Formulating the Alternating Current Optimal Power Flow problem

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Outline

The quantities

- charge (basic measure)
- current $I =$ charge per surface unit per second
- electric field $=$ force vector at point acting on unit charge
- voltage $V =$ potential energy of unit charge in electric field
- power $S =$ voltage $\times$ current (in units of measure)

[Bienstock 2016, p. 2]

Optimal Power Flow

- Decide power flows on electrical cables to minimize costs
- Alternating Current:
  generated by magnetic field induced by a 50-60 Hz mechanical rotation
- Current traverses grid 50 to 60 times per second
  consider average over time
- ACOPF: static approximation of a dynamic problem
  approximation yields modelling/numerical difficulties

  Example 1: lines directed for flow injection but undirected for admittance

  Example 2: voltage V, current I, power S are complex quantities

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- Different approximations by different stakeholders
  $\Rightarrow$ ambiguities, lack of accepted formal definitions

Notation

- Complex number: $x = x^{\mathsf{r}} + ix^{\mathsf{c}} \in \mathbb{C}$
  most ACOPF literature uses j instead of i, reserved for current
- Complex conjugate: $\mathsf{conj}(x) = x^{\mathsf{r}} - ix^{\mathsf{c}}$
  $x\,\mathsf{conj}(x) = (x^{\mathsf{r}})^2 + (x^{\mathsf{c}})^2 = |x|^2$
- Polar representation: $\alpha e^{i\vartheta} = \alpha \cos \vartheta + i\alpha \sin \vartheta$

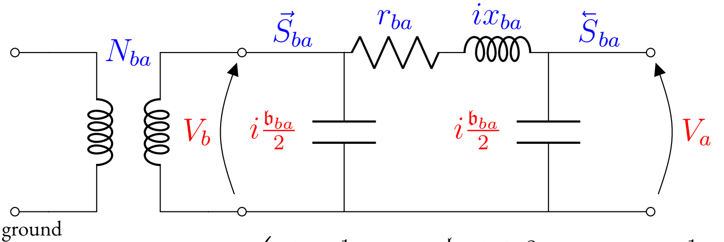
$$\begin{aligned} x^{\mathsf{r}} &= \alpha \cos \vartheta & \qquad \alpha &= \sqrt{(x^{\mathsf{r}})^2 + (x^{\mathsf{c}})^2} \\ x^{\mathsf{c}} &= \alpha \sin \vartheta & \qquad \vartheta &= \arccos(x^{\mathsf{r}}/\alpha) = \arcsin(x^{\mathsf{c}}/\alpha), \end{aligned}$$

α called "magnitude", ϑ "angle"/"phase"

On the word “flow”

- Power does not “flow” as does liquid or gas in a pipe
  electrons do not move much in cables: think more in terms of wave propagation
- For a line $\{b, a\}$, think of voltage difference between b and a as “influencing” the injection of power at b or a

The π-model of a line $\{b, a\}$



$$\mathbf{Y}_{ba} = \begin{pmatrix} Y_{bb} & Y_{ba} \\ Y_{ab} & Y_{aa} \end{pmatrix} = \begin{pmatrix} \left(\frac{1}{r_{ba}+ix_{ba}} + i\frac{\mathfrak{b}_{ba}}{2}\right)/\tau_{ba}^2 & -\frac{1}{(r_{ba}+ix_{ba})\tau e^{-i\theta_{ba}}} \\ -\frac{1}{(r_{ba}+ix_{ba})\tau_{ba} e^{i\theta_{ba}}} & \frac{1}{r_{ba}+ix_{ba}} + i\frac{\mathfrak{b}_{ba}}{2} \end{pmatrix}$$

- V_b, V_a: voltage differences with ground
- $\vec{S}_{ba}$: power injected on $\{b, a\}$ at b (vice versa for $\overleftarrow{S}_{ba}$)
- $\mathbf{Y}$ used in Ohm's law: current $= \mathbf{Y}\,(V_b, V_a)^\top$
- $r_{ba} + ix_{ba}$: series impedance of the line $\{b, a\}$
- $\mathfrak{b}_{ba}$: charging susceptange of the line $\{b, a\}$
- $N_{ba} = \tau_{ba} e^{i\theta_{ba}}$: tap ratio of transformer at b on line $\{b, a\}$

