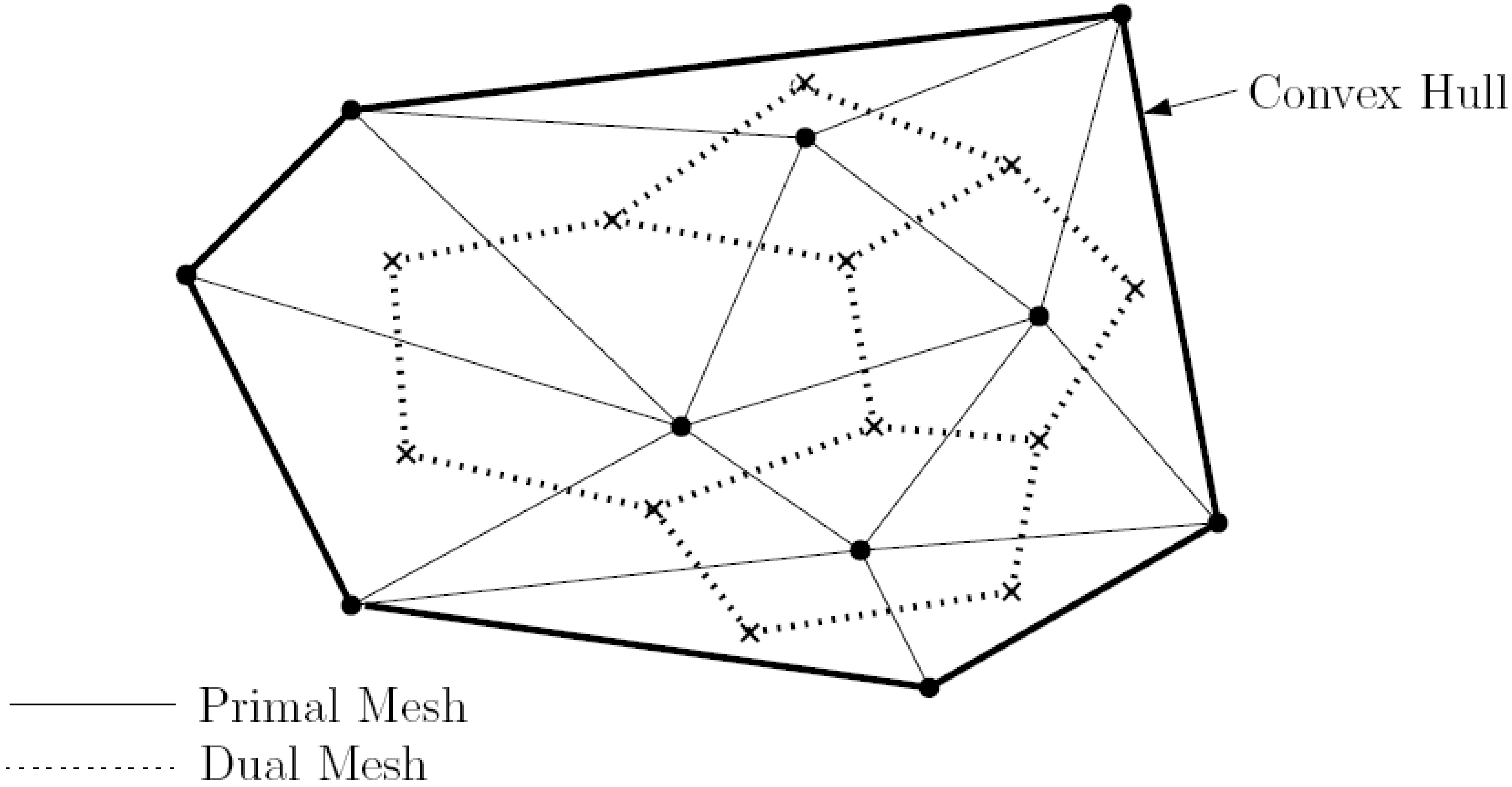
Mesh topology: Genus, polyhedra with holes



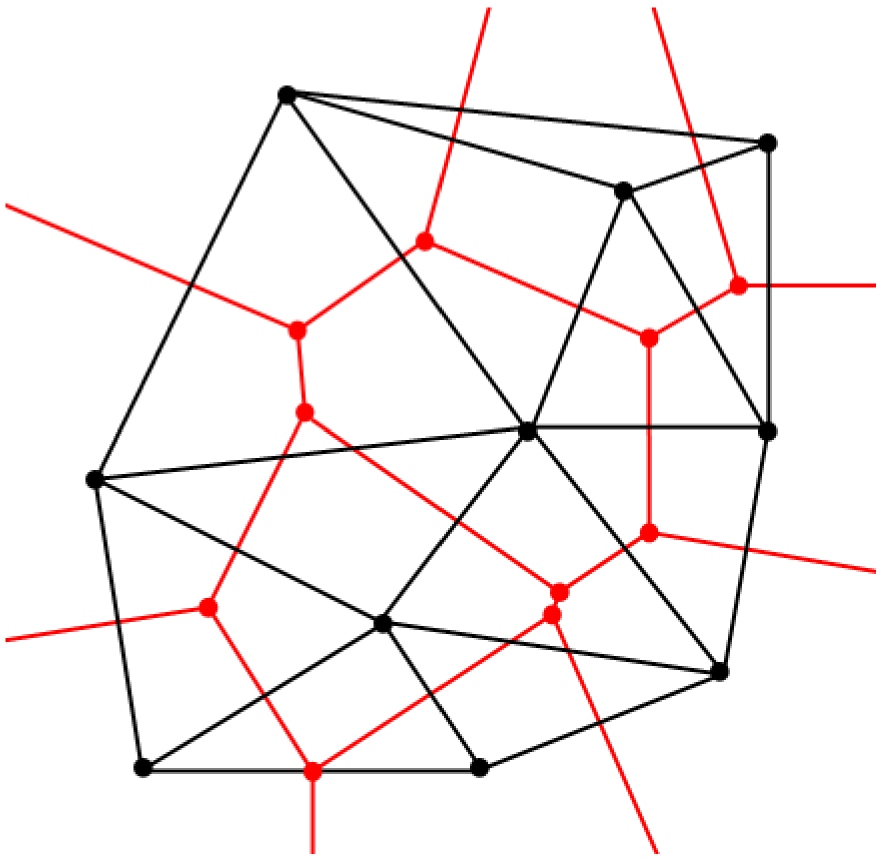
$$\#\mathrm{Vertices} - \#\mathrm{Edges} + \#\mathrm{Faces} = 2 - 2\#\mathrm{Genus}$$



