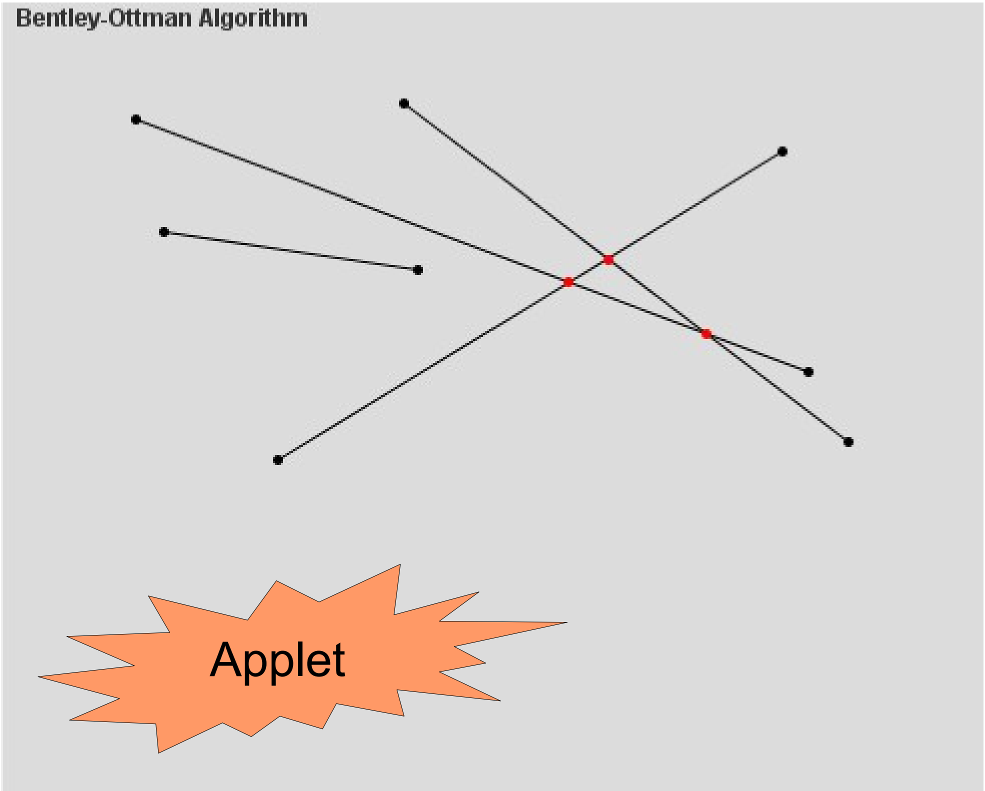
INF555

Fundamentals of 3D

Lecture 1 Follow-up/debriefing
Lecture 2: Convolutions and Filters
Matrix decompositions

Bentley-Ottman sweep line algorithm



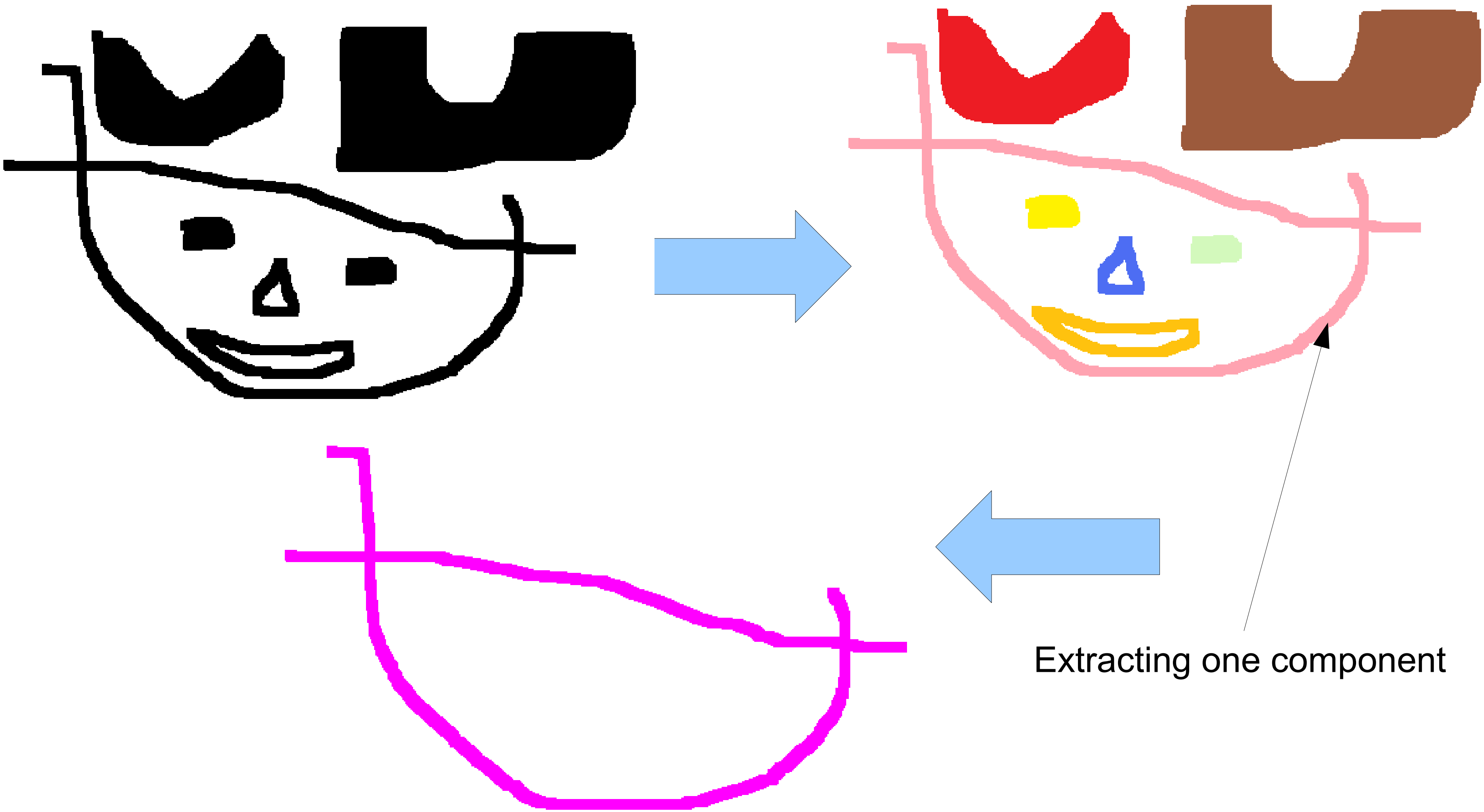
http://www.cs.uwaterloo.ca/~bjlafren/segint/index.htm

Complexity: Time O((n+I)log n), memory O(n+I)

(Augmented dictionary on sweep line)

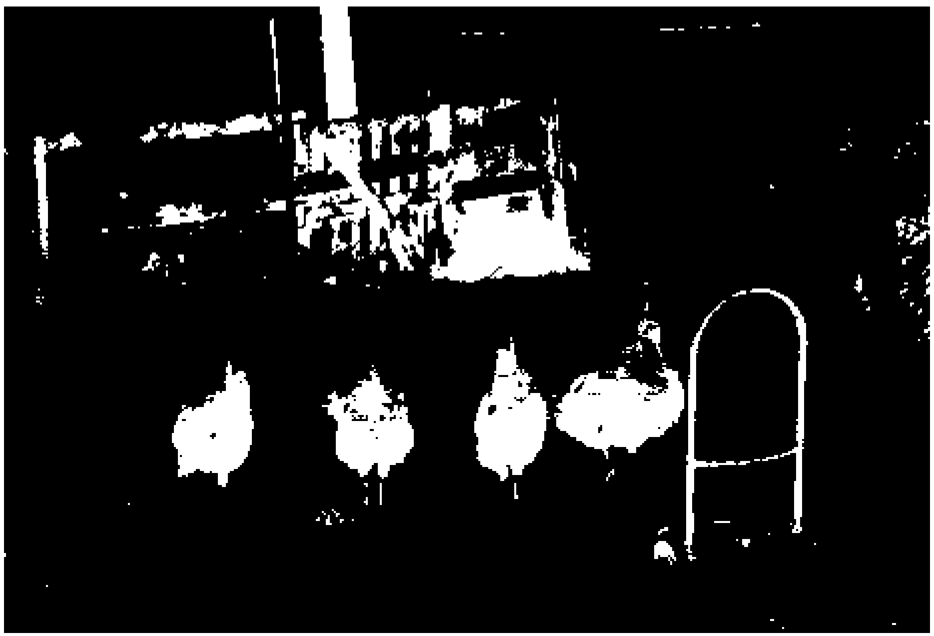
Memory/Time complexity of reporting intersections vs. enumerating them!

