

Optimal design of a biomass-based energy production process

MAURIZIO BRUGLIERI

INDACO, Politecnico di Milano, Via Durando 38/a, 20158 Milano, Italy
(maurizio.bruglieri@polimi.it)

LEO LIBERTI

CNRS LIX, Ecole Polytechnique, F-91128 Palaiseau, France
(liberti@lix.polytechnique.fr)

Abstract

We propose mathematical programming models for solving problems arising from planning and running an energy production process based on burning biomasses. The models take into account different aspects of the problem: determination of the biomasses to produce and/or buy, transportation decisions to convey the materials to the respective plants, and plant design. Whereas the “running model” is linear, the “planning model” is a Mixed-Integer Nonlinear Programming problem, which we solve by reformulating it to a Mixed-Integer Linear Programming problem.

Keywords: renewable energy, biomass exploitation, mathematical programming

1 Introduction

Producing energy derived from fossil carbon-based fuels is proving costly to both the environment (in terms of pollution) and society (in terms of monetary investment). As the prices of crude oil increase, governments and other institutions are researching the most cost-efficient ways to produce energy from alternative sources [Inyang, 2005]. One of the most popular contenders is energy produced by biomasses of several kinds [Regional Wood Energy Development Programme, 1998]. In [Adams et al., 1998] and [Adams et al., 2000] the competitiveness of biomass-based fuel for electrical energy opposed to carbon-based fuel is examined using a mathematical programming. Among the advantages of this type of energy production, there is the potential for employing wasted materials of biological origin, like used alimentary fats and oils, agricultural waste and so on. A factory producing energy with such materials would benefit from both the sales of the energy and the gains obtained by servicing waste [Arighieri et al., 2004]. In [Fiorese et al., 2005] a mathematical programming is proposed to localize both energy conversion plants and biomass catchments basins in provincial area. Other mathematical models for specific biomass discrete facility location problems are developed in [Freppaz et al., 2004] and [Voivontas et al., 2001].

This paper describes an optimization problem arising from the deployment of such an energy production process in different regions of Italy. The production process in question comprises several processing plants of different types (for example, a liquid biomass plant, a squeeze plant and a fermentation-distillation plant). Some of these plants (e.g. liquid biomass plant) produce energy; others (e.g. the fermentation-distillation plant) produce intermediate products which will then be routed to other plants for further processing. There are several possible input products (e.g. agricultural products, biological waste), obtained from different sources (e.g. direct farming or acquisition on the markets) at different unit costs. Apart from the energetic output, there may be other output products which are sold in different markets (e.g. bioethanol obtained from the fermentation-distillation plant and sold in the bioethanol market). See Fig. 1 for a typical process flowsheet.

There are in fact two optimization problems relating to this description. The first (and simplest) is that of modelling the production process as a net gain maximization supposing the type of plants involved and the end product demands are known. The second is that of deciding the type of plants to involve in

the process to maximize the net gain, subject to known end product demands. Although post-optimal sensitivity analysis may be used to gather heuristic hints on how to improve the plant design, a true design model furnishes the ultimate plant design tool.

Section 2 describes the model relating to the production process when the plant types are known (“running model”). Section 3 describes the model relating to the process design (“planning model”) and an exact mixed-integer linear reformulation thereof.

2 Modelling the production process

Modelling a flowsheet as that presented in Fig. 1 presents many difficulties. Notice that the products can be inputs, intermediate, outputs, or a both (like alcohol, which is both an output product and an intermediate product). Likewise, processes can be intermediate or final or a combination (like the fermentation-distillation plant). Consider also that the decision maker may choose to buy an intermediate product from a different source to cover demand needs, thus making the product a combination of intermediate and input. Of course the input products may be acquired or produced at different locations and at different prices. Moreover, each flow arrow has an associated transportation cost.

