## Extending SCIP for solving MIQCPs

**Mixed-Integer Quadratically Constrained Programming (MIQCP)**

We consider optimization problems of the form

\[
\begin{align*}
\text{minimize} & \quad b_0^T x + c_0 \\
\text{subject to} & \quad x^T A_j x + b_j^T x + c_j \leq 0, \quad j = 1, \ldots, m \\
& \quad x_i \in \mathbb{Z}, \quad \forall i \in I,
\end{align*}
\]

where \( A_j \in \mathbb{Q}^{n \times n}, b_j \in \mathbb{Q}^n, c_j \in \mathbb{Q}, j = 0, \ldots, m, \) and \( I \subseteq \{1, \ldots, n\}. \)

In general, \( A_j \) does not need to be positive semidefinite.

**Algorithm: LP-based branch-and-cut**

We extend the Constraint Integer Programming framework SCIP [1] by MINLP-specific plugins.

For nonlinear constraints, we generate a suitable linear outer approximation and apply domain propagation and primal heuristics.

### Currently implemented [5]

- reformulation for products with binary variables
- recognition of convex quadratic functions
- separation for convex and nonconvex quad. constraints
- domain propagation for quadratic constraints
- handling of second-order-cone (SOC) constraints
- local search heuristic: fix integers, solve sub-QCP locally
- interfaces to GAMS, MPS, and ZIMPL

### Computations Results

**Benchmark**

- 80 MIQCP benchmark instances from MINLPLib, H. Mittelmann, J.P. Vielma

**Impact of Single Components**

- Relaxation Enforced Neighborhood Search heuristic [2]:
  - fix integer variables that take integral value in optimal solution of LP relaxation and solve remaining sub-MIQCP

**Application: Mine Prod. Scheduling [6]**

- Stockpiles lead to mixing constraints:
  \[ \sum_{j} \alpha_{ij} \cdot \prod_{k \in I_{ij}} x_k \cdot \prod_{k \in \bar{I}_{ij}} (1 - x_k) \geq b_i \]

**Beyond MIQCP**

**Convex MINLP**

- QG-like algorithm (gradient-based cuts)

**Pseudo-Boolean Optimization [4]**

- Pseudo-Boolean Constraints:
  \[ \sum_{j=1}^{l_i} \alpha_{ij} \cdot \prod_{k \in I_{ij}} x_k \cdot \prod_{k \in \bar{I}_{ij}} (1 - x_k) \geq b_i \]

- replace multiplications by new variables and AND-constraints
- specialised separation and propagation for AND-constraints (small initial relax., separate strong ineq., always propagate)
- Winner in 3 out of 4 categories at Pseudo-Boolean Evaluation 2009

**Application: Gas Transport**

- IP: Network Configuration
- NLP: Physics of Gas
- SP: Legal Requirements

### References