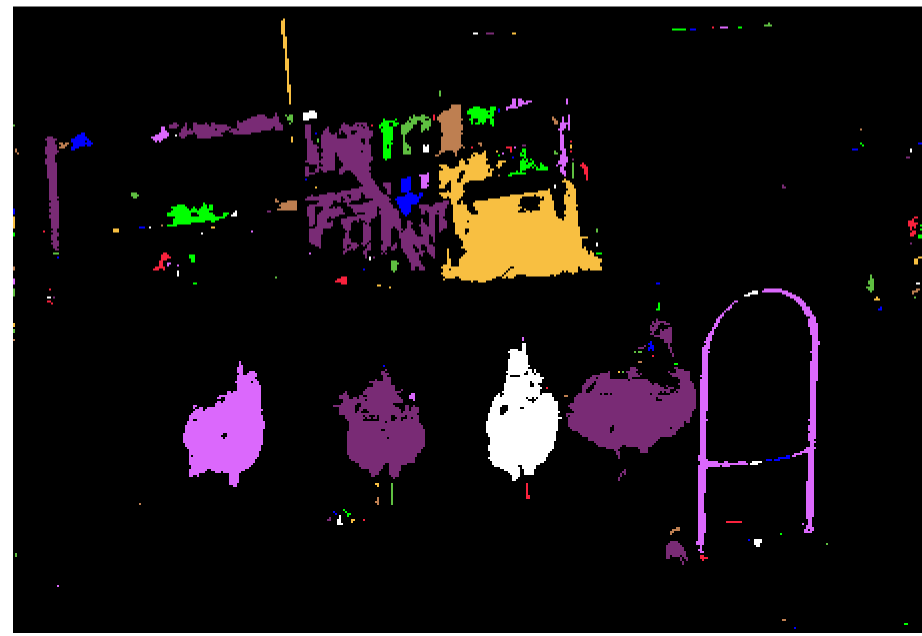
Labelling connected components (Union-Find)



Input: binary image Foreground/Background pixels
Output: Each connected component

Useful in computer vision...





How many (large) objects?

A simple algorithm

Initially each pixel defines its own region
(labelled by say the pixel index: x+y*width)

Scan the image from left to right and top to bottom:

If current pixel is background AND East pixel is background:
Merge their regions into one = « connect » them

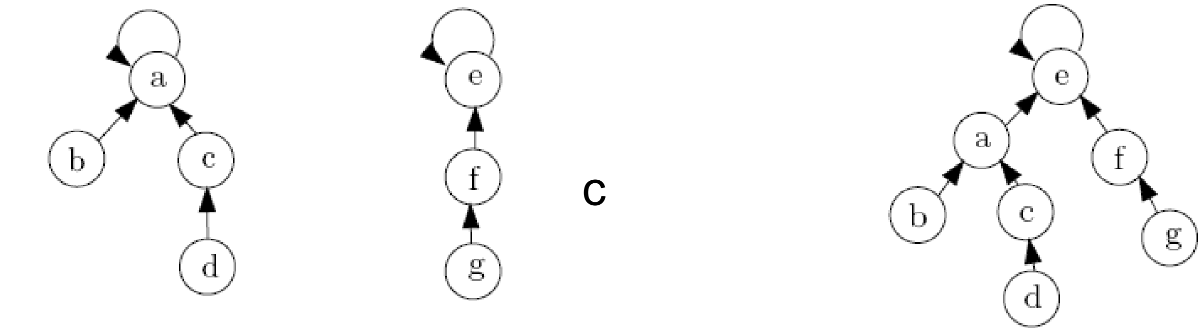
If current pixel is background AND South pixel is background:
Merge their regions into one = « connect » them

Extract regions (floodfilling or single pass algorithm)

How do we merge quickly disjoint sets?

Union-Find abstract data-structures

$\textsc{MakeSet}(x)$
1. $\text{parent}(x) \leftarrow x$
2. $\text{rank}(x) \leftarrow 0$

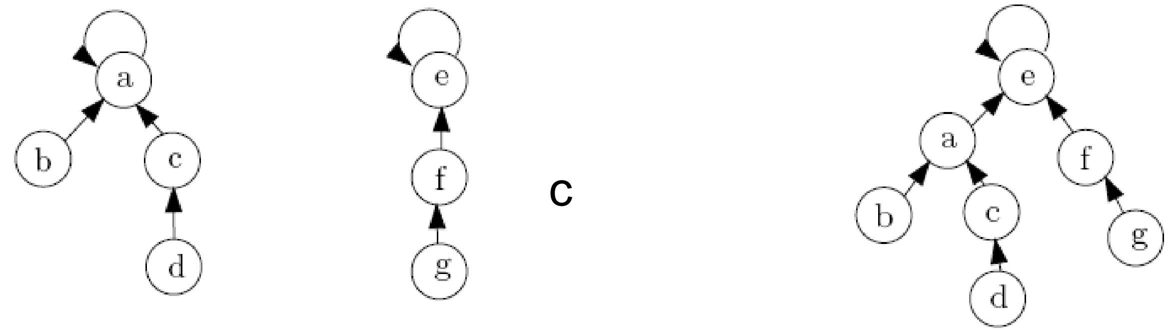


$\mathcal{S}_1 = \{a, b, c, d\}$ $\mathcal{S}_2 = \{e, f, g\}$ $\mathcal{S} = \mathcal{S}_1 \cup \mathcal{S}_2$

RANK=DEPTH

Visual Depiction

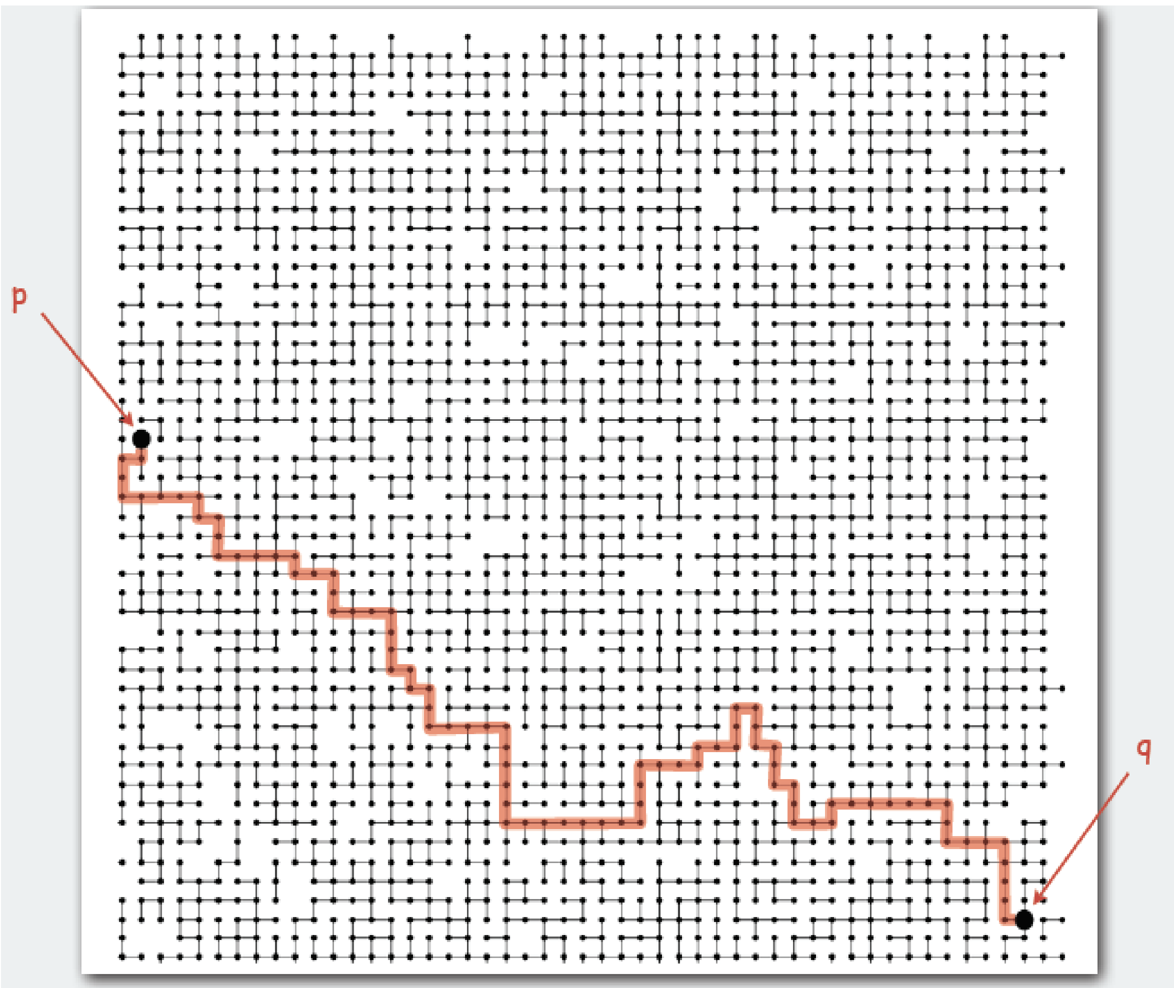
```
class UnionFind{
int  [ ] rank; int [ ] parent;

UnionFind(int n)
{int k;
parent=new int[n];
rank=new int[n];

for (k = 0; k < n; k++)
     {parent[k]   = k; rank[k] = 0;     }
}

int Find(int k)
{while (parent[k]!=k ) k=parent[k]; return k;}


int Union(int x, int y)
{
x=Find(x);y=Find(y);
if ( x == y ) return -1; // Do not perform union of same set

if (rank[x] > rank[y])
     {parent[y]=x;
       return x;}
     else
     { parent[x]=y;
      if (rank[x]==rank[y]) rank[y]++;return y;}
}
}
```



$\mathcal{S}_1 = \{a, b, c, d\}$ $\quad$ $\mathcal{S}_2 = \{e, f, g\}$ $\quad$ $\mathcal{S} = \mathcal{S}_1 \cup \mathcal{S}_2$