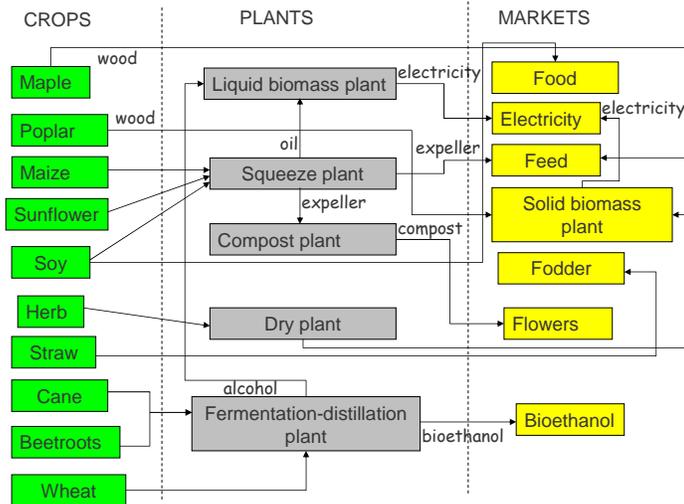


Figure 1: A typical process flowsheet.

The central concept in our model is the *process site*. A process site is a geographical location with at most one processing plant and/or various storage spaces for different types of goods (commodities). A place where production of a given commodity occurs is represented by a process site with a storage space. Thus, for example, a geographical location with two fields producing maize and sunflowers is a process site with two storage spaces and no processing plant. The fermentation-distillation plant is a process site with no storage spaces and one processing plant. Each output in Fig. 1 is represented by a process site with just one storage space for each output good. In this interpretation the concepts of input, output and intermediate products, and those of intermediate and final process, lose importance: this is appropriate because, as we have emphasized earlier, these distinctions are not always well-defined. Instead, we focus the attention on the material balance and on the transformation process in each process site. Furthermore, we are able to deal with the occurrence that a given commodity may be obtained at different costs depending on whether it is bought or produced directly.

We represent the process sites by a set V of vertices of a graph $G = (V, A)$ where the set of arcs A is given by the logistical connections among the locations. To each vertex $v \in V$ we associate a set of commodities $H^-(v)$ which may enter the process site, and a set of commodities $H^+(v)$ which may leave it. Thus, for example, the fermentation-distillation plant is a process site vertex where $H^-(\text{fermentation-distillation plant}) = \{\text{cane, beetroots}\}$ and $H^+(\text{fermentation-distillation plant}) = \{\text{alcohol}\}$. Furthermore, we let $H = \bigcup_{v \in V} (H^-(v) \cup H^+(v))$ be the set of all commodities involved in the production process, and we partition $V = V_0 \cup V_1$ into V_0 , the set of process sites with an associated processing plant, and $V_1 = V \setminus V_0$. Fig. 2 is the graph derived from the example in Fig. 1.

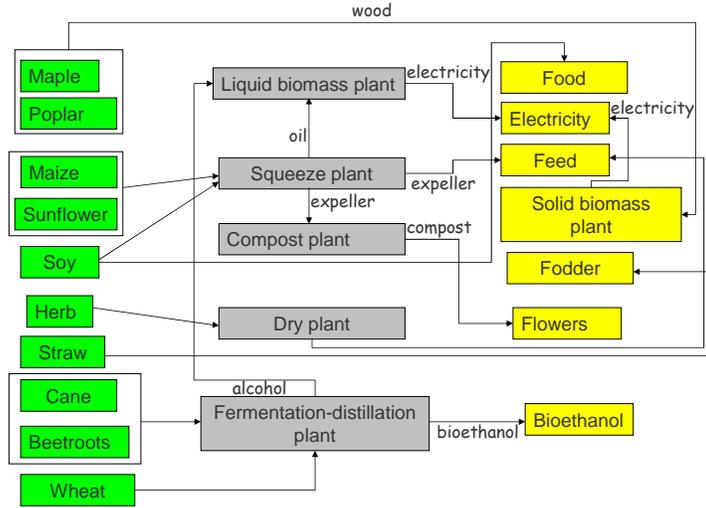


Figure 2: The graph derived from the example in Fig. 1.

