The price of equity in the Hazmat Transportation Problem

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1 Introduction

We consider the problem of the transportation of hazardous materials on a road network (Hazardous Materials Transportation Problem). We can figure it this way: there are N trucks which have to transport some kind of dangerous material from one or many production points to one or many garbage dumps and we have to select a set of paths which is optimal from the point of view of risk, cost and equity. The optimization of cost and risk on a network leads quite spontaneously to shortest path and flow problems which are milestones of Operational Research, but equity is somehow unusual and hard to define. We consider and compare two different ideas. The first approach simply requires that all the areas involved in the transportation network share the same level of risk. This is a fair and intuitive idea but it could also lead to "improper" solutions where risk is equal but uniformly high. The second (more interesting) definition of equity we use is inspired by the concept of fairness of J. Rawls [2,3]. Basically, in this context, the difference principle means that we may introduce disparities only if they advantage the worst-off, namely reduce the risk of the less favourite area (the most exposed to the risk).

The aim of this work is to provide rational elements to be able to estimate the cost of choosing a particular definition of equity (for hazmat trasportation). We investigate the relation between each definition of equity and the cost it generates. This can be used as a first criterion to make a choice.

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2 Mathematical programming formulation

Let $G = (V, A)$ be a directed graph, modelling a road network.
We consider many origin-destination pairs $(s, t) \in C \subseteq V \times V$. For every pair (s, t) there is a commodity to be trasported from a source s to a destination t to respond to a specific demand which we indicate with d_{st}. We look for a global route planning given by a multicommodity flow function $x : C \times A \to \mathbb{R}_+$ (the situation involving only one origin and one destination is a special case). Typically we can imagine that the road network covers a geographic area which is divided into zones; in particular each arc (road) belongs to a zone $\zeta \in Z$. For the sake of simplicity we assume that each arc belongs to only one zone. Each arc (i, j) has a positive traversal cost c_{ij}, a probability p_{ij} of an accident occurring on that arc, a value of damage (in monetary units) Δ_{ij} caused by a potential accident on that arc and a capacity χ_{ij}.

(1) **Sets**:
- $C \subseteq V \times V$ is the set of all pairs (s, t);
- Z is the set of all zones;
- $\zeta_l \subseteq A$ is a zone $(1 \leq l \leq |Z|)$;

(2) **Parameters**:
- $1 \leq l \leq |Z|$ = zone index
- p_{ij}^{st} : probability of accident on an arc;
- Δ_{ij}^{st} : *damage* (in monetary units) caused by an accident on an arc ;
- c_{ij}^{st} : cost on an arc;
- s : source;
- t : destination (target);
- d_{st} : demand of commodity (st);

  We call $p_{ij}^{st}\Delta_{ij}^{st}$ *traditional risk* and we indicate it, alternatively, as r_{ij}^{st}.

(3) **Decision variables**:
$\forall (i, j) \in A, \ \forall (s, t) \in C \ \ x_{ij}^{st}$: flow of the commodity (st) on the arc (i, j)

(4) **Constraints**.
- (capacity) $\sum_{(st)\in C} x_{ij}^{st} \leq c(ij)$
- (demand) $\sum_{(i)\in V} x_{it}^{st} = d_{st}$
- (flow conservation) $\forall (st) \in C \quad \sum\limits_{(i,j)\in A} x_{ij}^{st} - \sum\limits_{(j,i)\in A} x_{ij}^{st} = \begin{cases} 1 \text{ if } i = s \\ -1 \text{ if } i = t \\ 0 \text{ otherwise} \end{cases}$

(5) **Objective function 1 (Cost):** minimize total cost

$$\min \sum_{(i,j)\in A} \sum_{(s,t)\in C} c_{ij}^{st} x_{ij}^{st} \tag{1}$$

(6) **Objectives concerning equity, two possible versions:**

- **Objective function 2 (a) (Risk sharing):** minimize the difference of traditional risk between two zones

$$\min \left(\sum_{\forall(\zeta_l,\zeta_m)\in Z\times Z} \left| \left(\sum_{(i,j)\in\zeta_l} \sum_{(s,t)\in C} r_{ij}^{st} x_{ij}^{st} \right) - \left(\sum_{(h,l)\in\zeta_m} \sum_{(s,t)\in C} r_{hl}^{st} x_{ij}^{st} \right) \right| \right) \quad (2)$$

- **Objective function 2 (b) (Rawls' principle)**: minimize the traditional risk of the least advantaged zone

$$\min \left(\max_{\zeta\in Z} \sum_{(i,j)\in\zeta} \sum_{(s,t)\in C} r_{ij}^{st} x_{ij}^{st} \right) \quad (3)$$

3 Methodology and Tests

The problem we are considering belongs to the special class of optimization problems called Multicriteria Optimization Problems (MOP) [5,7,6]. Probably, the most common approaches are the *weighted sum method* and the *ϵ-constraint method*. Since the latter can be used either with convex or with non-convex objectives space, and we plan to introduce many sources of non-convexity in next refinements of our model, we adopt from the outset the *ϵ-constraint method*. The basic idea of this method consists in the trasformation of all the objectives in constraints, out of one which is minimized (or maximized). Varying ϵ_i, alternative solutions are obtained (even if it is known that it is difficult to chose proper values for the vector ϵ and arbitrarily ones produce no feasible solutions).

We consider first small instances of the problem based on networks composed by a kept down number of nodes and involving a few zones and shipments and only one origin and destination. We used the AMPL modelling environment and the off-the-shelf CPLEX 10.1 solver running on a 64-bit 2.1 GHz Intel Core2 CPU with 4GB RAM. The results we got using an instance composed by 15 nodes distributed in 2 zones, with 10 shipments show that both kind of equity have a negative impact on the cost and make it grow, which is the awaited outcome. The aim is to establish which one makes it increase most. In order to solve this question we introduce some methodological expedients. In fact, even if we solved either C_1 and C_2 applying the *ϵ-constraint method*, we can not simply compare the cost for a fixed equal threshold of ϵ because it has a different meaning in the two situations. We have to normalize the comparison to "equal levels" of equity and to map the cost to the *share* of equity instead of its absolute value. For example, we compare the cost we get when we have the peak of equity in Risk Sharing and Rawls sense, then when we get the 99% of equity (independently from the different corresponding values of ϵ which are different in C_1 and C_2), the 98% ... and so on. Thus we establish first the maximal possible level of equity and then we (can) define the

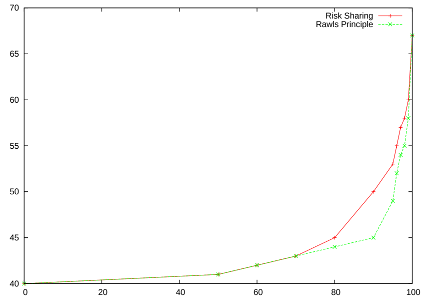

Fig. 1. Comparison between Risk Sharing and Rawls' Principle

values corresponding to its fractions. We measure the increment of cost while equity varies from its possible minimum to its possible maximum and we map it on the share of equity. We discover that the raise of cost induced by equity in the sense of Rawls is weaker that the one induced by the "naive" one. We report some sample results (in the format [Equity Share; Cost of Risk Sharing; Cost of Rawls' Principle] : [0;40;40], [50;41;41], [60;42;42], [70;43;43], [80;45;44], [90;50;45], [95;53;49], [96;55;52], [97;57;54], [98;58;55], [99;60;58], [100;67;67]). Fig.1 shows the corresponding plot. The tests are partial since we use small artificial instances. We plan to use real data and different multi objectives methods in future work to corroborate our conclusions.

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