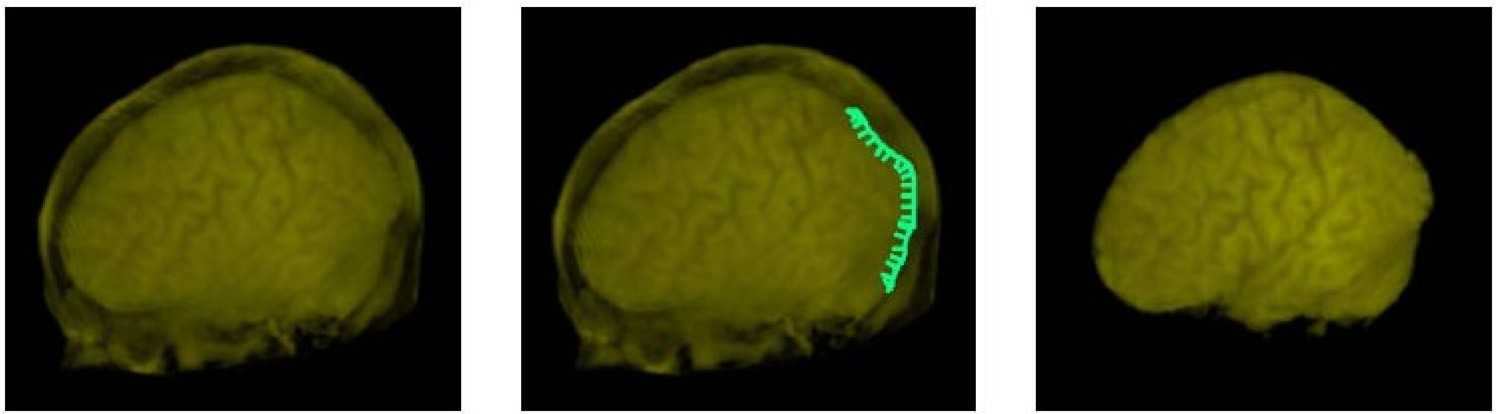
```
// Attach tree(y) to tree(x)
// else
// Attach tree(x) to tree(y)
       and if same depth, then increase depth of y by one
```

Many applications of the union-find data-structure



- Variable name aliases.
- Pixels in a digital photo.
- Computers in a network.
- Web pages on the Internet.
- Transistors in a computer chip.

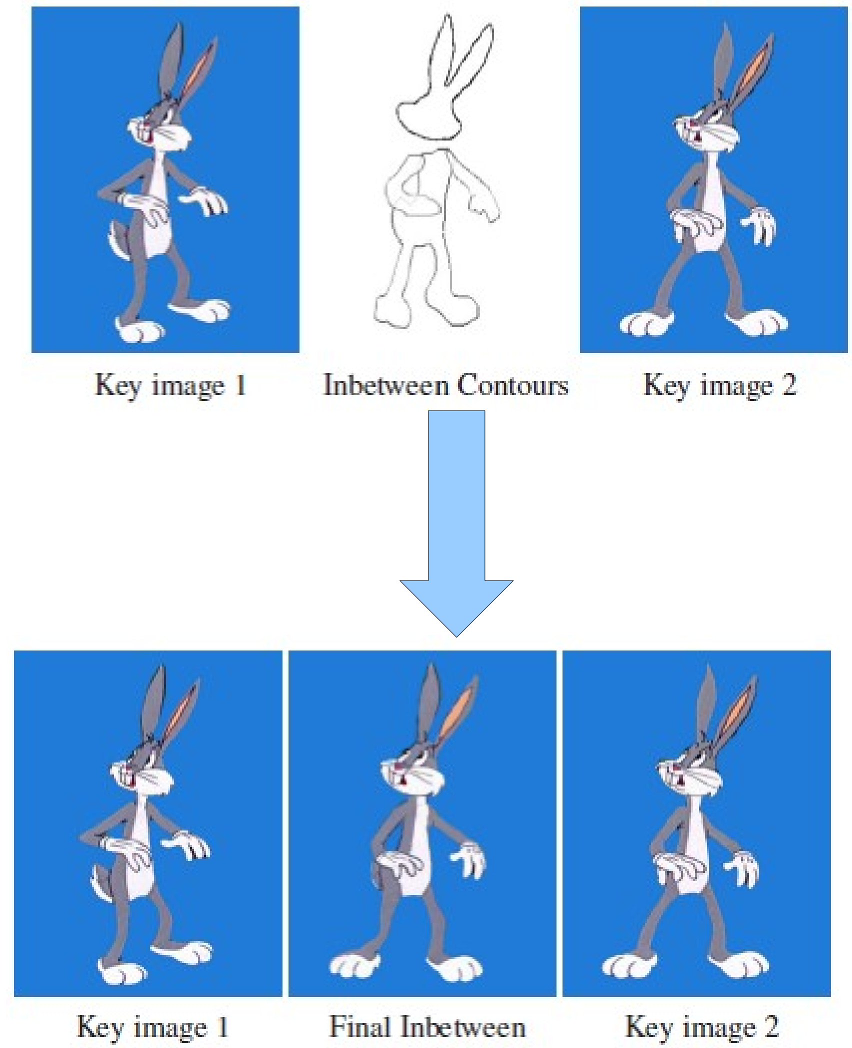
Another example of union-find/segmentation: Volume catcher



Video

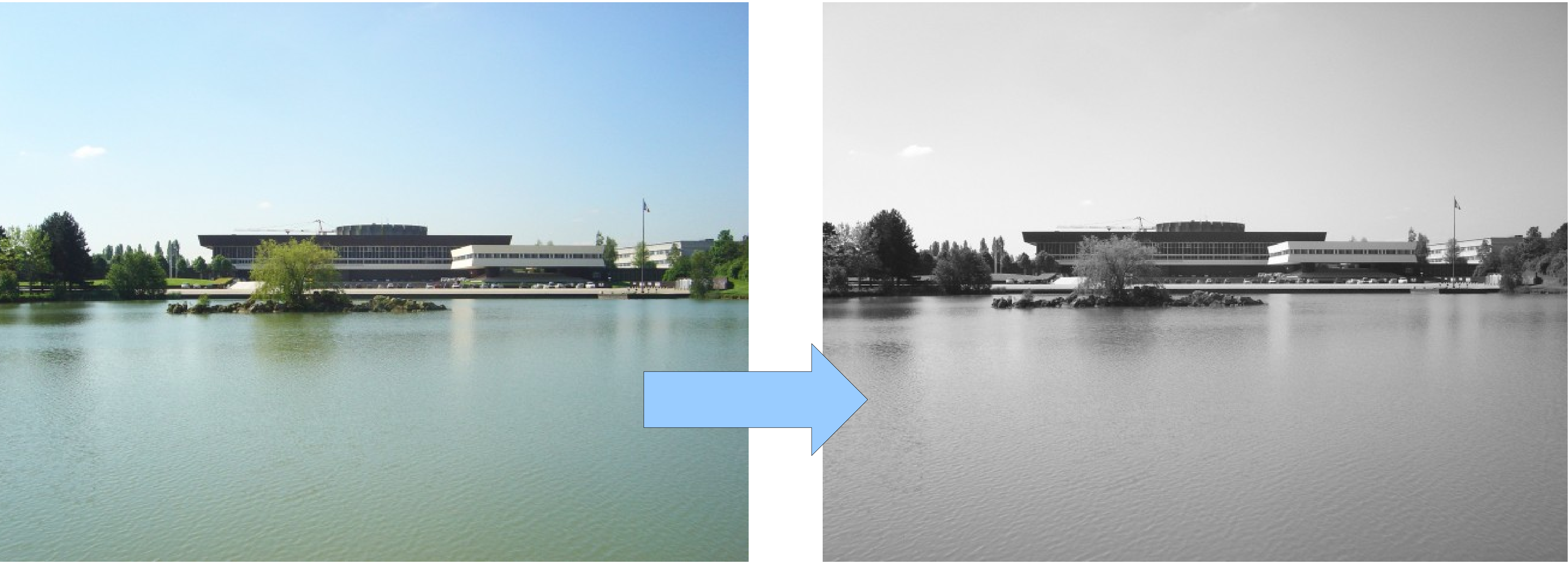
http://www-ui.is.s.u-tokyo.ac.jp/~o/VolumeCatcher/

Yet another example of UF/segmentation: Inbetweening for cell animation



http://www.vuse.vanderbilt.edu/~bobbyb/pubs/sca06.html

Convolutions et filtres



Intensity=0.3red+0.59green+0.11blue

Image Convolution

Roberts Cross (edge) detection

Discrete gradient (maximal response for edge at 45 degrees)

+1	0
0	-1

Gx

0	+1
-1	0

Gy

The two filters are 90 degrees apart (inner product is zero)

$$|G| = \sqrt{Gx^2 + Gy^2}$$

$$\theta = \arctan(Gy/Gx) - 3\pi/4$$

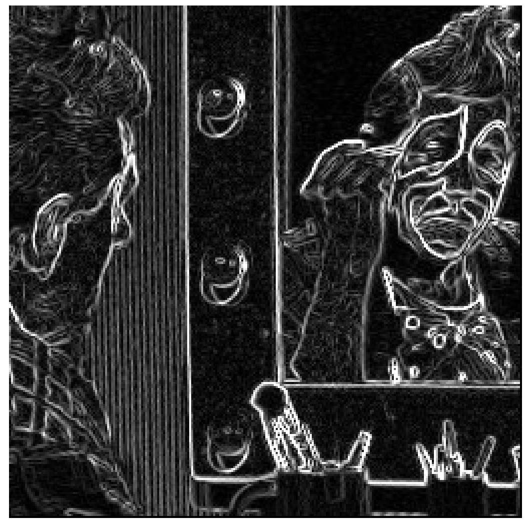
P_1	P_2
P_3	P_4

Approximated by

$$|G| = |P_1 - P_4| + |P_2 - P_3|$$

Roberts Cross edge detector

Input



Filter response (x5)