We assume the following to be known parameters:

- c_{vk} : cost of supplying vertex v with a unit of commodity k (a negative cost may be associated to waste, since waste disposal is a service commodity);
- C_{vk} : maximum quantity of commodity k in vertex v ;
- τ_{uvk} : transportation cost for a unit of commodity k on the arc (u, v) ;
- T_{uvk} : transportation capacity for commodity k on arc (u, v) ;
- λ_{vkh} : cost of processing a unit of commodity k into commodity h in vertex v ;
- π_{vkh} : yield of commodity h expressed as unit percentage of commodity k in vertex v ;
- d_{vk} : demand of commodity k in vertex v .

It is clear that certain parameters make sense only when associated to a particular subset of vertices, like e.g. the demands may only be applied to the vertices representing the outputs. In this case, the corresponding parameter should be set to 0 in all vertices for which it is not applicable.

The decision variables are:

- x_{vk} : quantity of commodity k in vertex v ;

- y_{uvk} : quantity of commodity k on arc (u, v) ;
- z_{vkh} : quantity of commodity k processed into commodity h in vertex v .

Since the output demands are known *a priori*, we would like to minimize the total operation costs subject to demand satisfaction. There are three types of costs:

- cost of supplying vertices with commodities:

$$\gamma_1 = \sum_{k \in H} \sum_{v \in V} c_{vk} x_{vk};$$

- transportation costs:

$$\gamma_2 = \sum_{k \in H} \sum_{(u,v) \in A} \tau_{uvk} y_{uvk};$$

- processing costs:

$$\gamma_3 = \sum_{v \in V} \sum_{k \in H^-(v)} \sum_{h \in H^+(v)} \lambda_{vkh} z_{vkh},$$

so the objective function is

$$\sum_{i=1}^3 \gamma_i(x, y, z). \quad (1)$$

We need to make sure that some material conservation equations are enforced in each process site where a plant is installed:

$$\sum_{k \in H^-(v)} \pi_{vkh} z_{vkh} = x_{vh}, \quad \forall v \in V_0, h \in H^+(v). \quad (2)$$

Notice that these constraints do not actually enforce a conservation of mass, for in most processing plants a percentage of the input quantities goes to waste; but it is nonetheless a conservation law subject to the yield properties of the particular transformation process of the plant.

Secondly, the quantity of processed commodity must not exceed the quantity of input commodity in each vertex:

$$\sum_{h \in H^+(v)} z_{vkh} \leq x_{vk}, \quad \forall v \in V_0, k \in H^-(v). \quad (3)$$

Furthermore, we need the quantity of input commodity in each vertex to be consistent with the quantity of commodity in the vertex itself, and similarly for output commodities:

$$\sum_{u \in V: (u,v) \in A} y_{uvk} = x_{vk}, \quad \forall v \in V, k \in H^-(v) \quad (4)$$

$$\sum_{u \in V: (v,u) \in A} y_{vuh} = x_{vh}, \quad \forall v \in V, h \in H^+(v). \quad (5)$$

Finally, we have the bounds on the variables:

$$d_{vk} \leq x_{vk} \leq C_{vk}, \quad \forall v \in V, k \in H \quad (6)$$

$$0 \leq y_{uvk} \leq T_{uvk}, \quad \forall (u, v) \in A, k \in H \quad (7)$$

$$z_{vkh} \geq 0, \quad \forall v \in V, k \in H^-(v), h \in H^+(v) \quad (8)$$

and some fixed variables for irrelevant vertices:

$$x_{vk} = 0, \forall v \in V_1, k \in H \setminus (H^-(v) \cup H^+(v)) \quad (9)$$

$$y_{uvk} = 0, \forall (u, v) \in A, k \in H \setminus H^-(v), \quad (10)$$

$$y_{uvk} = 0, \forall (u, v) \in A, k \in H \setminus H^+(u). \quad (11)$$

The main advantage to this model is that it can be easily extended to deal with more commodities and plants in a natural way, by adding appropriate vertices or changing the relevant $H^-(v), H^+(v)$ and related parameters.

2.1 Solution of the problem

The problem described in Section 2 is a Linear Programming (LP) problem, which can be solved by using one of several LP solvers. We generated a random instance defined over the graph of Fig. 2, which we solved to optimality using AMPL [Fourer and Gay] with the `lp_solve` [Berkelaar, 2004] solver.