Outline

Subsection 1

Natural formulation

Sets

- B: set of buses (nodes) of the power grid
- L: set of lines (links) in the power grid
  edge $\{b, a\}$ will index entity E as E_{ba}
- G: set of generators
  set $\mathscr{G}_b$ of generators at bus b, $\bigcup_b \mathscr{G}_b = G$

Decision variables

- Voltage $V_b \in \mathbb{C}$ at bus $b \in B$
- Current $\mathbf{I}_{ba} = (\vec{I}_{ba}, \overleftarrow{I}_{ba}) \in \mathbb{C}^2$ on line $\{b, a\} \in L$
- Power $\mathbf{S}_{ba} = (\vec{S}_{ba}, \overleftarrow{S}_{ba}) \in \mathbb{C}^2$ on line $\{b, a\} \in L$
- Power $\mathscr{S}_{bg} \in \mathbb{C}$ for a generator $g \in \mathscr{G}_b$ at bus $b \in B$

Parameters

- Voltage magnitude in $[\underline{V}_b, \overline{V}_b] \in \mathbb{IR}$ at each bus $b \in B$
- Phase difference in $[\underline{\omega}_{ba}, \overline{\omega}_{ba}] \subseteq [-\pi, \pi]$ at each line $\{b, a\} \in L$
- A *reference* bus $r \in B$ s.t. $V_b^{\mathsf{c}} = 0$ and $V_b^{\mathsf{r}} \geq 0$
- Power demand $\tilde{S}_b \in \mathbb{C}$ at bus $b \in B$
  there can be buses with negative demand
- Magnitude of power injected on a line $\{b, a\} \in L$ bounded above by $\bar{S}_{ba} = \bar{S}_{ab} \in \mathbb{R}$
- Power generated by $g \in \mathscr{G}_b$ installed at bus $b \in B$ in $[\underline{\mathscr{S}_{bg}}, \overline{\mathscr{S}_{bg}}] \in \mathbb{IC}$
- Admittance matrix $\mathbf{Y}_{ba} \in \mathbb{C}^{2\times 2}$ for a line $\{b, a\} \in L$
- Shunt admittance $A_b \in \mathbb{C}$ at bus $b \in B$

Bounds

- Bounds on power

$$\forall \{b,a\} \in L \quad |\mathbf{S}_{ba}| \leq \bar{S}_{ba}\mathbf{1}$$

where $|\mathbf{S}_{ba}| = (|\vec{S}_{ba}|, |\overleftarrow{S}_{ba}|)^\top$

- Bounds on generated power (enforced on real/imaginary parts)

$$\forall b \in B, g \in \mathscr{G}_b \quad \underline{\mathscr{S}}_{bg} \leq \mathscr{S}_{bg} \leq \overline{\mathscr{S}}_{bg}$$

- Bounds on voltage magnitude

$$\forall b \in B \quad \underline{V}_b \leq |V_b| \leq \overline{V}_b$$

- Reference bus

$$V_r^{\mathsf{c}} = 0 \quad \wedge \quad V_r^{\mathsf{r}} \geq 0$$

Phase difference bounds

- Constraints:
  $$\forall \{b,a\} \in L \quad \underline{\omega}_{ba} \leq \theta_b - \theta_a \leq \overline{\omega}_{ba} \qquad (\star)$$

- Issue: we don't use phase variables θ
  and cartesian→polar mapping is nonlinear

- Prop.
  $$(\star) \equiv \left[\tan(\underline{\omega}_{ba}) \leq \frac{(V_b \mathsf{conj}(V_a))^{\mathsf{c}}}{(V_b \mathsf{conj}(V_a))^{\mathsf{r}}} \leq \tan(\overline{\omega}_{ba}) \wedge (V_b \mathsf{conj}(V_a))^{\mathsf{r}} \geq 0\right]$$