Thresholded

P_1	P_2
P_3	P_4

Approximated by

$$|G| = |P_1 - P_4| + |P_2 - P_3|$$

Sobel edge detector

-1	0	+1
-2	0	+2
-1	0	+1

Gx

+1	+2	+1
0	0	0
-1	-2	-1

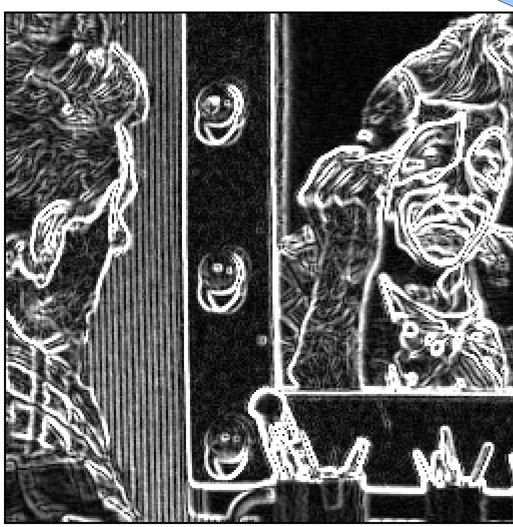
Gy

Approximated by

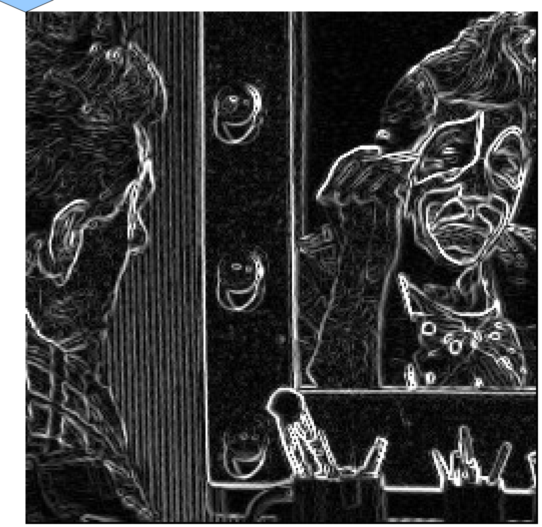
P_1	P_2	P_3
P_4	P_5	P_6
P_7	P_8	P_9

$$|G| = |(P_1 + 2 \times P_2 + P_3) - (P_7 + 2 \times P_8 + P_9)| + |(P_3 + 2 \times P_6 + P_9) - (P_1 + 2 \times P_4 + P_7)|$$

Sobel



Roberts Crossl

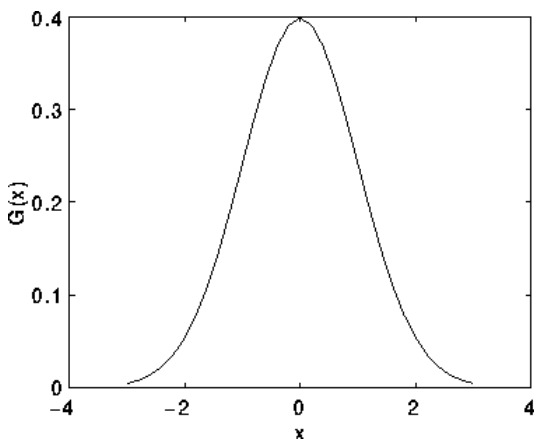


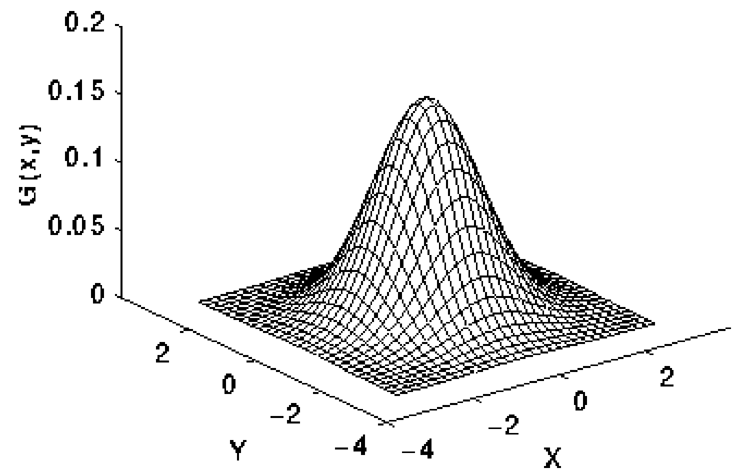
http://homepages.inf.ed.ac.uk/rbf/HIPR2/sobeldemo.htm

Gaussian smoothing

$$G(x,y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

$$G(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}}$$





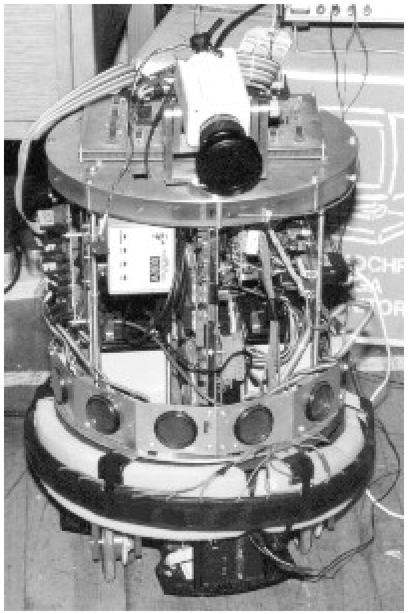
Normalize discrete Gaussian kernel to 1

Two parameters to tune:
- K, the dimension of the matric
- Sigma, the smoothing width...

$\frac{1}{273}$

1	4	7	4	1
4	16	26	16	4
7	26	41	26	7
4	16	26	16	4
1	4	7	4	1

Blurring, low-pass filter eliminates edges...



Source

Sigma=1, 5x5

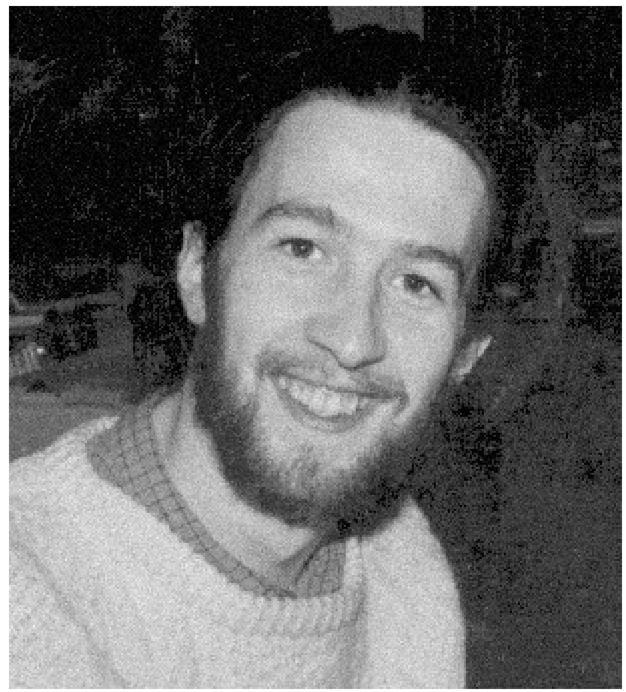
Sigma=2, 9x9

Sigma=4, 15x15

Mean filter= Uniform average

$$\begin{array}{|c|c|c|}\hline \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \hline \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \hline \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \hline \end{array}$$

Source



Corrupted, Gaussian noise (sigma=8)

Mean filter 3x3

Mean filter= Uniform average

$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$
$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$
$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$

Source

Corrupted, Gaussian noise (sigma=13)

Mean filter 3x3

Median filter

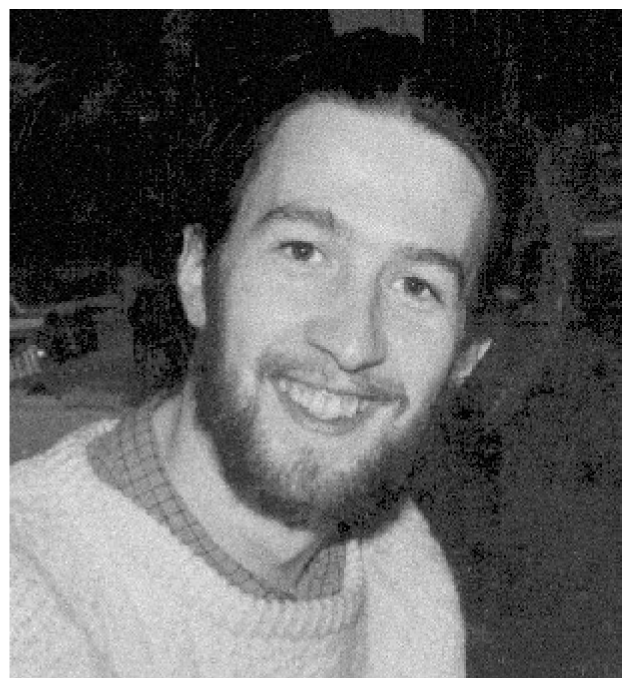
123	125	126	130	140
122	124	126	127	135
118	120	150	125	134
119	115	119	123	133
111	116	110	120	130

Neighbourhood values:

115, 119, 120, 123, 124,
125, 126, 127, 150

Median value: 124

Source



Corrupted, Gaussian noise (sigma=8)