Mesh: Primal/Dual graph representations

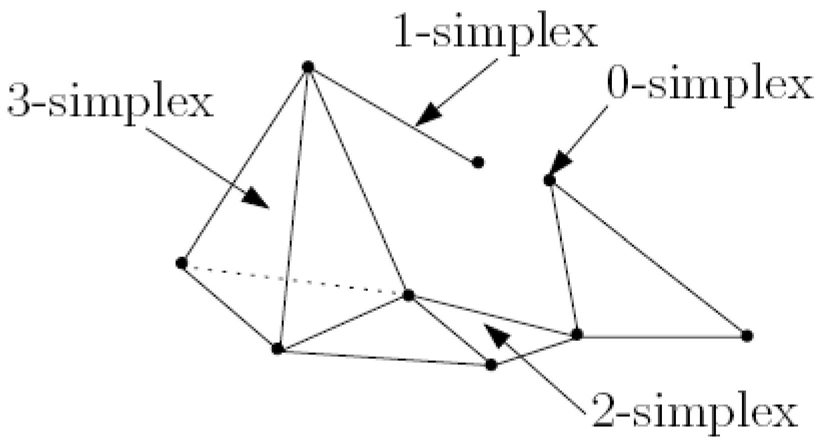


Mesh: Primal/Dual graph representations

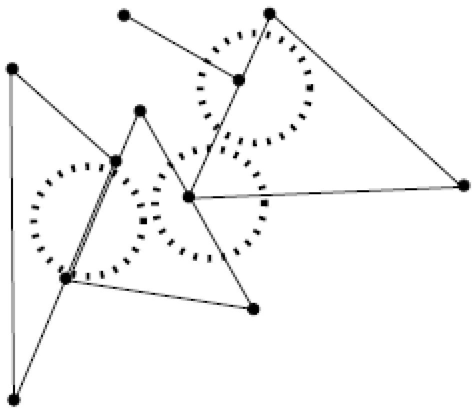


Primal Voronoi/Dual Delauny triangulation

Simplicial complexes

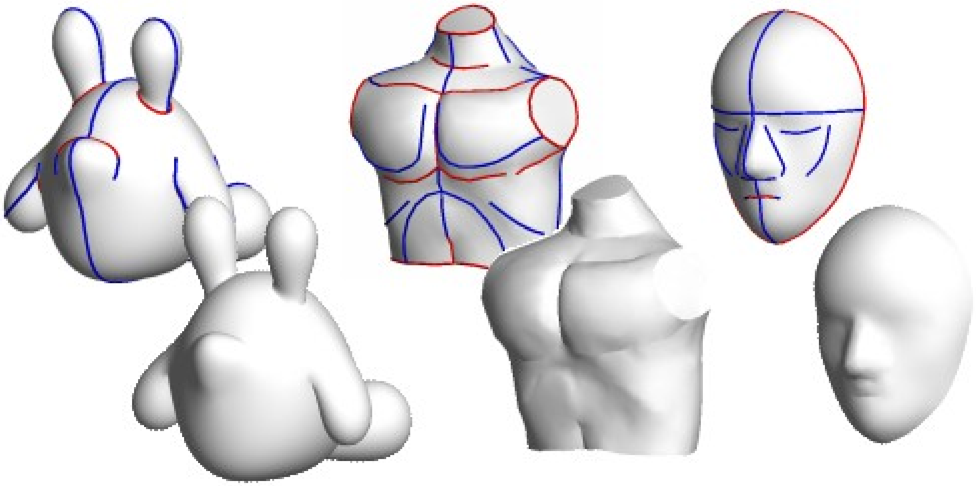


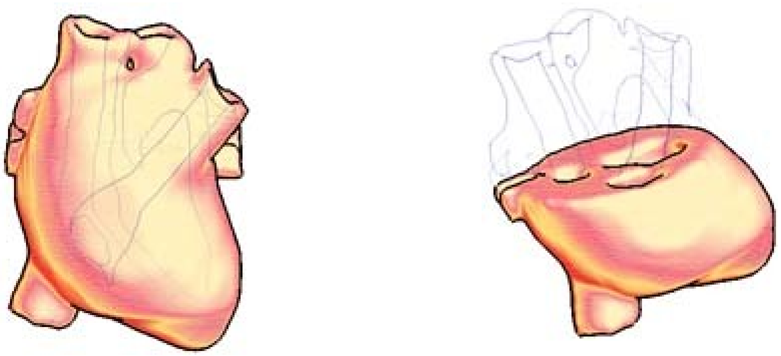
(a) Simplicial complex



(b) Nonsimplicial complex

Sketching meshes: Pen computing

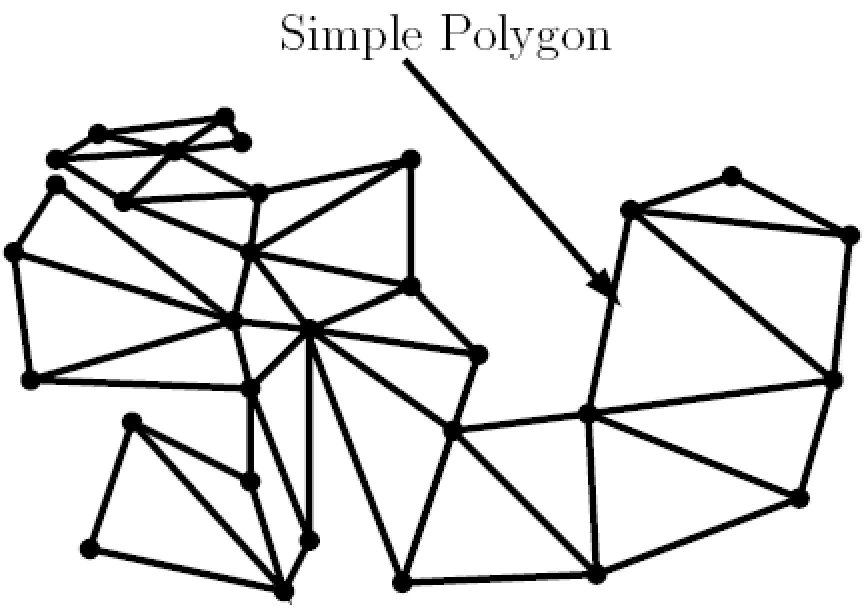


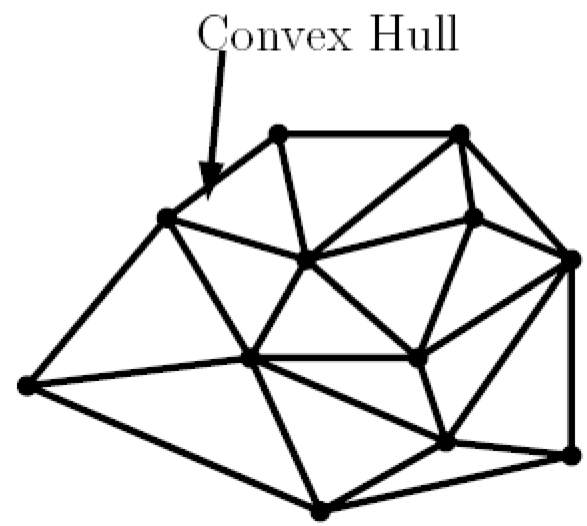


http://www.sonycsl.co.jp/person/nielsen/PT/vteddy/vteddy-desc.html

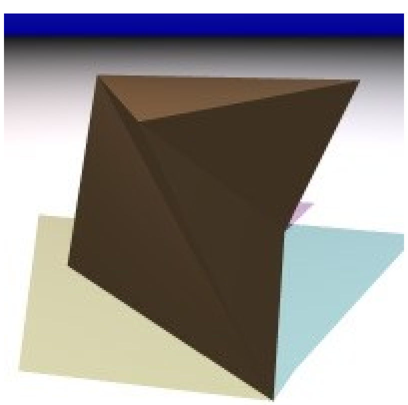
Triangulation meshes:

Always possible in 2D (but difficult)

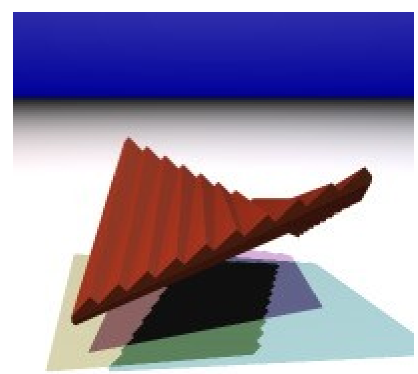




NOT Always possible in 3D!!! (require additional Steiner points)



Untetrahedralizable Objects



Schonhardt's polyhedron

Chazelle's polyhedron

Meshes: Procedural modeling/ city(buildings)

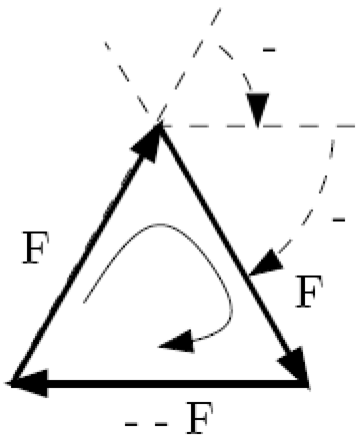
L-system (Lindenmayer) process

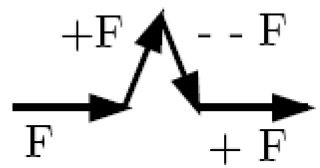
$$\mathrm{START} \longrightarrow A$$
$$A \longrightarrow B$$
$$B \longrightarrow AB$$

Fibonacci's sequences

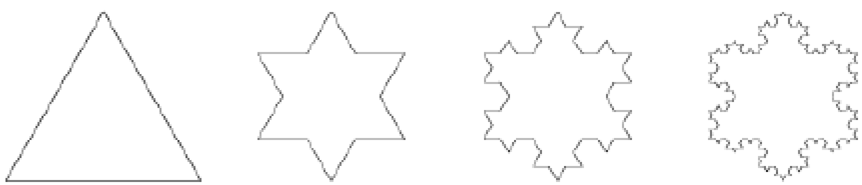
Step	String	String Length
0	A	1
1	B	1
2	AB	2
3	BAB	3
4	ABBAB	5
5	BABABBAB	8
6	ABBABBABABBAB	13
7	BABABBABABBABBABABBAB	21