  Pf.
  $$\begin{aligned}\tan(\theta_b - \theta_a) &= \frac{\sin(\theta_b - \theta_a)}{\cos(\theta_b - \theta_a)} = \frac{|V_b|\,|V_a|\sin(\theta_b - \theta_a)}{|V_b|\,|V_a|\cos(\theta_b - \theta_a)} \\ &= \frac{|V_b|\sin\theta_b|V_a|\cos\theta_a - |V_b|\cos\theta_b|V_a|\sin\theta_a}{|V_b|\cos\theta_b|V_a|\cos\theta_a + |V_b|\sin\theta_b|V_a|\sin\theta_a} \\ &= \frac{V_b^{\mathsf{c}} V_a^{\mathsf{r}} - V_b^{\mathsf{r}} V_a^{\mathsf{c}}}{V_b^{\mathsf{r}} V_a^{\mathsf{r}} + V_b^{\mathsf{c}} V_a^{\mathsf{c}}} = \frac{(V_b \mathsf{conj}(V_a))^{\mathsf{c}}}{(V_b \mathsf{conj}(V_a))^{\mathsf{r}}}\end{aligned}$$

  and tan is monotonically increasing

Constraints

- Power flow equations

$$\forall b \in B \quad \sum_{\{b,a\}\in L} \vec{S}_{ba} + \tilde{S}_b = -\mathsf{conj}(A_b)|V_b|^2 + \sum_{g \in \mathscr{G}_b} \mathscr{S}_g$$

- Power in terms of voltage and current

$$\forall \{b,a\} \in L \quad \mathbf{S}_{ba} = \mathbf{V}_{ba} \odot \mathsf{conj}(\mathbf{I}_{ba})$$

where $\odot \equiv$ *entrywise prod. and* $\mathsf{conj}(\mathbf{I})_{ba} = (\mathsf{conj}(\vec{I}_{ba}), \mathsf{conj}(\overleftarrow{I}_{ba}))^\top$

- Ohm's law

$$\forall \{b,a\} \in L \quad \mathbf{I}_{ba} = \mathbf{Y}_{ba}\mathbf{V}_{ba}$$

where $\mathbf{V}_{ba} = (V_b, V_a)^\top$ *for* $\{b,a\} \in L$

Constraints

- Power flow equations

$$\forall b \in B \quad \sum_{\{b,a\} \in L} \vec{S}_{ba} + \tilde{S}_b = -\mathsf{conj}(A_b)|V_b|^2 + \sum_{g \in \mathscr{G}_b} \mathscr{S}_g$$

- Power in terms of voltage and current

$$\forall \{b,a\} \in L \quad \vec{S}_{ba} = V_b \, \mathsf{conj}(\vec{I}_{ba})$$
$$\forall \{b,a\} \in L \quad \overleftarrow{S}_{ba} = V_a \, \mathsf{conj}(\overleftarrow{I}_{ba})$$

- Ohm's law

$$\forall \{b,a\} \in L \quad \vec{I}_{ba} = Y_{bb}V_b + Y_{ba}V_a$$
$$\forall \{b,a\} \in L \quad \overleftarrow{I}_{ba} = Y_{ab}V_b + Y_{aa}V_a$$

Objective function

- Depends on application setting
- Often: cost of generated power

$$\min \mathscr{S}^{\mathsf{H}} Q \mathscr{S} + (c^{\mathsf{H}} \mathscr{S})^{\mathsf{r}} + {c_0}^{\mathsf{r}}$$

- Q hermitian $\Rightarrow \mathscr{S}^{\mathsf{H}} Q \mathscr{S} \in \mathbb{R}$

Subsection 2

Edge formulation

Differences

- A line is an edge $\ell = \{b, a\}$
- Entities with a direction are indexed with b or a

$$\begin{aligned} \vec{E}_{ba} &\equiv E^b_\ell \\ \overleftarrow{E}_{ba} &\equiv E^a_\ell \end{aligned}$$

- This formulation is used by MATPOWER

Sets, parameters, variables

- $\forall b \in B$ let $\delta(b) = \{\ell \in L \mid \ell = \{b, a\}\}$
  set of lines adjacent to b