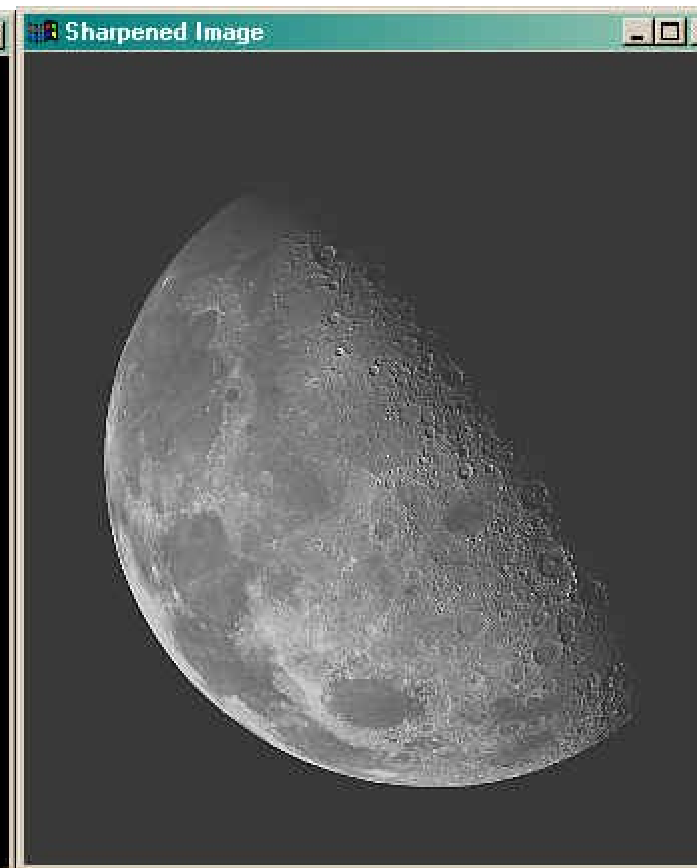
Median filter 3x3

Sharpening: Identity+Laplacian kernel

-1	-1	-1
-1	8	-1
-1	-1	-1

$$\mathbf{S} = \begin{bmatrix} 0 & -\lambda & 0 \\ -\lambda & 1+4\lambda & -\lambda \\ 0 & -\lambda & 0 \end{bmatrix}$$

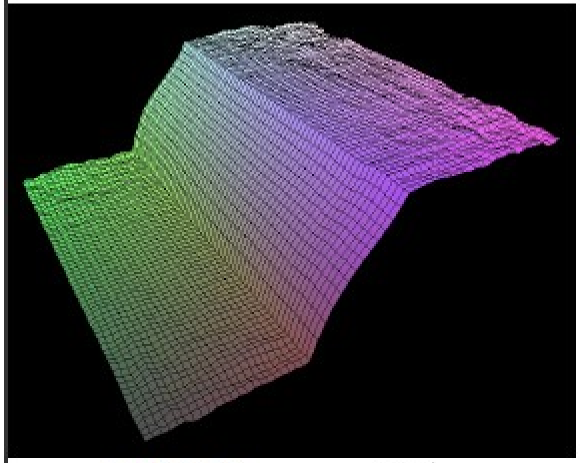
Source image

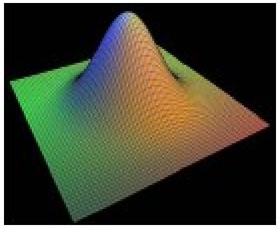


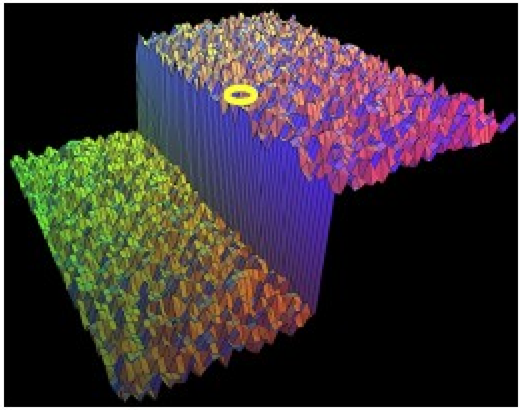
Bilateral filtering

Traditional spatial gaussian filtering

$$J(x) = \sum_{\xi} f(x,\xi) \, I(\xi)$$





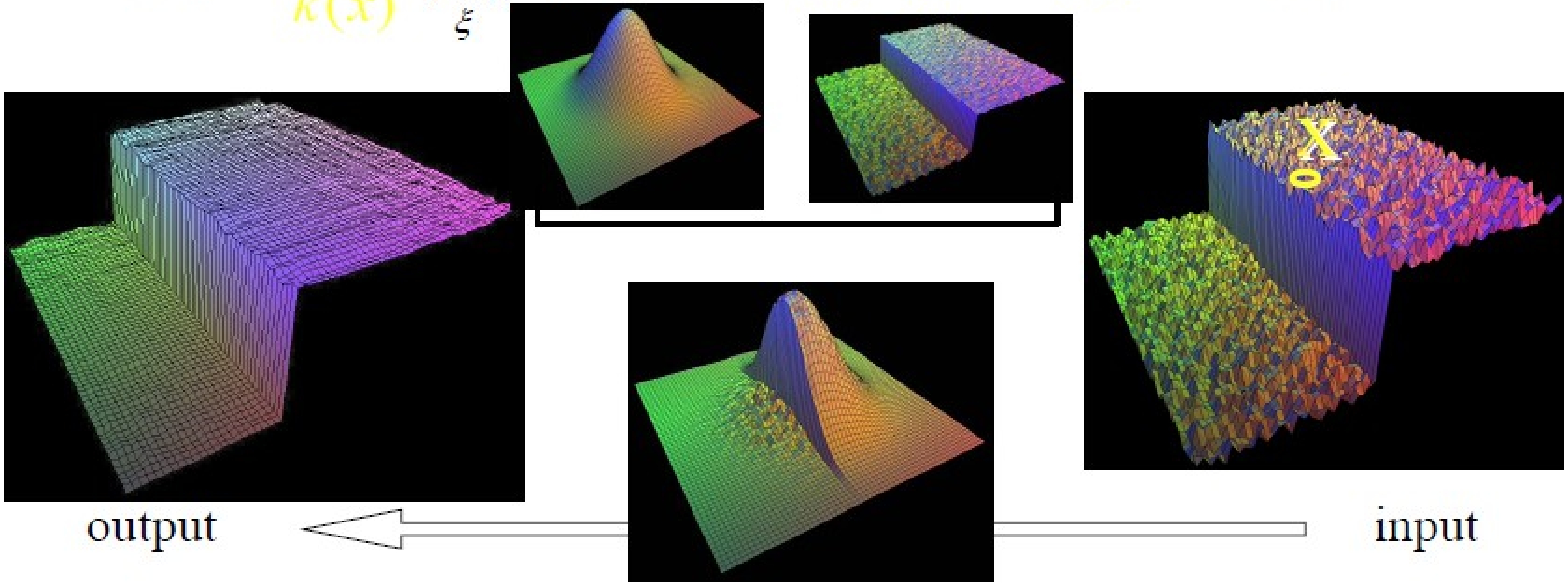


output ⟸ input

Bilateral filtering

New! gaussian on the intensity difference filtering

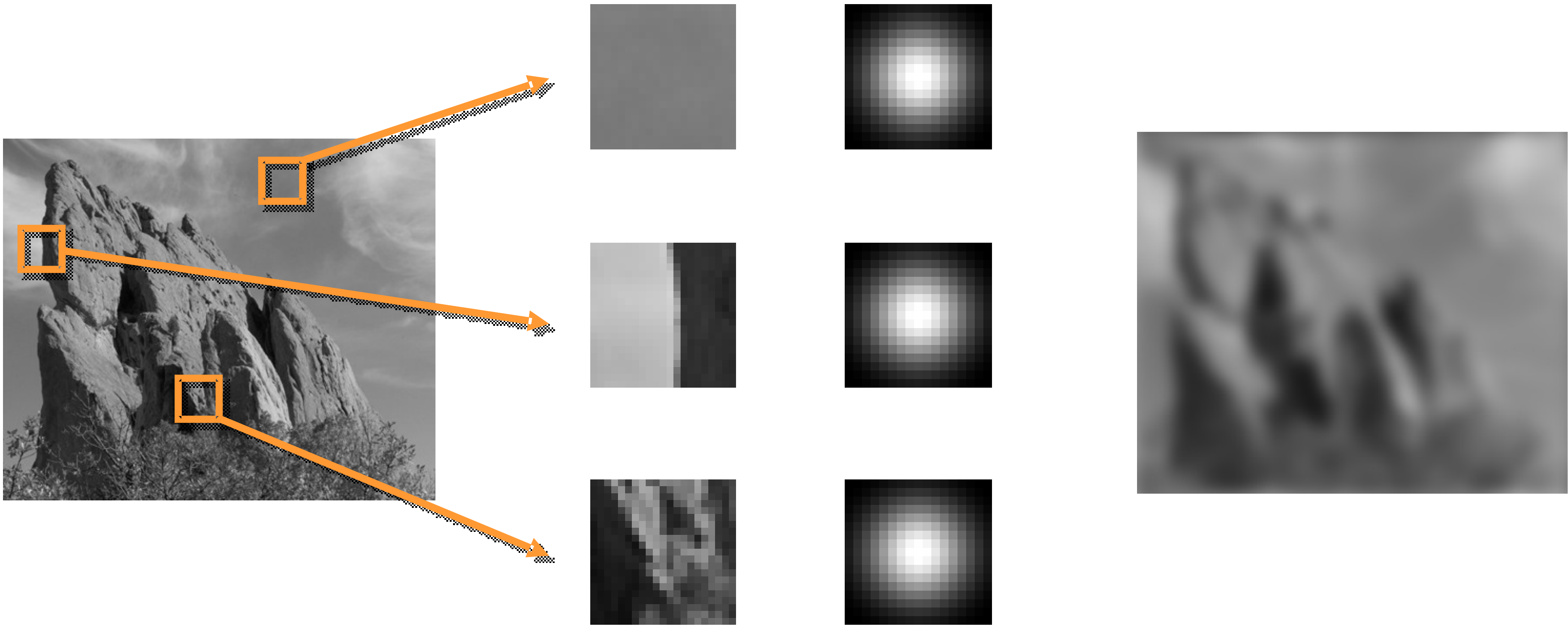
$$J(x) = \frac{1}{k(x)} \sum_{\xi} f(x,\xi) \quad g(I(\xi)-I(x)) \quad I(\xi)$$