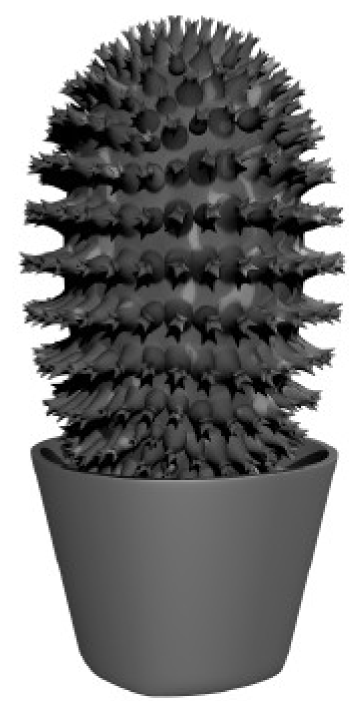
L-system (Lindenmayer) process / LOGO





$$\text{START} \longrightarrow F--F--F$$
$$F \longrightarrow F+F--F+F$$

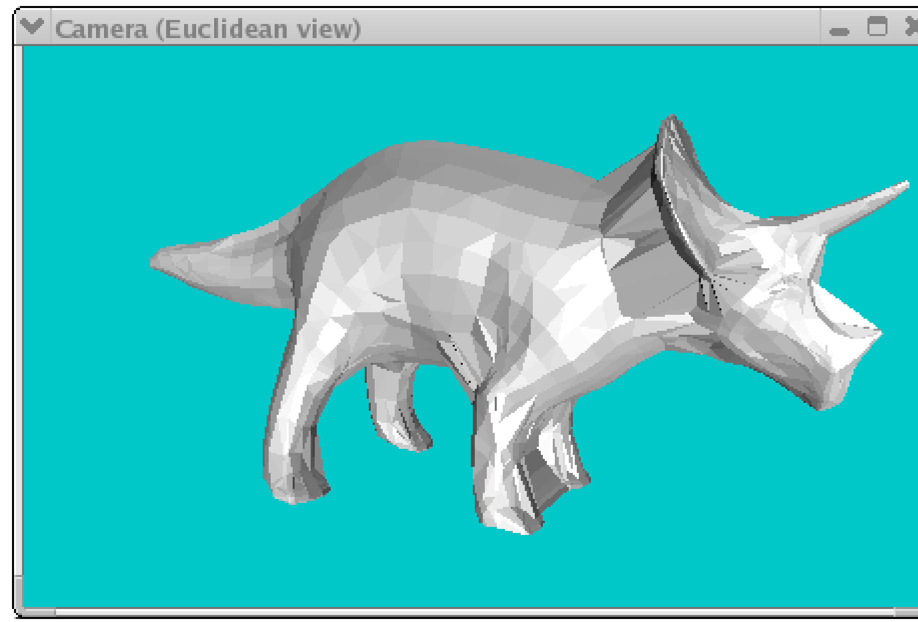




Data-structures for meshes: Indexed face list

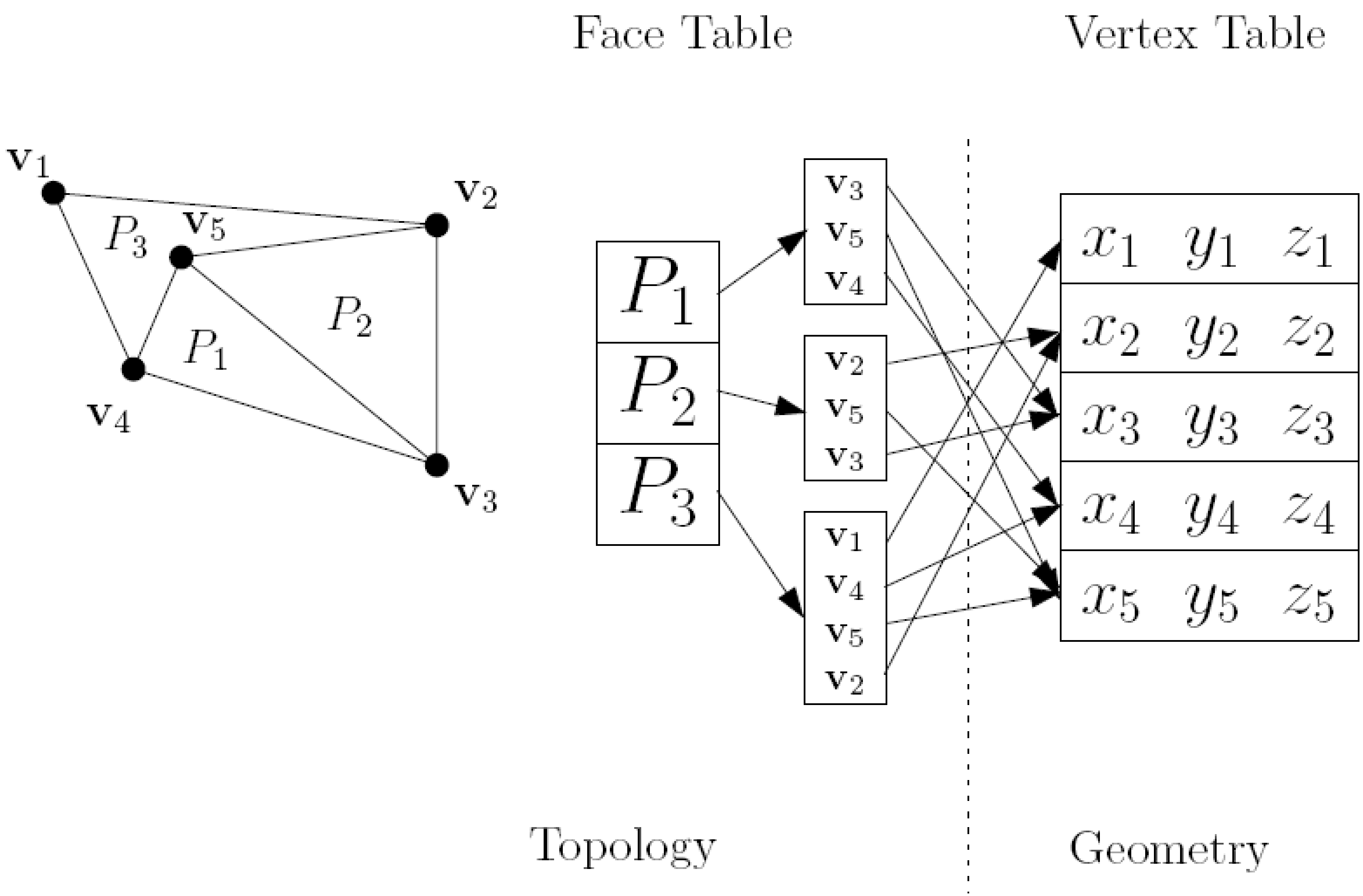
Object Oriented Graphics Library (OOGL) / OFF format

```
OFF
# Geomview OOGL format cube.off
# #Vertices #Faces #Edge
8 6 0
# Vertex table
−0.500000 −0.500000 0.500000
0.500000 −0.500000 0.500000
−0.500000 0.500000 0.500000
0.500000 0.500000 0.500000
−0.500000 0.500000 −0.500000
0.500000 0.500000 −0.500000
−0.500000 −0.500000 −0.500000
0.500000 −0.500000 −0.500000
# Face index table (first vertex index: 0)
4 0 1 3 2
4 2 3 5 4
4 4 5 7 6
4 6 7 1 0
4 1 7 5 3
4 6 0 2 4
```



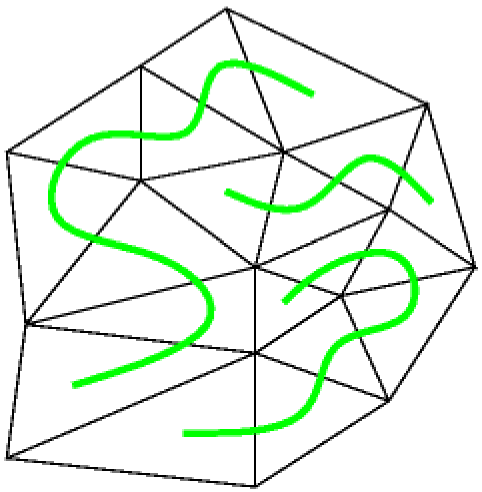
Geomview.org viewer

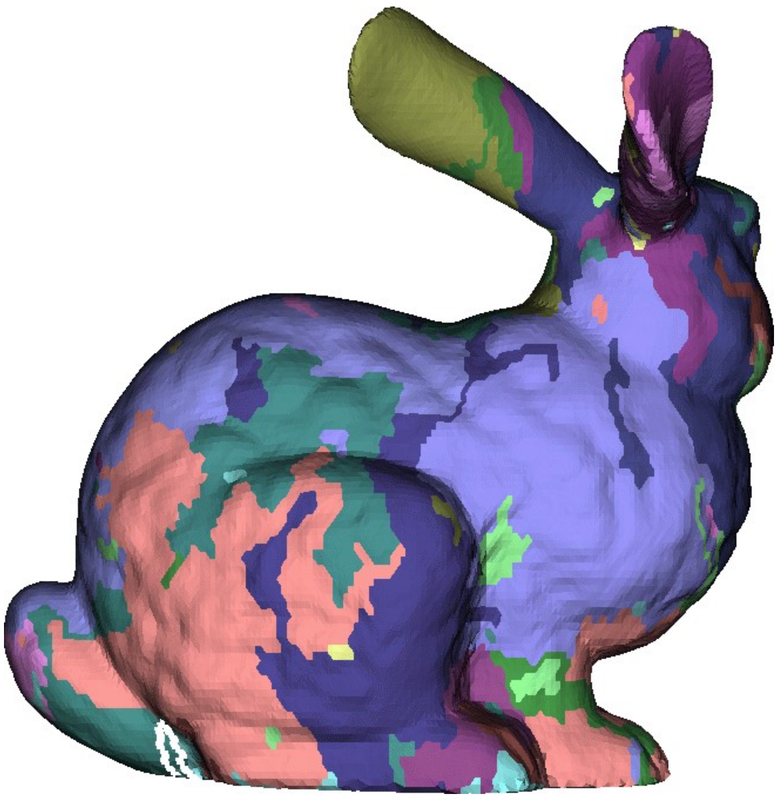
Data-structures for meshes: Indexed face list