3 Planning the process design

In this section, we suppose no processing plants are yet present at the process sites. At each process site, we wish to install an appropriate processing plant chosen from a set P of possible plants (e.g. there may be different types of liquid biomass plants, each having different yield levels on the input commodities). We therefore wish to take discrete decisions as regards the plant installation, feasible with the material balance constraints as in Section 2, which minimizes the total operation costs.

We re-define the parameters λ, π to make them dependent on a processing plant $p \in P$ as follows:

- $\lambda_{vkh p}$: cost of using plant p to transform a unit of commodity k into commodity h in vertex v ;
- $\pi_{vkh p}$: yield of commodity h , using plant p , expressed as unit percentage of commodity k in vertex v .

Furthermore, we consider the following additional binary decision variables:

$$w_{vp} = \begin{cases} 1 & \text{if plant } p \text{ is installed in vertex } v \\ 0 & \text{otherwise} \end{cases}$$

The objective function (1) changes in the γ_3 term, which becomes:

$$\gamma'_3 = \sum_{v \in V} \sum_{k \in H^-(v)} \sum_{h \in H^+(v)} \left(\sum_{p \in P} \lambda_{vkh p} w_{vp} \right) z_{vkh}.$$

The material conservation constraints (2) become:

$$\sum_{k \in H^-(v)} \left(\sum_{p \in P} \pi_{vkh p} w_{vp} \right) z_{vkh} = x_{vh}, \forall v \in V, h \in H^+(v). \quad (12)$$

The following constraints enforce consistency on the assignment variables:

$$\sum_{p \in P} w_{vp} = 1, \forall v \in V_0, \quad (13)$$

$$\sum_{p \in P} w_{vp} = 0, \forall v \in V_1. \quad (14)$$

Finally, constraints (3)-(8) are also part of the formulation.

3.1 Solution of the problem

The model described in Section 3 is a Mixed-Integer Nonlinear Programming problem (MINLP) with nonconvex terms in both the objective function and the constraints. Problems of this type are solved either by employing heuristic methods, like Multi Level Single Linkage (MLSL) [Kucherenko and Sytsko, 2004, Liberti and Kucherenko, 2005] or Variable Neighbourhood Search (VNS) [Hansen and Mladenović, 2001, L. Liberti and M. Dražić, 2005], or by using an ε -approximate method called spatial Branch-and-Bound (sBB) [Smith and Pantelides, 1999, Adjiman et al., 1998], which also provides a proof of ε -optimality. It turns out, however, that by exploiting the special structure of the MINLP at hand, we can reformulate it exactly to a MILP and solve it with a state-of-the-art MILP solver (e.g. CPLEX [ILO02]).

The reformulation process is as follows.

1. Distribute products over sums, so that all the bilinear terms can be expressed as $w_{vp}z_{vkh}$ (also see [Tawarmalani and Sahinidis, 2002]).
2. Replace each bilinear term $w_{vp}z_{vkh}$ by a new added variable ζ_{vkhp} called a *linearization variable*. More precisely, the bilinear terms in γ'_3 in the objective and in constraints (12) should be replaced by the corresponding linearization variable, yielding:

$$\gamma''_3 = \sum_{v \in V} \sum_{k \in H^-(v)} \sum_{h \in H^+(v)} \sum_{p \in P} \lambda_{vkhp} \zeta_{vkhp}$$

and

$$\sum_{k \in H^-(v)} \sum_{p \in P} \pi_{vkhp} \zeta_{vkhp} = x_{vh}, \quad \forall v \in V, h \in H^+(v). \quad (15)$$

Naturally, to keep the reformulation exact, we must add the bilinear constraints

$$(\zeta_{vkhp} = w_{vp}z_{vkh}), \quad \forall v \in V, k, h \in H, p \in P \quad (16)$$

to the formulation. These are called *defining constraints*.