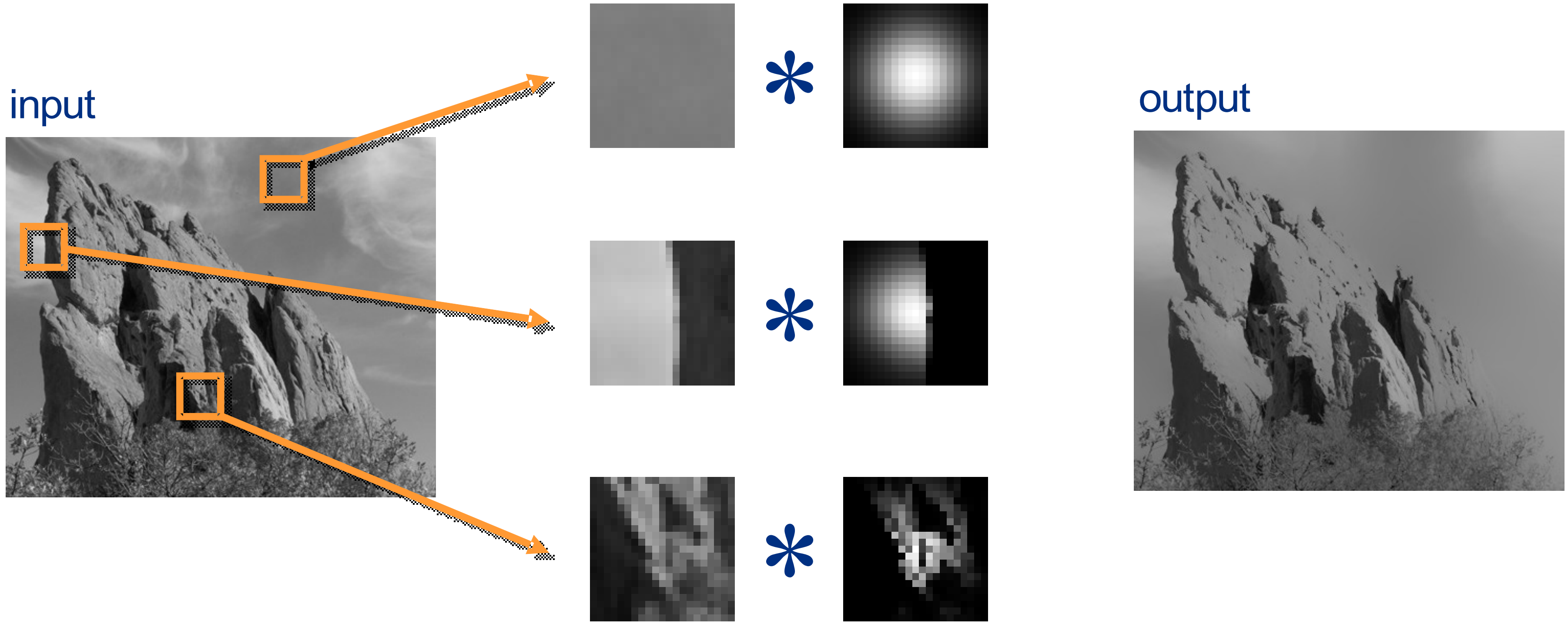


Bilateral Filtering for Gray and Color Images, **Tomasi and Manduchi 1998**
.... SUSAN feature extractor...

Bilateral filtering

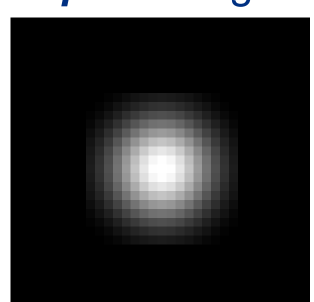
Traditional spatial gaussian filtering

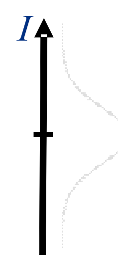




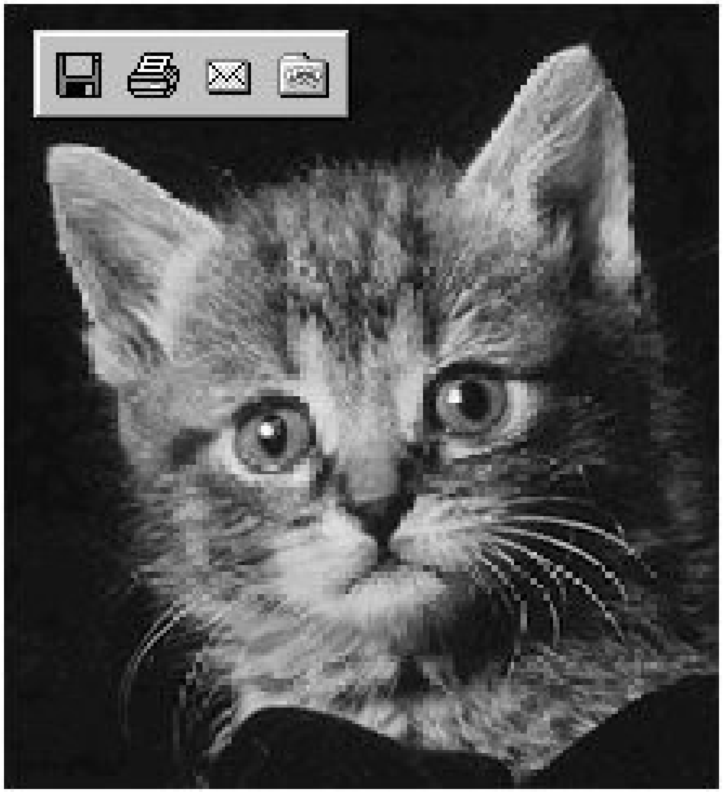
The kernel shape depends on the image content.

$$BF[I]_{\mathbf{p}} = \underbrace{\overbrace{\frac{1}{W_{\mathbf{p}}}}^{\text{new}}}_{\text{normalization factor}} \sum_{\mathbf{q}\in S} \underbrace{\overbrace{G_{\sigma_s}(\|\mathbf{p}-\mathbf{q}\|)}^{\text{not new}}}_{\textbf{\textit{space}}\text{ weight}} \underbrace{\overbrace{G_{\sigma_r}(|I_{\mathbf{p}}-I_{\mathbf{q}}|)}^{\text{new}}}_{\textbf{\textit{range}}\text{ weight}} I_{\mathbf{q}}$$





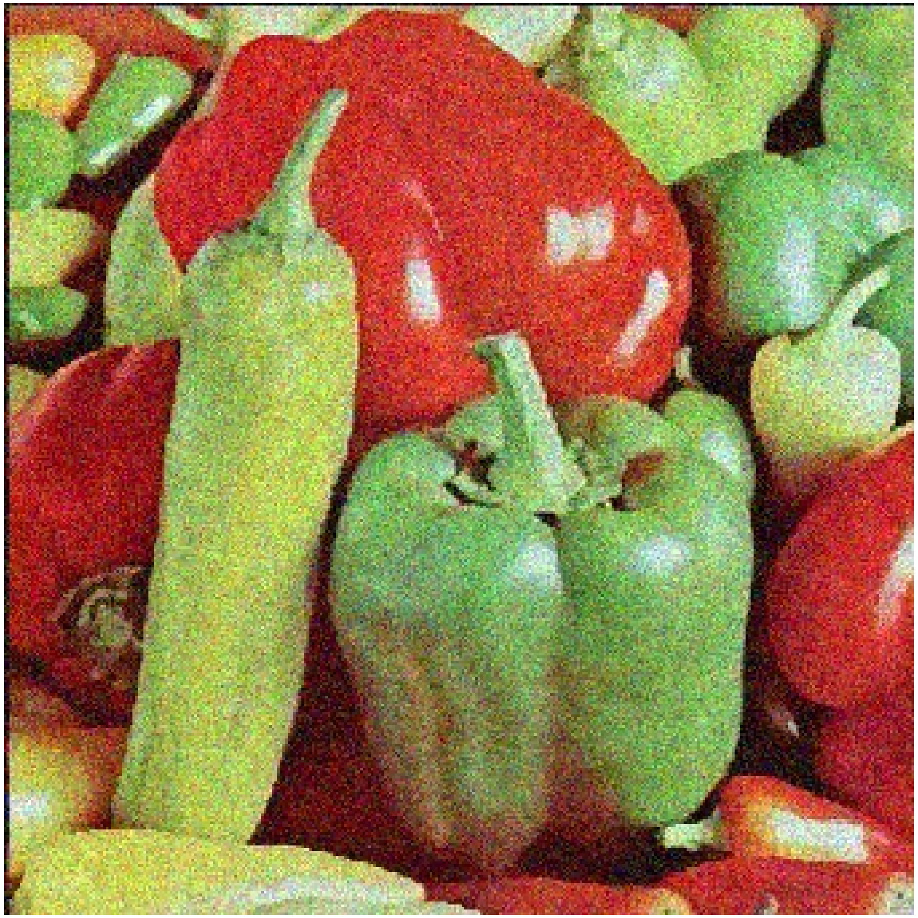
Bilateral filtering



- Example of Bilateral filtering
- Low contrast texture has been removed

•Brute-force implementation is slow > 10min (but real-time with GPU and better algorithms)

Bilateral filtering



Origin image

Bilateral (3)

Results

Origin Image

One iteration

five iterations

Bootstrapping

http://people.csail.mit.edu/sparis/siggraph07_course/

Bilateral filtering extends to meshes

Source

Bilateral mesh denoising

Harris-Stephens edge detector

Aim at finding good feature

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

Gradient with respect to x, times gradient with respect to y

Sum over image region – area we are checking for corner

Harris-Stephens edge detector

Measure the corner response as

$$\boxed{R = \det M - k\left(\operatorname{trace} M\right)^2}$$

$$\boxed{\begin{aligned}\det M &= \lambda_1 \lambda_2 \\ \operatorname{trace} M &= \lambda_1 + \lambda_2\end{aligned}}$$

Avoid computing eigenvalues themselves.