Optimizing bandwidth: Triangle/quad strips

Compress mesh vertices
Compress mesh connectivity



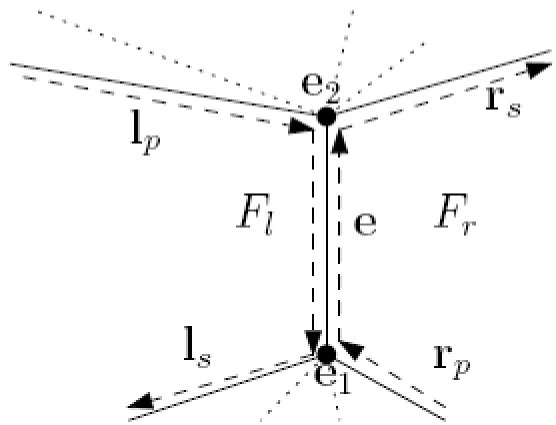


Bunny with 150 strips

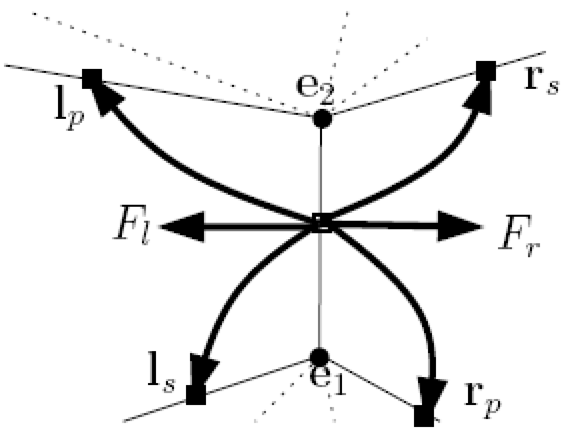
```
GREEDYSTRIPMESH(M)
◁ Overview of the greedy stripping method ▷
while there remains triangles in M
    do Pick a triangle T of M that has minimum number of adjacent triangles
        For each edge e of T, build the strip passing through T and e
        Choose the longuest strip and remove its triangles from M
```

Many data-structures for meshes....