- Upper bound $\bar{S}_\ell$ to power magnitude
- Bounds $[\underline{\omega}_\ell, \overline{\omega}_\ell]$ to phase difference

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- Current $\mathbf{I}_\ell = (I_\ell^b, I_\ell^a) \in \mathbb{C}^2$ on line $\ell = \{b, a\} \in L$
- Power $\mathbf{S}_\ell = (S_\ell^b, S_\ell^a) \in \mathbb{C}^2$ on line $\ell = \{b, a\} \in L$

Constraints

- Power flow equations

$$\forall b \in B \quad \sum_{\ell \in \delta(b)} S_\ell^b + \tilde{S}_b = -\mathsf{conj}(A_b)|V_b|^2 + \sum_{g \in \mathscr{G}_b} \mathscr{S}_g$$

- Power in terms of voltage and current

$$\begin{aligned} \forall \ell \in L \quad S_\ell^b &= V_b\,\mathsf{conj}(I_\ell^b) \\ \forall \ell \in L \quad S_\ell^a &= V_a\,\mathsf{conj}(I_\ell^a) \end{aligned}$$

- Ohm's law

$$\begin{aligned} \forall \ell \in L \quad I_\ell^b &= Y_{bb}V_b + Y_{ba}V_a \\ \forall \ell \in L \quad I_\ell^a &= Y_{ab}V_b + Y_{aa}V_a \end{aligned}$$

Subsection 3

Arc formulation

Differences

- A line is a pair of anti-parallel arcs $\{(b,a),(a,b)\}$
- Entities with a direction are indexed by (b,a) or (a,b)

$$\vec{E}_{ba} \equiv E^{b}_{\{b,a\}} \quad \equiv \quad E_{ba}$$
$$\overleftarrow{E}_{ba} \equiv E^{a}_{\{b,a\}} \quad \equiv \quad E_{ab}$$

- This formulation is easier to code in `AMPL`

Sets, parameters, variables

- set $L' = \{(b,a),(a,b) \mid \{b,a\} \in L\}$ of all arcs
- set $L_0 \subset L'$ of arcs given in data
  s.t. $\forall \{b,a\} \in L \quad (b,a) \in L_0$ xor $(a,b) \in L_0$

- Upper bounds $\bar{S}_{ba} = \bar{S}_{ab}$ to power magnitude
- Bounds $[\underline{\omega}_{ba} = \underline{\omega}_{ab}, \overline{\omega}_{ba} = \overline{\omega}_{ab}]$ to phase difference
- If $(b,a) \in L_0$ has a transformer, it is on the side of $b \in B$

- Current $I_{ba} \in \mathbb{C}$ for each $(b,a) \in L'$ injected at $b \in B$
- Power $S_{ba} \in \mathbb{C}$ on each $(b,a) \in L'$ injected at $b \in B$

Constraints

- Power flow equations

$$\forall b \in B \quad \sum_{(b,a)\in L'} S_{ba} + \tilde{S}_b = -\mathsf{conj}(A_b)|V_b|^2 + \sum_{g\in\mathscr{G}_b} \mathscr{S}_g$$

- Power in terms of current

$$\forall (b,a) \in L' \quad S_{ba} \quad = \quad V_b \, \mathsf{conj}(I_{ba})$$

- Ohm's law

$$\forall (b,a) \in L_0 \quad I_{ba} \quad = \quad Y_{bb}V_b + Y_{ba}V_a$$
$$\forall (b,a) \in L_0 \quad I_{ab} \quad = \quad Y_{ab}V_b + Y_{aa}V_a$$

- Upper bounds on power magnitude

$$\forall (b,a) \in L' \quad |S_{ba}| \quad \leq \quad \bar{S}_{ba}$$

Outline

Ohm's law matrix

Diagonal components of $\mathbf{Y}_{ba} \in \mathbb{C}^{2\times 2}$

$$\begin{aligned}
Y_{bb} &= \left(\frac{1}{r+ix} + i\frac{\mathfrak{b}}{2}\right)/\tau^2 = \frac{2(r-ix) + i\mathfrak{b}(r^2+x^2)}{2(r+ix)(r-ix)\tau^2} \\
&= \frac{r}{(r^2+x^2)\tau^2} + i\frac{\mathfrak{b}(r^2+x^2) - 2x}{2(r^2+x^2)\tau^2}
\end{aligned}$$

$$\begin{aligned}
Y_{aa} &= \frac{1}{r+ix} + i\frac{\mathfrak{b}}{2} = \frac{2(r-ix) + i\mathfrak{b}(r^2+x^2)}{2(r+ix)(r-ix)} \\
&= \frac{r}{r^2+x^2} + i\frac{\mathfrak{b}(r^2+x^2) - 2x}{2(r^2+x^2)}
\end{aligned}$$