3. Since w_{vp} is a binary variable, ζ_{vkhp} can only take values in $\{0, z_{vkh}\}$; this allows us to reformulate the defining constraints (16) exactly by adding the following linear constraints:

$$(\zeta_{vkhp} \geq 0), \quad \forall v \in V, k, h \in H, p \in P \quad (17)$$

$$(\zeta_{vkhp} \geq z_{vkh} + z_{vkh}^U(w_{vp} - 1)), \quad \forall v \in V, k, h \in H, p \in P \quad (18)$$

$$(\zeta_{vkhp} \leq z_{vkh}^U w_{vp}), \quad \forall v \in V, k, h \in H, p \in P \quad (19)$$

$$(\zeta_{vkhp} \leq z_{vkh}), \quad \forall v \in V, k, h \in H, p \in P \quad (20)$$

to the formulation and dropping the bilinear defining constraints (16), where z_{vkh}^U is a tight upper bound to z_{vkh} for each $v \in V, k, h \in H$. It is important to choose these upper bounds as tight as possible, as the difficulty of finding feasible solutions during the MILP Branch-and-Bound process (and hence the overall running time) may well depend on this. Constraints (17)-(20) are known as McCormick envelopes [McCormick, 1976].

3.1.1 Reformulation-Linearization Technique cuts

Possible Reformulation-Linearization Technique (RLT) cuts may be derived as in [Sherali and Adams, 1986, Sherali and Alameddine, 1992, Sherali and Adams, 1999] by multiplying constraints by appropriate variables and then linearizing the resulting bilinear terms, as detailed below:

- constraints (3) by bound factors w_{vp} and $(1 - w_{vp})$ for all $p \in P$;
- constraints (13) and (14) by variables z_{vkh} for all k, h , to obtain:

$$\sum_{p \in P} \zeta_{vkh p} = z_{vkh}, \forall v \in V_0, k, h \in H \quad (21)$$

$$\sum_{p \in P} \zeta_{vkh p} = 0, \forall v \in V_1, k, h \in H. \quad (22)$$

Constraints (21) and (22) are a particular subclass of RLT constraints ([Sherali and Alameddine, 1992, Sherali and Adams, 1999]) called *reduction constraints* [Liberti, 2004b, Liberti, 2004c, Liberti, 2004a], with very interesting properties. In particular, although the bilinear defining constraints (16) are not in the formulation, it can be shown that in consequence of (21) and (22), a certain subset of them still hold at the relaxed solution. To sum up, the LP relaxation at each Branch-and-Bound node consists in minimizing $\gamma_1 + \gamma_2 + \gamma_3''$ subject to (15) and (3) with the RLT cuts derived from them, (4), (5), (6)-(8), (9)-(11), (13) and (14) with the reduction constraints (21) and (22) derived from them, and the McCormick envelopes (17)-(20).

4 Conclusion

In this paper we described a Linear Programming (LP) model for running a biomass-based energy production process, and Mixed-Integer Nonlinear Programming model for planning the installation of processing plants types used in the production process. We solve the first model using a commercial LP solver (CPLEX), whereas the second model is solved by way of an exact reformulation to a mixed-integer linear programming model.

References

- [Aringhieri et al., 2004] R. Aringhieri, M. Bruglieri, F. Malucelli, and M. Nonato. An asymmetric vehicle routing problem arising in the collection and disposal of special waste. In L. Liberti and F. Maffioli, editors, *CTW04 Workshop on Graphs and Combinatorial Optimization*, volume 17 of *Electronic Notes in Discrete Mathematics*, pages 41–46, Amsterdam, 2004. Elsevier.
- [Adams et al., 1998] D.M. Adams, R.J. Alig, J.T. Chmelik, B.A. McCarl, Analysis of Biomass Fueled Electrical Powerplants: Implications in the Agricultural and Forestry Sectors, *Annals of Operations Research*, 1998.
- [Adams et al., 2000] D.M. Adams, R.J. Alig, J.T. Chmelik, B.A. McCarl, Competitiveness of biomass-fueled electrical power plants *Annals of Operations Research*, 94: 37-55, 2000.
- [Adjiman et al., 1998] C.S. Adjiman, S. Dallwig, C.A. Floudas, and A. Neumaier. A global optimization method, *abb*, for general twice-differentiable constrained nlp: I. theoretical advances. *Computers & Chemical Engineering*, 22(9):1137–1158, 1998.
- [Berkelaar, 2004] M. Berkelaar. *LP_SOLVE: Linear Programming Code*. <http://www.cs.sunysb.edu/algorithm/algorithm/impl/implsolve/implsolve.shtml>, 2004.
- [Fiorese et al., 2005] G. Fiorese, G. Marino, G. Guariso Environmental and economic evaluation of biomass as an energy resource: application to a farming provincial area. Manuscript of the authors, 2005.
- [Fourer and Gay] R. Fourer and D. Gay. *The AMPL Book*.