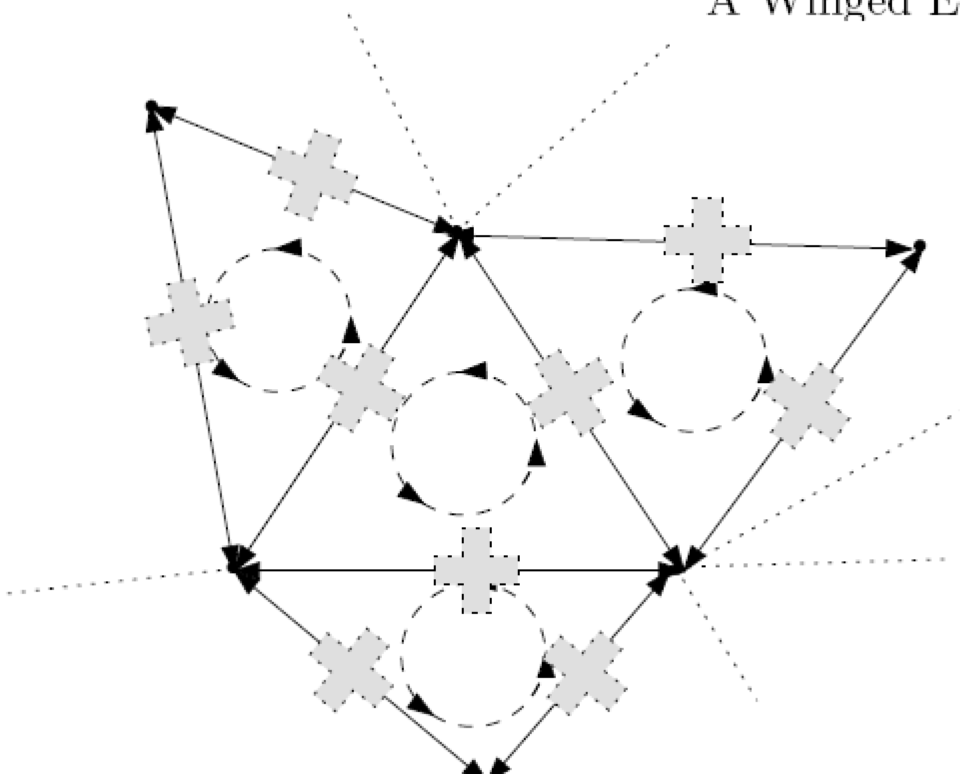
Winged edges
Half edges
Quad edges

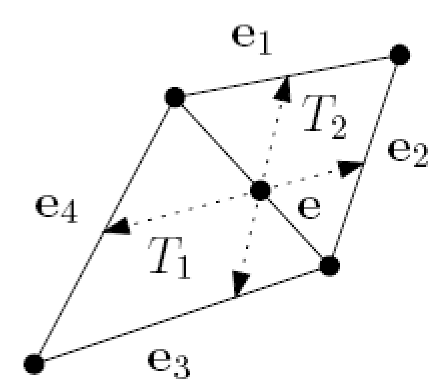


A Winged Edge



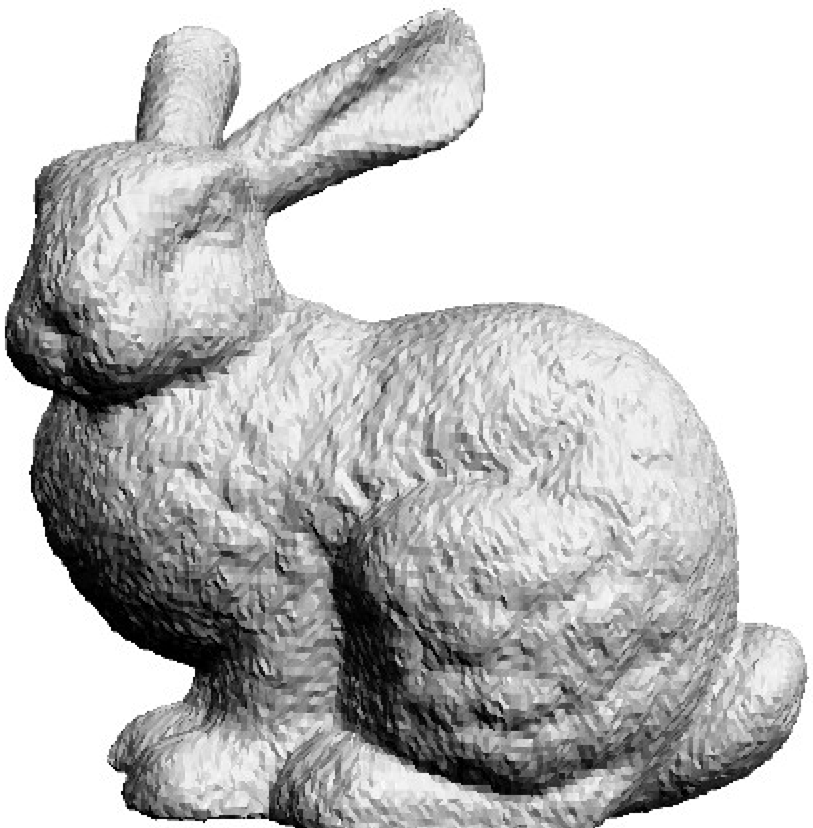
Pointer References

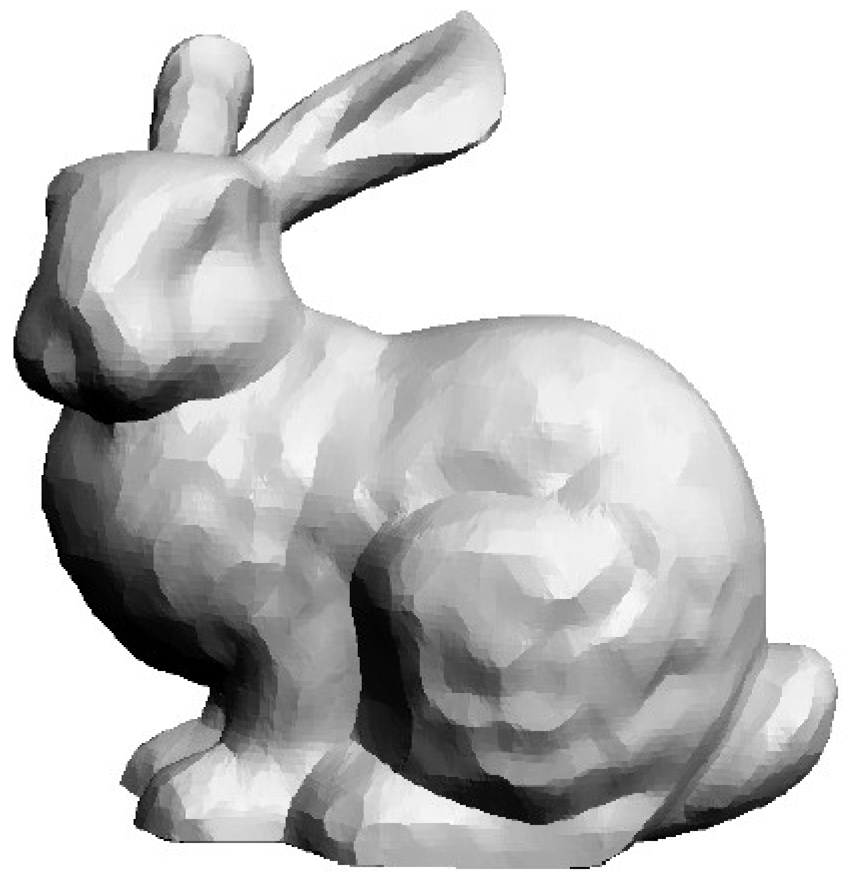




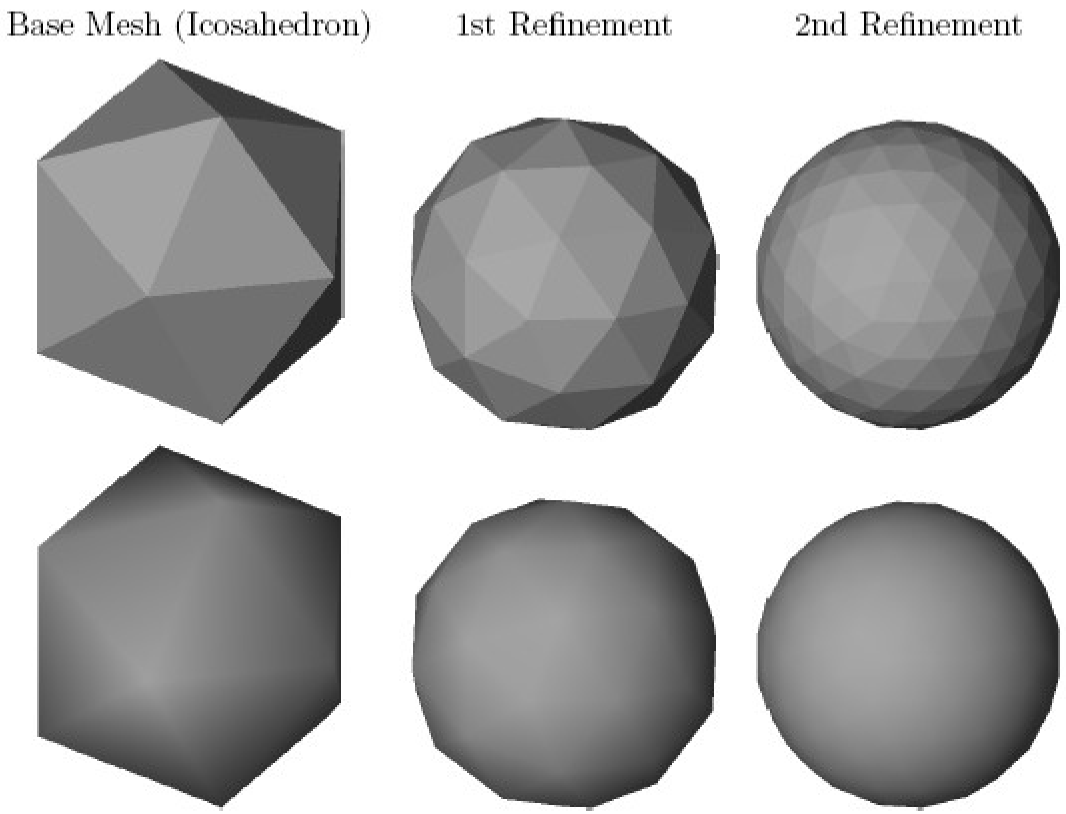
Laplacian smoothing on meshes

$$\mathbf{v} \leftarrow \mathbf{v} + \frac{\lambda}{|N(\mathbf{v})|} \sum_{i=1}^{|N(\mathbf{v})|} (N(\mathbf{v}, i) - \mathbf{v})$$



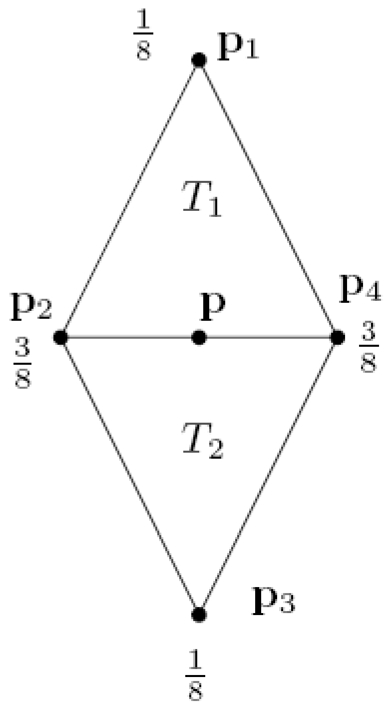


Surface subdivision



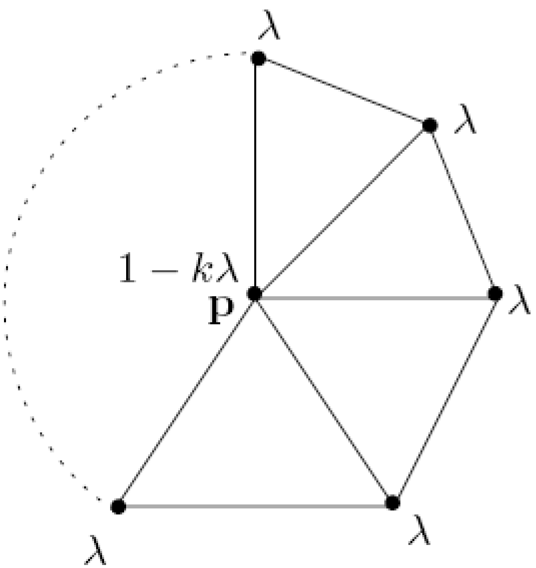
Surface subdivision: Loop scheme

New vertex creation



$$\mathbf{p} = \frac{1}{8}\mathbf{p}_1 + \frac{3}{8}\mathbf{p}_2 + \frac{1}{8}\mathbf{p}_3 + \frac{3}{8}\mathbf{p}_4$$

Vertex relocation



$$\mathbf{p} = (1 - k\lambda)\mathbf{p} + \lambda \sum_{i=1}^{k} N(\mathbf{p}, i)$$

$$\lambda = \frac{1}{k}\left(\frac{5}{8} - \left(\frac{3}{8} + \frac{1}{4}\cos\frac{2\pi}{k}\right)^2\right)$$

Simplified to

$$\lambda = \begin{cases} \frac{3}{16} & k = 3, \\ \frac{3}{8k} & k > 3. \end{cases}$$

SUBDIVISION($\mathcal{M}, k$)

◁ General subdivision framework ▷
◁ Depend on whether the scheme is approximating/interpolating and primal/dual ▷
for $\mathbf{e} \in \mathcal{M}$
  do ◁ for each edge ▷
    Create new vertex using the vertex creation mask
    ◁ Could be several for n-adic subdivision ▷
for $\mathbf{v} \in \mathcal{M}$
  do Move original vertex using the vertex displacement mask
Reconnect all vertices as a triangular mesh based on $\mathcal{M}$

Limit position, smoothness properties

$$\lambda_\infty = \frac{1}{\frac{3}{8}\lambda + k}$$

Kobbelt's subdivision (+edge flipping)

$$\mathbf{p} = (1 - \lambda n)\mathbf{p} + \lambda \sum_{i=1}^{|N(\mathbf{p})|} N(\mathbf{p}, i). \qquad \lambda = \frac{4 - 2\cos\frac{2\pi}{n}}{9n}$$