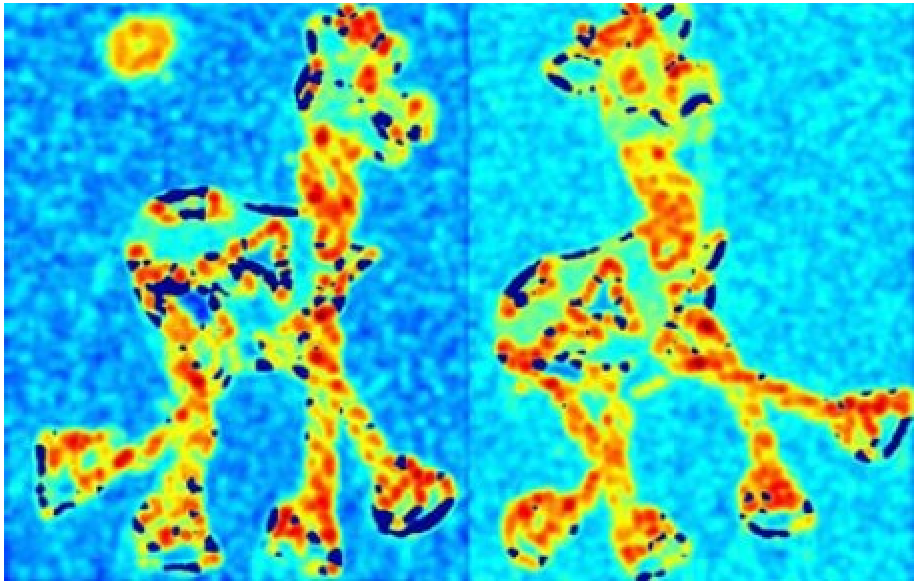
(k – empirical constant, k = 0.04-0.06)

Algorithm:

– Find points with large corner response function R (R > threshold)

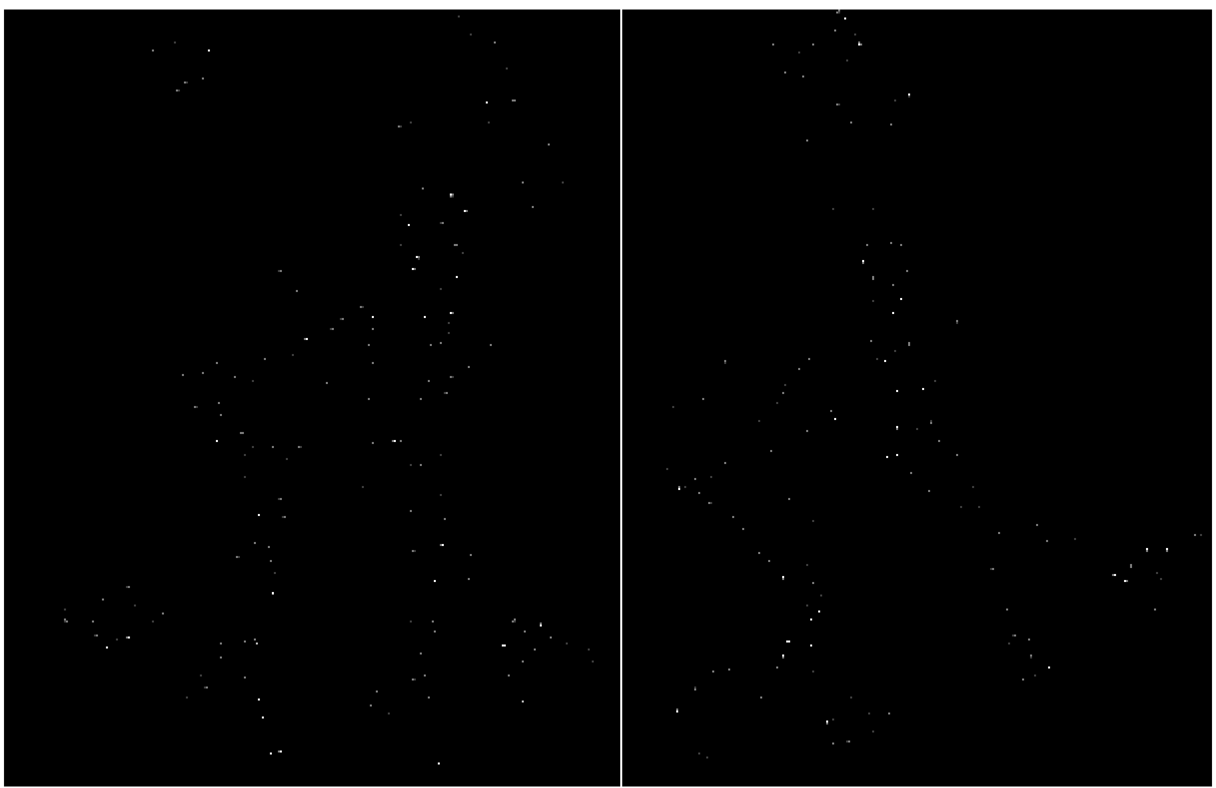
– Take the points of **local maxima** of R