Ohm's law matrix

Off-diagonal components of $\mathbf{Y}_{ba} \in \mathbb{C}^{2\times 2}$

$$
\begin{aligned}
Y_{ba} &= -\frac{1}{(r+ix)\tau e^{-i\theta}} = -\frac{1/\tau}{(r\cos\theta + x\sin\theta) + i(x\cos\theta - r\sin\theta)} \\
&= -\frac{1}{\tau}\frac{r\cos\theta + x\sin\theta - i(x\cos\theta - r\sin\theta)}{(r\cos\theta + x\sin\theta)^2 + (x\cos\theta - r\sin\theta)^2} \\
&= -\frac{r\cos\theta + x\sin\theta}{\tau(r^2+x^2)} - i\frac{r\sin\theta - x\cos\theta}{\tau(r^2+x^2)}
\end{aligned}
$$

$$
\begin{aligned}
Y_{ab} &= -\frac{1}{(r+ix)\tau e^{i\theta}} = -\frac{1/\tau}{(r\cos\theta - x\sin\theta) + i(x\cos\theta + r\sin\theta)} \\
&= -\frac{1}{\tau}\frac{r\cos\theta - x\sin\theta - i(x\cos\theta + r\sin\theta)}{(r\cos\theta + x\sin\theta)^2 + (x\cos\theta - r\sin\theta)^2} \\
&= \frac{x\sin\theta - r\cos\theta}{\tau(r^2+x^2)} + i\frac{r\sin\theta + x\cos\theta}{\tau(r^2+x^2)}
\end{aligned}
$$

Subsection 1

Arc formulation

Constraints

- Linear constraint: separate real and imaginary parts
- Power in terms of voltage and current

$$\begin{aligned}
\forall (b,a) \in L' \quad S^{\mathsf{r}}_{ba} &= V^{\mathsf{r}}_b I^{\mathsf{r}}_{ba} + V^{\mathsf{c}}_b I^{\mathsf{c}}_{ba} \\
\forall (b,a) \in L' \quad S^{\mathsf{c}}_{ba} &= -V^{\mathsf{r}}_b I^{\mathsf{c}}_{ba} + V^{\mathsf{c}}_b I^{\mathsf{r}}_{ba}
\end{aligned}$$

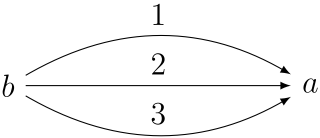
- Ohm's law

$$\begin{aligned}
\forall (b,a) \in L_0 \quad I^{\mathsf{r}}_{ba} &= Y^{\mathsf{r}}_{bb} V^{\mathsf{r}}_b - Y^{\mathsf{c}}_{bb} V^{\mathsf{c}}_b + Y^{\mathsf{r}}_{ba} V^{\mathsf{r}}_a - Y^{\mathsf{c}}_{ba} V^{\mathsf{c}}_a \\
\forall (b,a) \in L_0 \quad I^{\mathsf{c}}_{ba} &= Y^{\mathsf{r}}_{bb} V^{\mathsf{c}}_b + Y^{\mathsf{c}}_{bb} V^{\mathsf{r}}_b + Y^{\mathsf{r}}_{ba} V^{\mathsf{c}}_a + Y^{\mathsf{c}}_{ba} V^{\mathsf{r}}_a \\
\forall (b,a) \in L_0 \quad I^{\mathsf{r}}_{ab} &= Y^{\mathsf{r}}_{ab} V^{\mathsf{r}}_b - Y^{\mathsf{c}}_{ab} V^{\mathsf{c}}_b + Y^{\mathsf{r}}_{aa} V^{\mathsf{r}}_a - Y^{\mathsf{c}}_{aa} V^{\mathsf{c}}_a \\
\forall (b,a) \in L_0 \quad I^{\mathsf{c}}_{ab} &= Y^{\mathsf{r}}_{ab} V^{\mathsf{c}}_b + Y^{\mathsf{c}}_{ab} V^{\mathsf{r}}_b + Y^{\mathsf{r}}_{aa} V^{\mathsf{c}}_a + Y^{\mathsf{c}}_{aa} V^{\mathsf{r}}_a
\end{aligned}$$