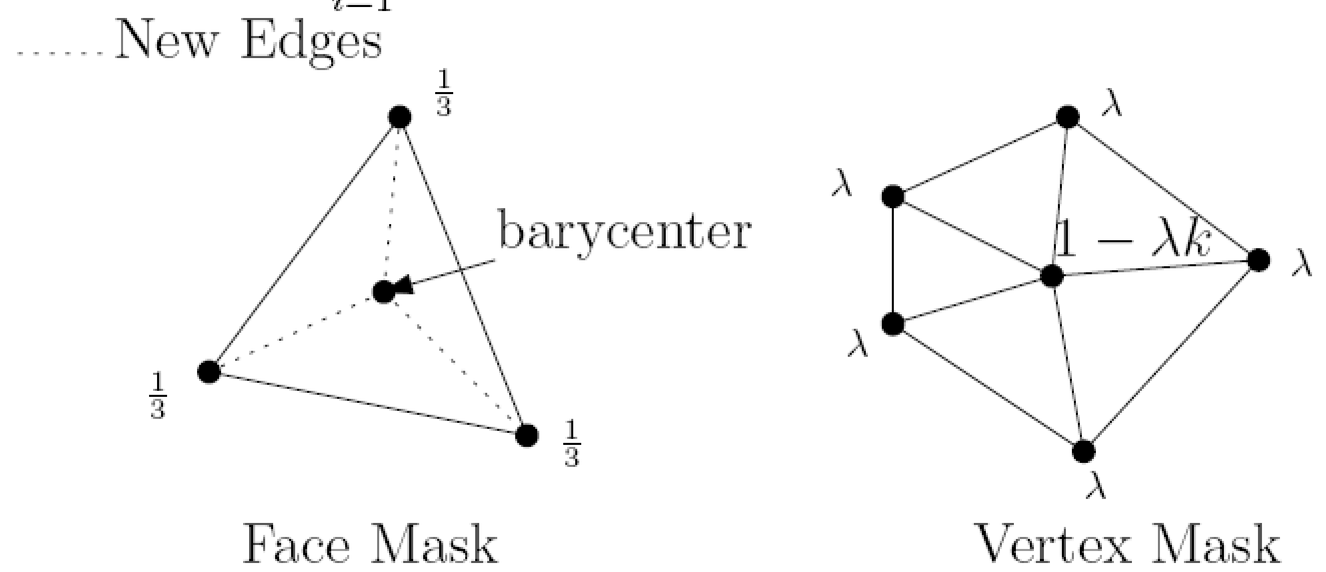
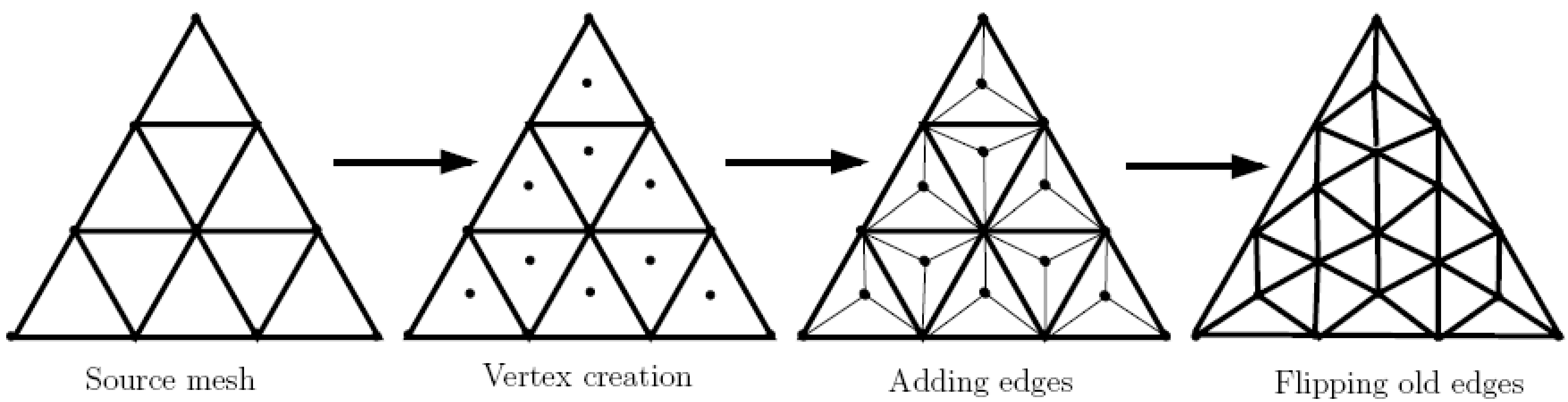
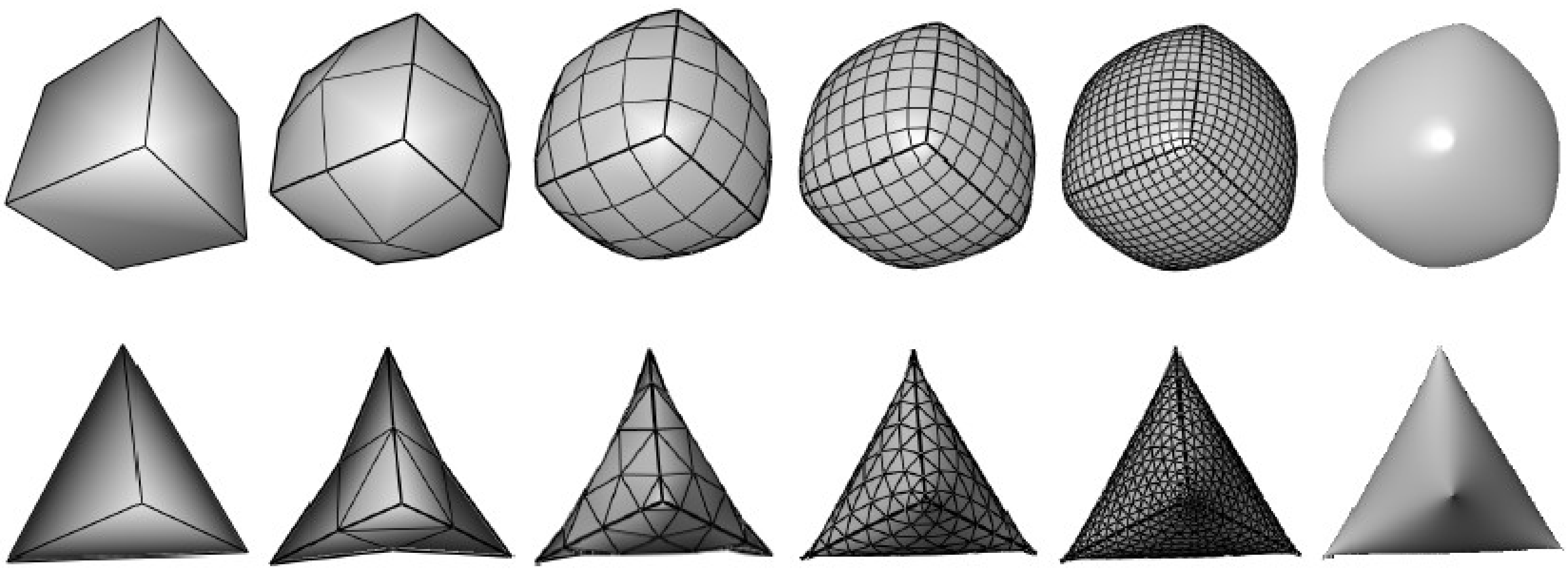


FIGURE 5.41 *The face and vertex masks for the $\sqrt{3}$-subdivision.*



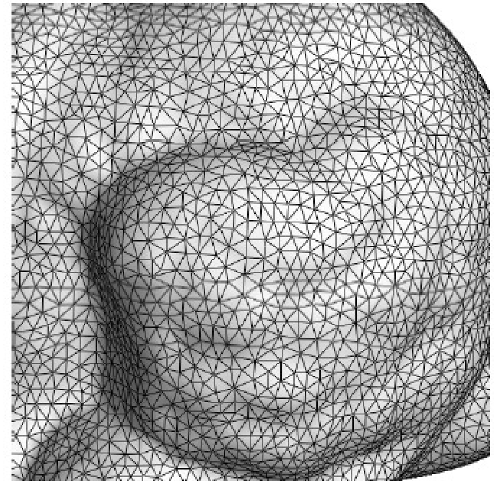
Mesh subdivision

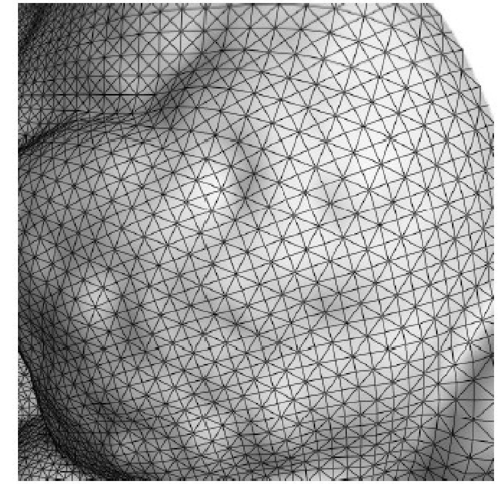
	Cube			Tetrahedron		
Level	#Vertices	#Faces	#Edges	#Vertices	#Faces	#Edges
0	8	6	12	4	4	6
1	26	24	48	10	16	24
2	98	96	192	34	64	96
3	386	384	768	130	256	384
4	1538	1536	3072	514	1024	1536
⋮	⋮	⋮	⋮	⋮	⋮	⋮
8	393218	393216	786432	131074	262144	393216