Harris-Stephens edge detector

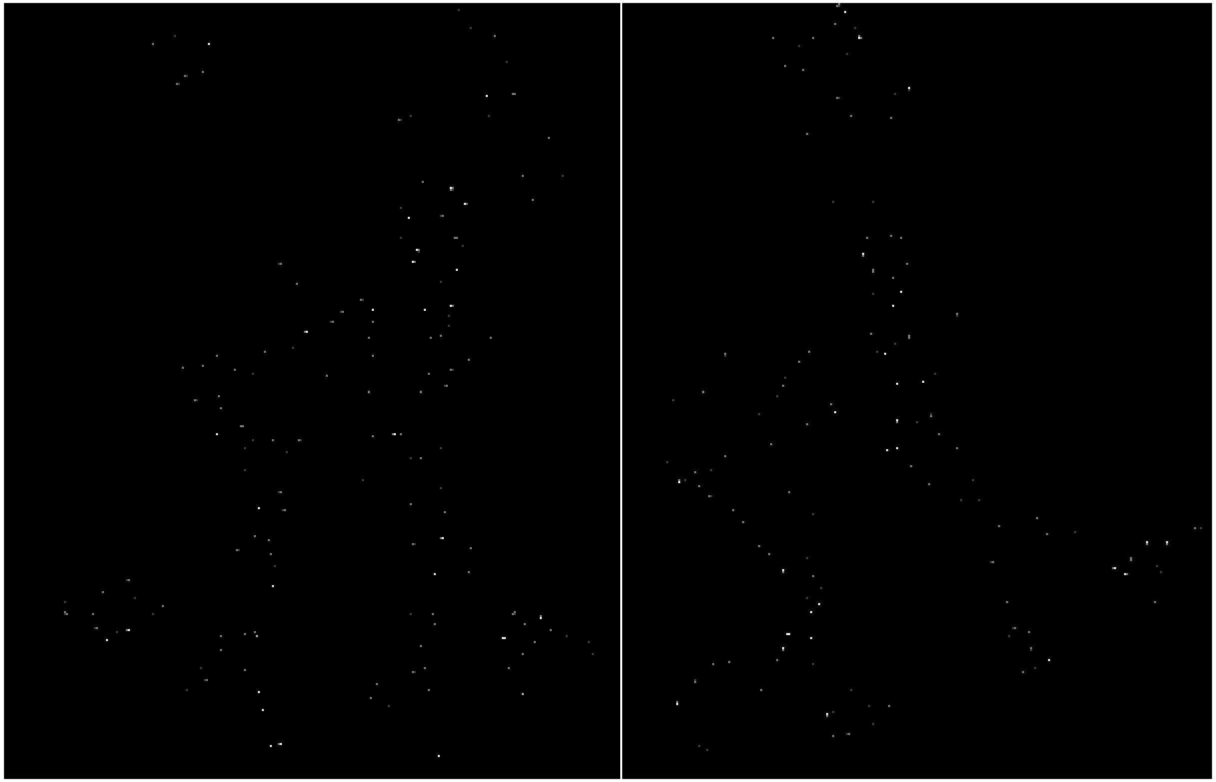


Corner response R

Thresholding R>cste



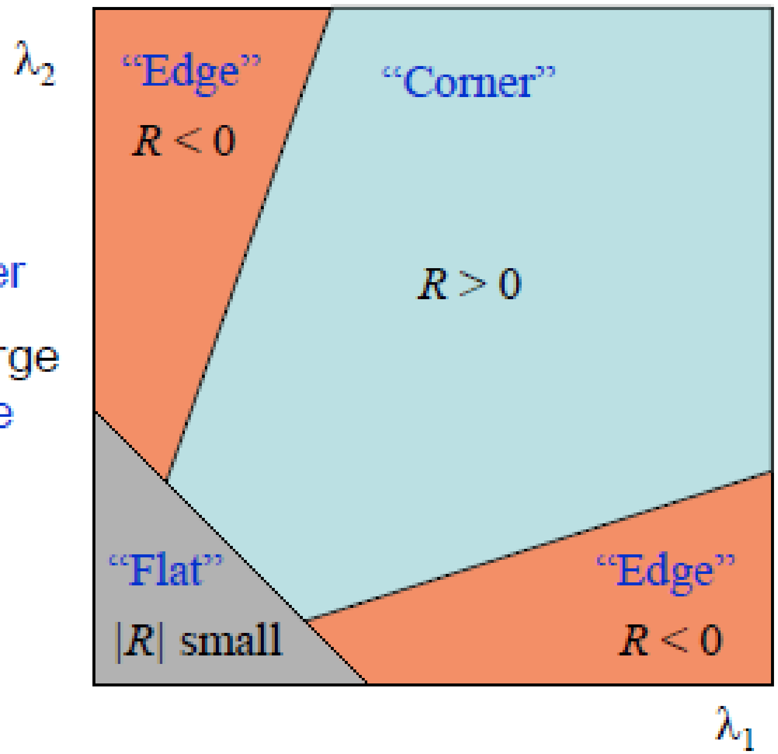
Local maxima of R

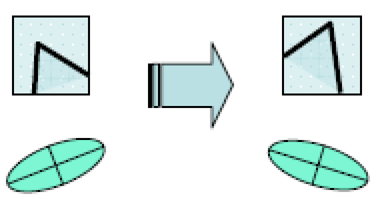


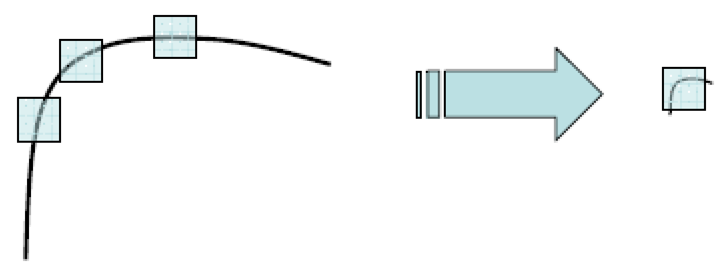
Superposing local maxima on source images

Invariant to orientation but
Depends on the scaling of the image

- R depends only on eigenvalues of M
- R is large for a corner
- R is negative with large magnitude for an edge
- $|R|$ is small for a flat region







Application to feature matching for image panorama tools

http://www.ptgui.com/download.html

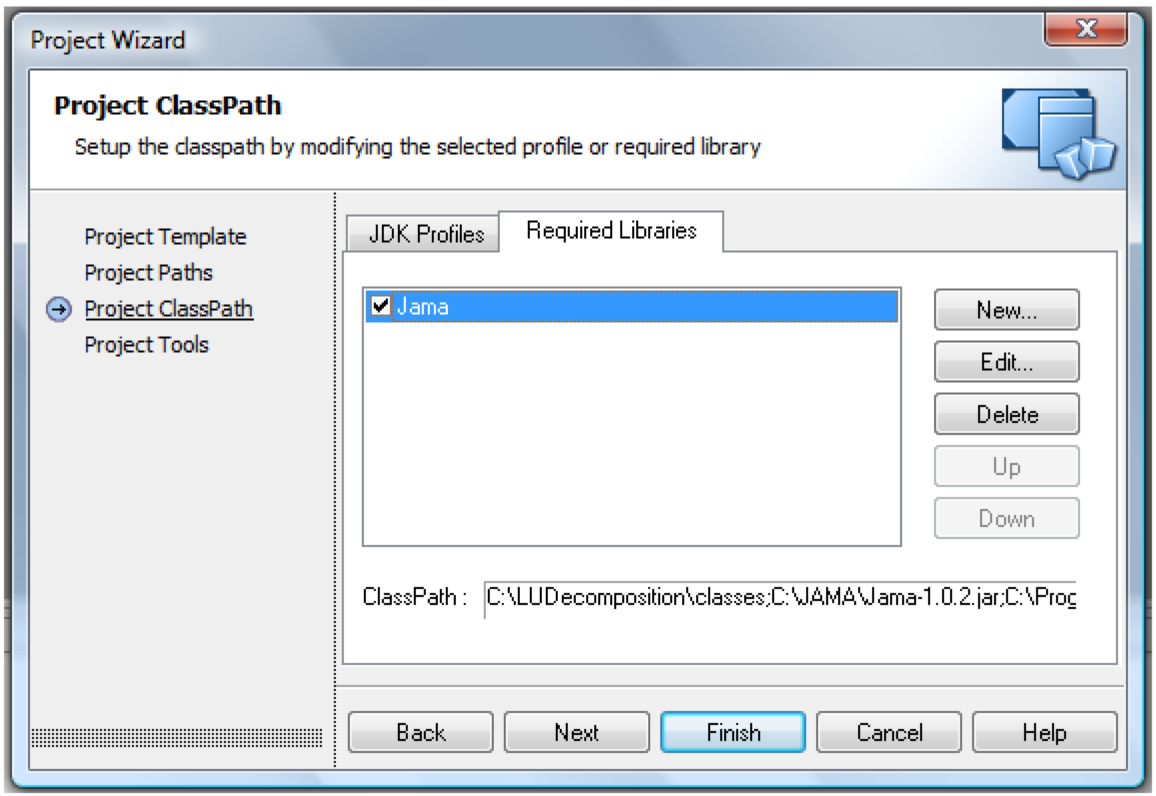
Matrix operations using JAMA

JAMA : A Java Matrix Package

Use JAMA library

javac -*classpath Jama-1.0.2.jar* filename.java

Using JCreator



http://math.nist.gov/javanumerics/jama/

Summary of JAMA Capabilities

Object Manipulation	constructors set elements get elements copy clone
Elementary Operations	addition subtraction multiplication scalar multiplication element-wise multiplication element-wise division unary minus transpose norm
Decompositions	Cholesky LU QR SVD symmetric eigenvalue nonsymmetric eigenvalue
Equation Solution	nonsingular systems least squares
Derived Quantities	condition number determinant rank inverse pseudoinverse

Write a class wrapper around Jama

Essential operations are:

CholeskyDecomposition	Cholesky Decomposition.
EigenvalueDecomposition	Eigenvalues and eigenvectors of a real matrix.
LUDecomposition	LU Decomposition.
Matrix	Jama = Java Matrix class.
QRDecomposition	QR Decomposition.
SingularValueDecomposition	Singular Value Decomposition.

http://www.bluebit.gr/matrix-calculator/

LU Decomposition of rectangular matrices

Product of a lower and upper triangular matrices

$$A = LU$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}.$$

$$\begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ -1/2 & 1 & 0 \\ 0 & -2/3 & 1 \end{pmatrix} . \begin{pmatrix} 2 & -1 & 0 \\ 0 & 3/2 & -1 \\ 0 & 0 & 4/3 \end{pmatrix}$$

LU Decomposition of rectangular matrices

$$P^{-1}A = LU$$

P permutation matrix

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

LU Decomposition for solving linear systems:

$$A\{x\} = \{b\}$$

$$LU\{x\} = \{b\}$$

$$L\{U.x\} = \{b\}$$

$$\{U.x\} = \{y\}$$

$$L\{y\} = \{b\}$$

Trivial to solve since L is lower triangular matrix

```java
Matrix A=Matrix.random(3,3);
Matrix b = Matrix.random(3,1);
Matrix x= A.solve(b);

System.out.println("A="); A.print(6,3);
System.out.println("b="); b.print(6,3);
System.out.println("x="); x.print(6,3);

LUDecomposition luDecomp=A.lu();
Matrix L=luDecomp.getL();
Matrix U=luDecomp.getU();
int [ ] pivot= luDecomp.getPivot();

System.out.println("L="); L.print(6,3);
System.out.println("U="); U.print(6,3);
for(int i=0;i<pivot.length;i++)
    System.out.print(pivot[i]+" ");
System.out.println("");
```

```
A=

  0.345   0.090   0.599
  0.932   0.428   0.703
  0.420   0.089   0.439

b=

  0.342
  0.192
  0.806

x=

  4.432
 -7.913
 -0.791

L=

  1.000   0.000   0.000
  0.450   1.000   0.000
  0.370   0.660   1.000

U=

  0.932   0.428   0.703
  0.000  -0.103   0.122
  0.000   0.000   0.258

1 2 0
```

QR Decomposition

$$A = QR,$$

Q: Orthogonal matrix
R: Upper triangular matrix

Useful for solving least squares problem

```
import Jama.*;

class QRDecompositionTest
{
public static void main(String args[])
{
Matrix A=Matrix.random(3,3);
QRDecomposition qr=new QRDecomposition(A);
System.out.println("A=");A.print(6,3);
Matrix Q=qr.getQ();
System.out.println("Q=");Q.print(6,3);
Matrix R=qr.getR();
System.out.println("R=");R.print(6,3);
}
}
```

```
A=

  0.821  0.098  0.374
  0.057  0.151  0.036
  0.994  0.150  0.703

Q=

 -0.637  0.131  0.760
 -0.044 -0.990  0.134
 -0.770 -0.052 -0.636

R=

 -1.290 -0.185 -0.781
  0.000 -0.145 -0.023
  0.000  0.000 -0.158
```

Cholesky decomposition

$$\mathbf{A} = \mathbf{L}\mathbf{L}^*,$$

For a **symmetric positive definite matrix** A,
Cholesky decomposition yields:

L is a lower triangular matrix
L* is the conjugate transpose

```java
import Jama.*;

class CholeskyDecompositionTest
{
public static void main(String args[])
{
double [][] array=new double[3][3];
int i,j;

for(i=0;i<3;i++)
	for(j=0;j<=i;j++){
		array[i][j]=Math.random();
		array[j][i]=array[i][j];}

Matrix A=new Matrix(array);
CholeskyDecomposition cd=new CholeskyDecomposition(A);

System.out.println("A=");A.print(6,3);
Matrix L=cd.getL();
System.out.println("L=");L.print(6,3);

if (cd.isSPD())
	{L.times(L.transpose()).print(6,3);}



}
}
```

```
A=

  0.688   0.342   0.046
  0.342   0.411   0.345
  0.046   0.345   0.962

L=

  0.829   0.000   0.000
  0.412   0.491   0.000
  0.055   0.658   0.725


  0.688   0.342   0.046
  0.342   0.411   0.345
  0.046   0.345   0.962
```