Subsection 2

Parallel lines

The situation



quantify over
(bus, bus, counter)

Can't easily merge properties of separate cables on a single line

Modelling

- Natural formulation: $\bar{L} \subset L \times \mathbb{N}$
  quantification: $\forall(\{b, a\}, i) \in \bar{L}$

- Edge formulation: $\bar{L} \subset L \times \mathbb{N}$
  quantification: $\forall(\ell, i) \in \bar{L}$

- Arc formulation: $\bar{L}' \subset L' \times \mathbb{N}$
  quantification: $\forall(b, a, i) \in \bar{L}'$

Indexing entities

- Natural formulation: $\mathbf{S}_{bai}, \mathbf{I}_{bai}, \underline{\omega}_{bai}, \overline{\omega}_{bai}, \bar{S}_{bai}, \mathbf{Y}_{bai}, \ldots$
- Edge formulation: $\mathbf{S}_{\ell i}, \mathbf{I}_{\ell i}, \underline{\omega}_{\ell i}, \overline{\omega}_{\ell i}, \bar{S}_{\ell i}, \mathbf{Y}_{\ell i}, \ldots$
- Arc formulation: $S_{bai}, I_{bai}, \underline{\omega}_{bai}, \overline{\omega}_{bai}, \bar{S}_{bai}, \mathbf{Y}_{bai}, \ldots$

Outline

Subsection 1

MatPower

Generalities

- Matlab package
- Designed by electrical engineers not optimizers/computer scientists
- Very popular/mature code, works well
- Provides an instance library
- Data coded in a counterintuitive way

Table B-1: Bus Data (`mpc.bus`)

name	column	description
BUS_I	1	bus number (positive integer)
BUS_TYPE	2	bus type (1 = PQ, 2 = PV, 3 = ref, 4 = isolated)
PD	3	real power demand (MW)
QD	4	reactive power demand (MVAr)
GS	5	shunt conductance (MW demanded at $V = 1.0$ p.u.)
BS	6	shunt susceptance (MVAr injected at $V = 1.0$ p.u.)
BUS_AREA	7	area number (positive integer)
VM	8	voltage magnitude (p.u.)
VA	9	voltage angle (degrees)
BASE_KV	10	base voltage (kV)
ZONE	11	loss zone (positive integer)
VMAX	12	maximum voltage magnitude (p.u.)
VMIN	13	minimum voltage magnitude (p.u.)
LAM_P†	14	Lagrange multiplier on real power mismatch (u/MW)
LAM_Q†	15	Lagrange multiplier on reactive power mismatch (u/MVAr)
MU_VMAX†	16	Kuhn-Tucker multiplier on upper voltage limit (u/p.u.)
MU_VMIN†	17	Kuhn-Tucker multiplier on lower voltage limit (u/p.u.)

† Included in OPF output, typically not included (or ignored) in input matrix. Here we assume the objective function has units u.