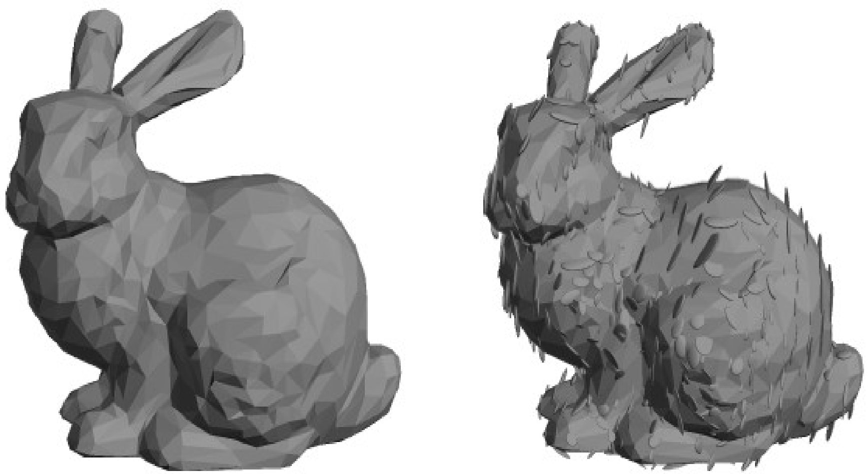


Remeshing

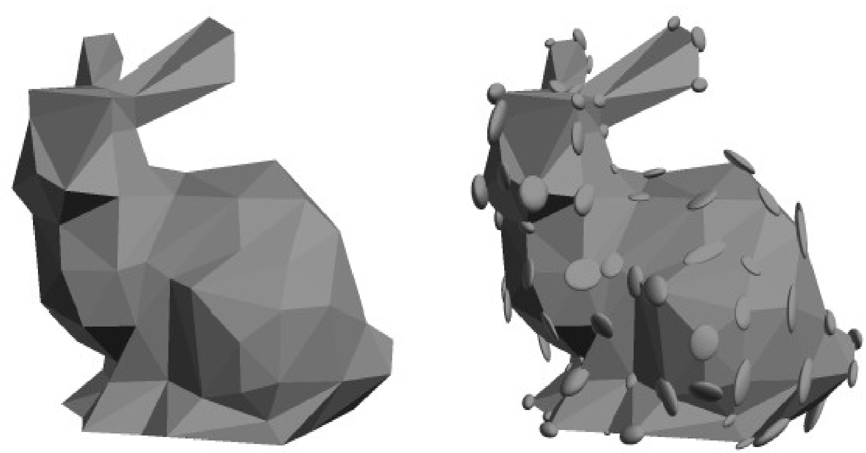
Decimating mesh



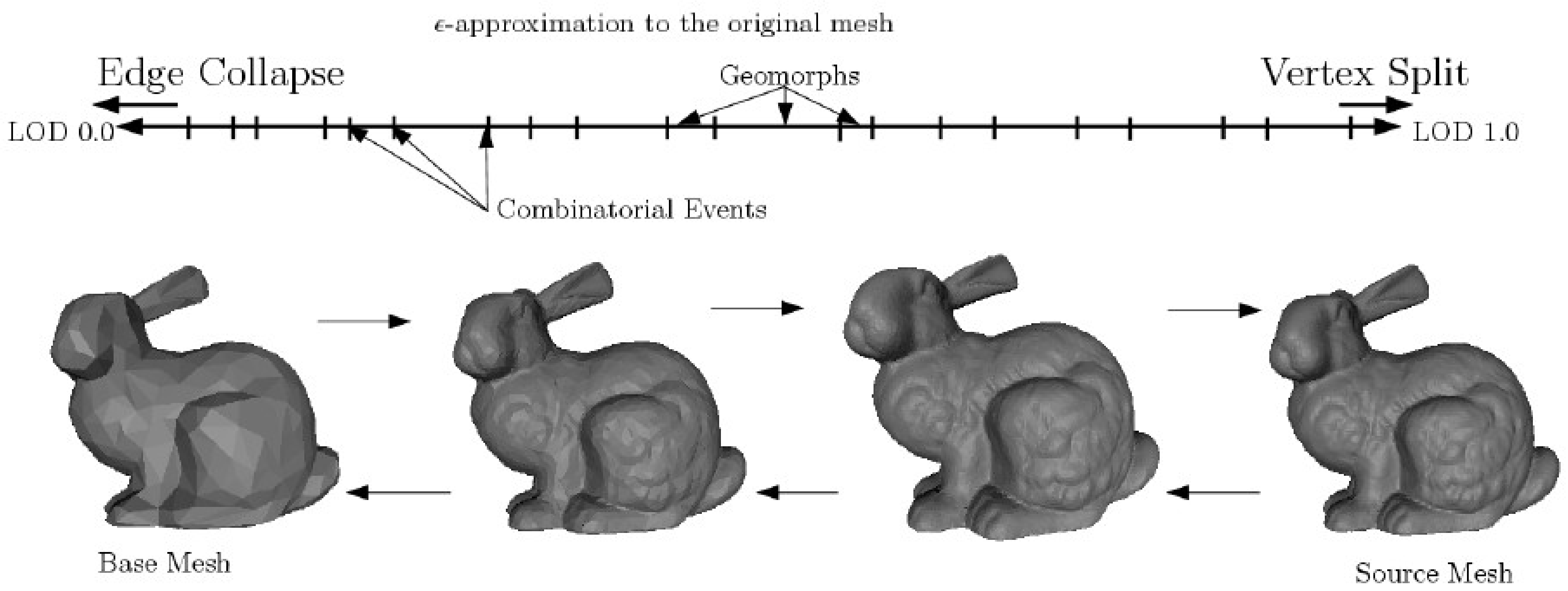




(a) (b)

