Here follows a list of OR exercises of the difficulty and type that is likely to arise in the exam.

1 Optimization on graphs

1. Apply Prim’s algorithm to the graph $G = (V, E)$ below to find the spanning tree of minimum cost. Describe each step of the algorithm graphically. Now suppose the cost $c_{45}$ on the edge $\{4, 5\}$ is set to the value 12. How can you modify the spanning tree so that it is optimal with respect to this edge cost change, without having to re-apply Prim’s algorithm from the start? [Liberti]

2. Compute the cost of the Fundamental Cycle Basis associated to both spanning trees found in Exercise 1. Can you find a Fundamental Cycle Basis with lower cost? [Liberti]

3. Dijkstra’s algorithm finds all shortest paths from a given root vertex to all other vertices on a directed graph. How do you proceed to apply it to an undirected graph? Apply Dijkstra’s algorithm from root vertex 1 to the graph $G = (V, E)$ of Exercise 1. [Liberti]

4. Describe an algorithmic procedure to find a shortest path between two given vertices $s, t \in V$ on a weighted undirected graph $G = (V, E)$, and apply it to the graph of Exercise 1, first with $s = 1, t = 5$, then with $s = 1, t = 7$. [Liberti]

5. In the graph $G = (V, E)$ of Exercise 1, let the cost $c_{25}$ on the edge $\{2, 5\}$ take the value $-6$. What is the shortest path from vertex 1 to vertex 6? [Liberti]

2 Dynamic programming

No mock question was readied in this section. Either contact Philippe Baptiste directly, or just carry out some exercises on this subject in any operations research book.
3 Linear programming

1. Give the mathematical programming formulation of an optimization problem with exactly two distinct local minima whose objective function values are 0 and (respectively) 1. [Liberti]

2. Give the mathematical programming formulation of an optimization problem with variables vector $x \in \mathbb{R}^3$ whose feasible region has volume $\sqrt{2}$. [Liberti]

3. Write a linear programming formulation of a problem with infinitely many local minima. Are these minima also global? [Liberti]

4. Write a linear programming formulation of a problem in canonical form with variables vector $x \in \mathbb{R}^2$ where all the vertices of the feasible region are degenerate. [Liberti]

5. Given a polyhedron $K = \{ x \in \mathbb{R}^n \mid Ax = b \land x \geq 0 \}$ in standard form, reformulate it to the corresponding polyhedron $K'$ in canonical form. [Liberti]

6. Find an example of a linear programming problem where the simplex algorithm passes from a basic feasible solution $x$ to a feasible solution $x'$ where both $x, x'$ correspond to the same vertex of the feasible polyhedron. [Liberti]

7. Solve the linear programming problem

$$\begin{align*}
\text{max } & \quad 7x_1 + 8x_2 \\
\text{s.t. } & \quad x_1 + 2x_2 + x_3 = 2 \\
& \quad 2x_1 + x_2 \leq 2 \\
& \quad 3x_1 + x_2 \leq 3 \\
& \quad \forall i \leq 3 \quad x_i \geq 0.
\end{align*}$$

Draw a picture in $\mathbb{R}^2$ of its feasible region. [Liberti]

8. Write the dual of problem (1)-(5). Using the Karush-Kuhn-Tucker conditions, prove that the point $(0, 0, 2)$ of the problem is not optimal. [Liberti]

4 Integer Programming

1. Total unimodularity. Are the following matrices totally unimodular or not?

$$A_1 = \begin{pmatrix}
1 & 0 & 1 & 0 & 1 \\
0 & 1 & 1 & 1 & 0 \\
0 & 0 & 0 & 1 & 1 \\
1 & 1 & 0 & 0 & 0
\end{pmatrix}, \quad A_2 = \begin{pmatrix}
-1 & 1 & -1 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1
\end{pmatrix}.$$

[Sadykov]

2. Convex hull. Consider the set $X = \{ x \in \mathbb{Z}^2_+ \mid \quad x_1 - x_2 \geq -1, 2x_1 + 6x_2 \leq 15, x_1 - x_2 \leq 3, 2x_1 + 4x_2 \leq 7 \}$. List and represent graphically the set of feasible points. Use this to find inequalities which describe the convex hull of $X$. [Sadykov]

3. Gomory inequalities. Prove that $x_2 + x_3 + 2x_4 \leq 6$ is valid for

$$X = \{ x \in \mathbb{Z}^4_+ \mid 4y_1 + 5y_2 + 9y_3 + 12y_4 \leq 34 \}.$$

[Sadykov]
5 Modelling

Look at the exercises in Chapter 4 of the course’s Exercise Book.