Table B-3: Branch Data (mpc.branch)

name	column	description
F_BUS	1	"from" bus number
T_BUS	2	"to" bus number
BR_R	3	resistance (p.u.)
BR_X	4	reactance (p.u.)
BR_B	5	total line charging susceptance (p.u.)
RATE_A	6	MVA rating A (long term rating), set to 0 for unlimited
RATE_B	7	MVA rating B (short term rating), set to 0 for unlimited
RATE_C	8	MVA rating C (emergency rating), set to 0 for unlimited
TAP	9	transformer off nominal turns ratio, if non-zero (taps at "from" bus, impedance at "to" bus, i.e. if $r = x = b = 0$, $tap = \frac{\lvert V_f \rvert}{\lvert V_t \rvert}$; $tap = 0$ used to indicate transmission line rather than transformer, i.e. mathematically equivalent to transformer with $tap = 1$)
SHIFT	10	transformer phase shift angle (degrees), positive ⇒ delay
BR_STATUS	11	initial branch status, 1 = in-service, 0 = out-of-service
ANGMIN*	12	minimum angle difference, $\theta_f - \theta_t$ (degrees)
ANGMAX*	13	maximum angle difference, $\theta_f - \theta_t$ (degrees)
PF†	14	real power injected at "from" bus end (MW)
QF†	15	reactive power injected at "from" bus end (MVAr)
PT†	16	real power injected at "to" bus end (MW)
QT†	17	reactive power injected at "to" bus end (MVAr)
MU_SF‡	18	Kuhn-Tucker multiplier on MVA limit at "from" bus (u/MVA)
MU_ST‡	19	Kuhn-Tucker multiplier on MVA limit at "to" bus (u/MVA)
MU_ANGMIN‡	20	Kuhn-Tucker multiplier lower angle difference limit (u/degree)
MU_ANGMAX‡	21	Kuhn-Tucker multiplier upper angle difference limit (u/degree)

* Not included in version 1 case format. The voltage angle difference is taken to be unbounded below if ANGMIN ≤ -360 and unbounded above if ANGMAX ≥ 360. If both parameters are zero, the voltage angle difference is unconstrained.

† Included in power flow and OPF output, ignored on input.

‡ Included in OPF output, typically not included (or ignored) in input matrix. Here we assume the objective function has units u.

Table B-2: Generator Data (mpc.gen)

name	column	description
GEN_BUS	1	bus number
PG	2	real power output (MW)
QG	3	reactive power output (MVAr)
QMAX	4	maximum reactive power output (MVAr)
QMIN	5	minimum reactive power output (MVAr)
VG‡	6	voltage magnitude setpoint (p.u.)
MBASE	7	total MVA base of machine, defaults to baseMVA
GEN_STATUS	8	machine status, > 0 = machine in-service; ≤ 0 = machine out-of-service
PMAX	9	maximum real power output (MW)
PMIN	10	minimum real power output (MW)
PC1*	11	lower real power output of PQ capability curve (MW)
PC2*	12	upper real power output of PQ capability curve (MW)
QC1MIN*	13	minimum reactive power output at PC1 (MVAr)
QC1MAX*	14	maximum reactive power output at PC1 (MVAr)
QC2MIN*	15	minimum reactive power output at PC2 (MVAr)
QC2MAX*	16	maximum reactive power output at PC2 (MVAr)
RAMP_AGC*	17	ramp rate for load following/AGC (MW/min)
RAMP_10*	18	ramp rate for 10 minute reserves (MW)
RAMP_30*	19	ramp rate for 30 minute reserves (MW)
RAMP_Q*	20	ramp rate for reactive power (2 sec timescale) (MVAr/min)
APF*	21	area participation factor
MU_PMAX†	22	Kuhn-Tucker multiplier on upper P_g limit (u/MW)
MU_PMIN†	23	Kuhn-Tucker multiplier on lower P_g limit (u/MW)
MU_QMAX†	24	Kuhn-Tucker multiplier on upper Q_g limit (u/MVAr)
MU_QMIN†	25	Kuhn-Tucker multiplier on lower Q_g limit (u/MVAr)

* Not included in version 1 case format.

† Included in OPF output, typically not included (or ignored) in input matrix. Here we assume the objective function has units u.

‡ Used to determine voltage setpoint for optimal power flow only if opf.use_vg option is non-zero (0 by default). Otherwise generator voltage range is determined by limits set for corresponding bus in bus matrix.