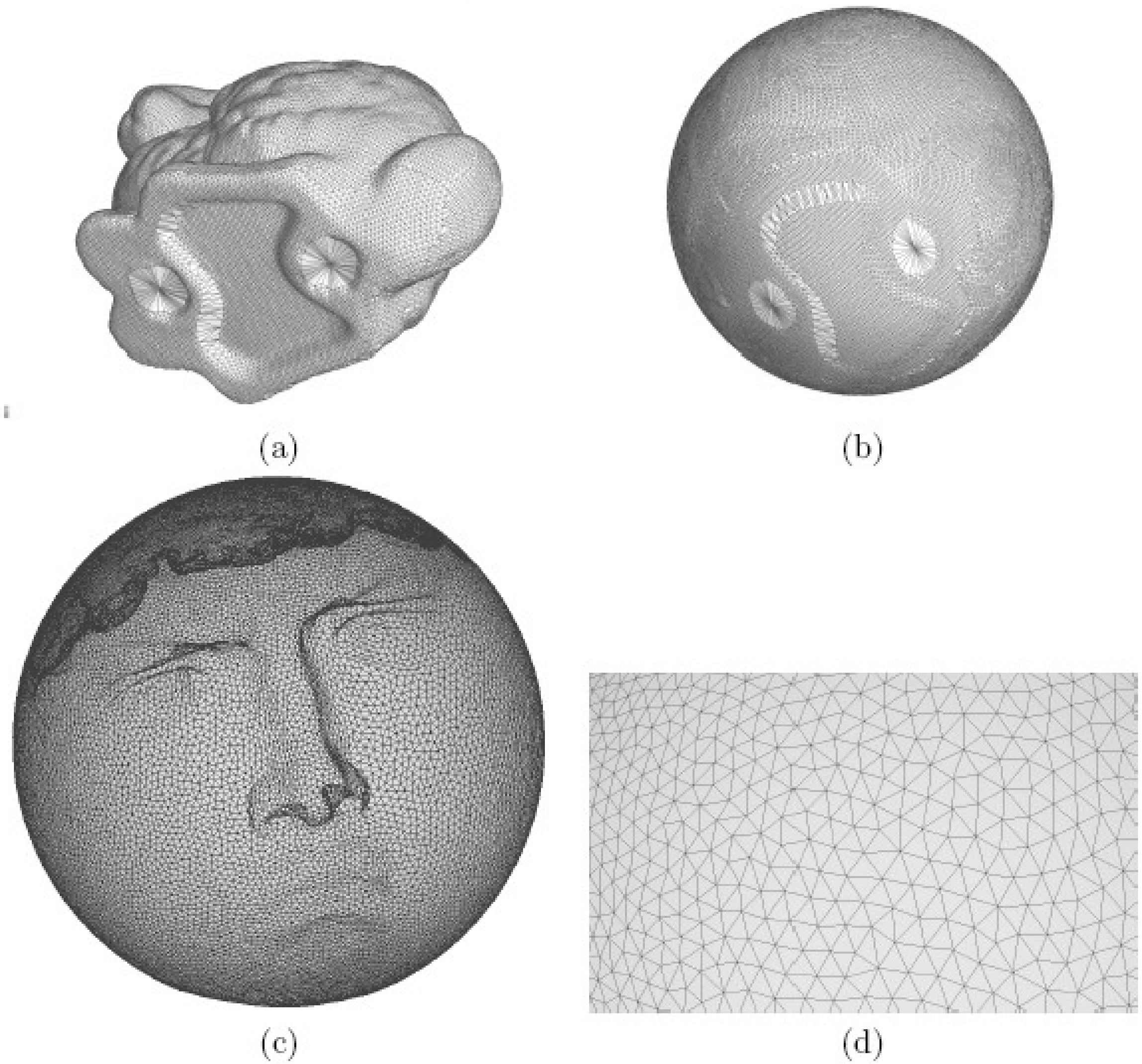


Progressive mesh representations

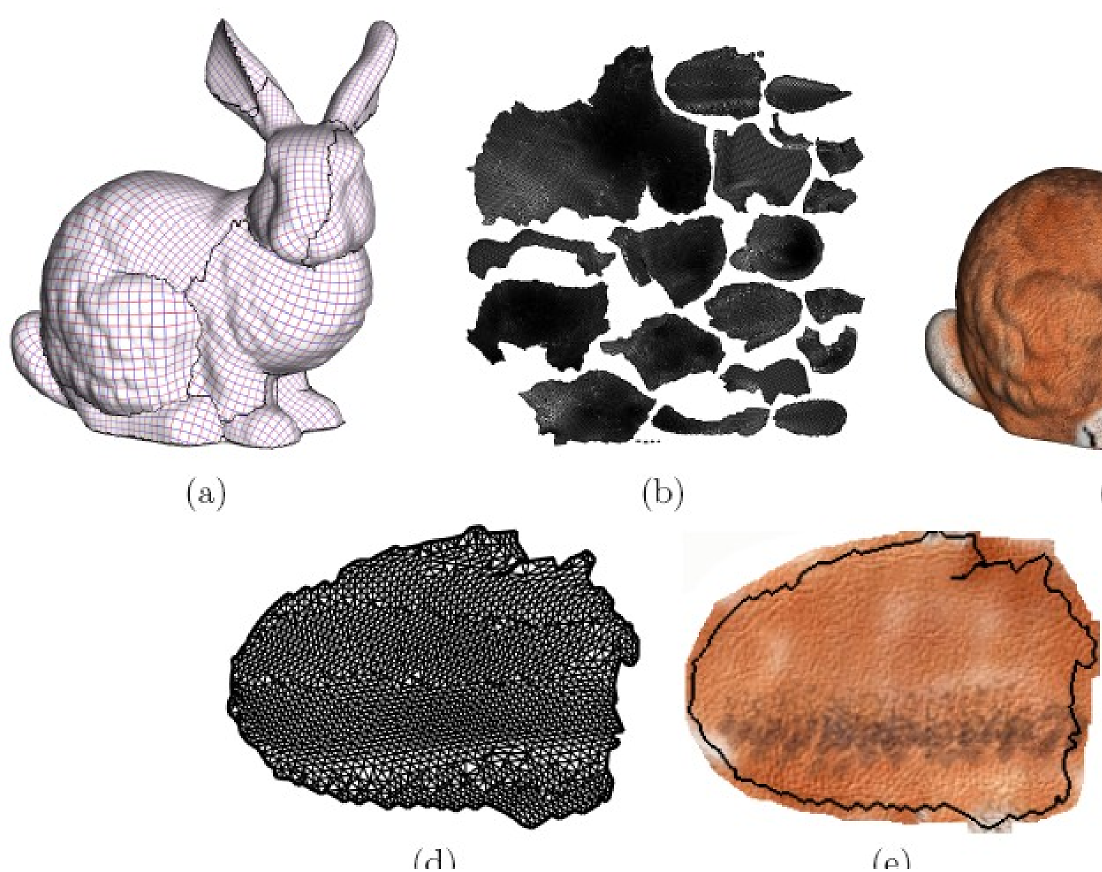


Level of details

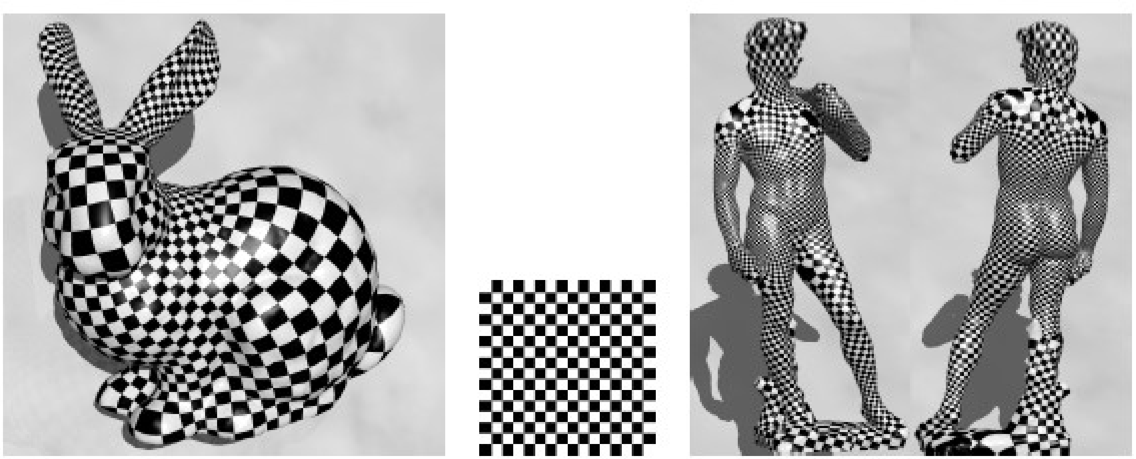
Parameterization and texture mapping



Parameterization and texture mapping



Minimize distortion

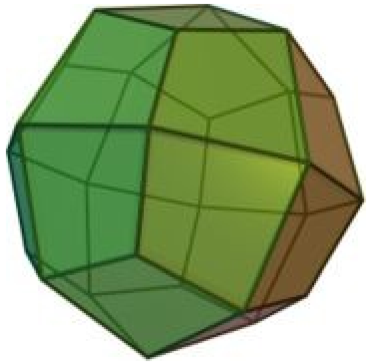


Conformal mapping
(preserve angles)

H- and V- representations of polytopes

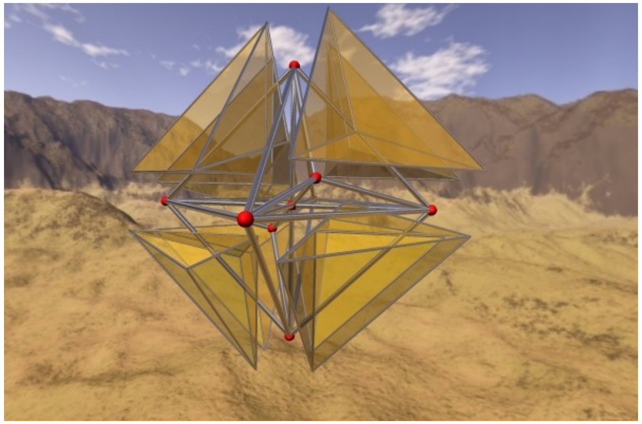
Half-spaces representation

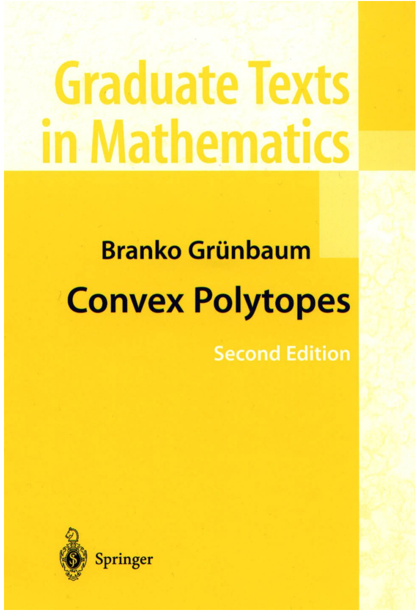
$$P = \{x \in \mathbb{R}^n : \ Ax \leq b\}$$



Vertex representation (convex hull)

$$\bar{P} = \{x \in \mathbb{R}^n : x = \sum_i \lambda_i x_i, \sum_i \lambda_i = 1, \lambda_i \geq 0\}$$





http://www.math.tu-berlin.de/polymake/index.html