- [Freppaz et al., 2004] D. Freppaz, R. Minciardi, M. Robba, M. Rovatti, R. Sacile, A. Taramasso Optimizing forest biomass exploitation for energy supply at a regional level *Biomass and Bioenergy*, 26:15–25, 2004.
- [Voivontas et al., 2001] D. Voivontas, D. Assimacopoulos, E.G. Koukios Assessment of biomass potential for power production: a GIS based method *Biomass and Bioenergy*, 20:101–112, 2001.
- [Hansen and Mladenović, 2001] P. Hansen and N. Mladenović. Variable neighbourhood search: Principles and applications. *European Journal of Operations Research*, 130:449–467, 2001.
- [ILO02] ILOG. *ILOG CPLEX 8.0 User’s Manual*. ILOG S.A., Gentilly, France, 2002.
- [Inyang, 2005] H.I. Inyang, editor. *Bridging the gaps for global sustainable development*. International Conference on Energy, Environment and Disasters, Charlotte, NC, July 2005.
- [Kucherenko and Sytsko, 2004] S. Kucherenko and Yu. Sytsko. Application of deterministic low-discrepancy sequences to nonlinear global optimization problems. *Computational Optimization and Applications*, 30(3):297–318, 2004.
- [L. Liberti and M. Dražić, 2005] L. Liberti and M. Dražić. Variable neighbourhood search for the global optimization of constrained nlps. *Proceedings of GO Workshop, Almeria, Spain*, 2005.
- [Liberti, 2004a] L. Liberti. Automatic reformulation of bilinear minlps. *DEI, Politecnico di Milano, Technical report n. 2004.24*, July 2004.
- [Liberti, 2004b] L. Liberti. Reduction constraints for the global optimization of nlps. *International Transactions in Operations Research*, 11(1):34–41, 2004.
- [Liberti, 2004c] L. Liberti. Linearity embedded in nonconvex programs. *Journal of Global Optimization*, (to appear) 2004.
- [Liberti and Kucherenko, 2005] L. Liberti and S. Kucherenko. Comparison of deterministic and stochastic approaches to global optimization. *International Transactions in Operations Research*, 12:263–285, 2005.
- [McCormick, 1976] G.P. McCormick. Computability of global solutions to factorable nonconvex programs: Part i — convex underestimating problems. *Mathematical Programming*, 10:146–175, 1976.
- [Regional Wood Energy Development Programme, 1998] Regional Wood Energy Development Programme. *Proceedings of the regional expert consultation on modern applications of biomass energy*. Food and Agricultural Organization of the UN, Bangkok, January 1998.
- [Sherali and Adams, 1986] H.D. Sherali and W.P. Adams. A tight linearization and an algorithm for 0-1 quadratic programming problems. *Management Science*, 32(10):1274–1290, 1986.
- [Sherali and Adams, 1999] H.D. Sherali and W.P. Adams. *A Reformulation-Linearization Technique for Solving Discrete and Continuous Nonconvex Problems*. Kluwer Academic Publishers, Dodrecht, 1999.
- [Sherali and Alameddine, 1992] H.D. Sherali and A. Alameddine. A new reformulation-linearization technique for bilinear programming problems. *Journal of Global Optimization*, 2:379–410, 1992.
- [Smith and Pantelides, 1999] E.M.B. Smith and C.C. Pantelides. A symbolic reformulation/spatial branch-and-bound algorithm for the global optimisation of nonconvex minlps. *Computers & Chemical Engineering*, 23:457–478, 1999.
- [Tawarmalani and Sahinidis, 2002] M. Tawarmalani and N.V. Sahinidis. Exact algorithms for global optimization of mixed-integer nonlinear programs. 2:65–86, 2002.