Table B-4: Generator Cost Data† (`mpc.gencost`)

name	column	description
`MODEL`	1	cost model, 1 = piecewise linear, 2 = polynomial
`STARTUP`	2	startup cost in US dollars*
`SHUTDOWN`	3	shutdown cost in US dollars*
`NCOST`	4	number of cost coefficients for polynomial cost function, or number of data points for piecewise linear
`COST`	5	parameters defining total cost function $f(p)$ begin in this column, units of f and p are \$/hr and MW (or MVAr), respectively (`MODEL` = 1) $\Rightarrow \quad p_0, f_0, p_1, f_1, \ldots, p_n, f_n$ where $p_0 < p_1 < \cdots < p_n$ and the cost $f(p)$ is defined by the coordinates $(p_0, f_0), (p_1, f_1), \ldots, (p_n, f_n)$ of the end/break-points of the piecewise linear cost (`MODEL` = 2) $\Rightarrow \quad c_n, \ldots, c_1, c_0$ $n+1$ coefficients of n-th order polynomial cost, starting with highest order, where cost is $f(p) = c_n p^n + \cdots + c_1 p + c_0$

† If `gen` has n_g rows, then the first n_g rows of `gencost` contain the costs for active power produced by the corresponding generators. If `gencost` has $2n_g$ rows, then rows $n_g + 1$ through $2n_g$ contain the reactive power costs in the same format.

* Not currently used by any MATPOWER functions.

Rosetta stone: buses

$\forall b \in B$:

- $\tilde{S}_b = (\texttt{PD} + i\texttt{QD})/\texttt{baseMVA} \in \mathbb{C}$
- $-\mathsf{conj}(A_b) = (-\texttt{GS} + i\texttt{BS})/\texttt{baseMVA} \in \mathbb{C}$
- $[\underline{V}_b, \overline{V}_b] = [\texttt{VMIN}, \texttt{VMAX}] \in \mathbb{IR}$

I've never seen `baseMVA` *being anything other than* 100

Rosetta stone: branches

$\forall \{b, a\} \in L$:

- $\bar{S}_{ba} = \texttt{RATE\_A}/\texttt{baseMVA} \in \mathbb{R}$
- $r_{ba} = \texttt{BR\_R} \in \mathbb{R}$
- $x_{ba} = \texttt{BR\_X} \in \mathbb{R}$
- $\mathfrak{b}_{ba} = \texttt{BR\_B} \in \mathbb{R}$
- $\tau_{ba} = \texttt{TAP} \in \mathbb{R}$
- $\theta_{ba} = \frac{\pi}{180}\texttt{SHIFT} \in \mathbb{R}$
- $\underline{\omega}_{ba} = \frac{\pi}{180}\texttt{ANGMIN} \in \mathbb{R}$
- $\overline{\omega}_{ba} = \frac{\pi}{180}\texttt{ANGMAX} \in \mathbb{R}$

Rosetta stone: generators

$\forall b \in B, g \in \mathscr{G}_b$:

- $[\underline{\mathscr{S}}_{bg}, \overline{\mathscr{S}}_{bg}] = \texttt{GEN\_STATUS}\left([\texttt{PMIN}, \texttt{PMAX}] + i[\texttt{QMIN}, \texttt{QMAX}]\right)$

Subsection 2

AMPL

The ACOPF in `AMPL`

- Natural, edge, arc: which formulation?
- Translating data from MatPower `.m` to `AMPL` `.dat` files
  many defaults are wrong: e.g. `RATE_A` $= 0$ *means* $\bar{S}_{ba} = +\infty$, `TAP` $= 0$ *means* $\tau = 1$, `ANGMIN` < -90 *means* `ANGMIN` $= 90$, `ANGMAX` > 90 *means* `ANGMAX` $= 90$
- Comparing results: polar vs. cartesian formulation

Implementation pitfalls

- Implementing the $\mathbf{Y}$ matrix is error-prone
- Power is $S = V\,\mathsf{conj}(I)$: don't forget the conjugate
- Many papers enforce upper bounds on power magnitude, some have current
- The shunt admittance A_b is never used *per se*; instead, we use $-\mathsf{conj}(A_b) = -A_b^{\mathsf{r}} + iA_b^{\mathsf{c}}$, i.e. we flip the sign of the real part
- The bus indexing is not a progressive counter
- Some text file lines are commented in some instances