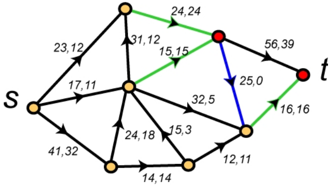
ARS Workshop

Markov Random Fields minimization and minimal cuts in image restoration

October 31, 2008



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Outline

1. **Context**
2. **Exact total variation minimization**
   - Total variation and regularization
   - *TV* models
   - Minimization
3. **Minimal cut (graph cut) as energy minimization**
   - Notations
   - General principle
   - Maximum flow / minimal cut
   - Energy representation
   - Results
   - More results
   - Further results for 3D images
4. **Conclusion**
   - Conclusion
   - Perspectives

Context

Main context

Image degradation

$$v = Hu + \eta$$

- $v \longrightarrow$ Observed image
- $u \longrightarrow$ Original image
- $\eta \longrightarrow$ Noise
- $H \longrightarrow$ Linear degradation

Goal

Obtain the best estimation $\bar{u}$ from u when $H = \text{identity}$.

Energy minimization

First approach

Restoration corresponds to find the minimum of

$$E(u,v) = \sum_{p \in \Omega} F_p(u_p, v_p)$$

with $\Omega \subset \mathbb{R}^2$.

- Inverse problem (Hadamard) $\Rightarrow$ noise amplification (when H $\neq$ Id).
- Need to regularize the solution.

$$E(u,v) = \sum_{p \in \Omega} \underbrace{F_p(u_p, v_p)}_{\text{Data fidelity term}} + \beta \cdot \sum_{\substack{p,q \in \Omega \\ \{p,q\} \in \mathcal{N}}} \underbrace{G_{p,q}(u_p, u_q)}_{\text{Regularization}} \qquad \forall \beta \in \mathbb{R}_*^+$$

Energy minimization

Standard minimization methods

→ **Continuous**

- Gradient descent.
- Graduated Non Convexity (GCN).

→ **Discrete**

- Dynamic programming (only in 1D).
- Simulated annealing.
- Iterated Conditional Modes.

Problems

- No or poor convergence guarantees.
- Solution not ever optimal.

Exact total variation minimization

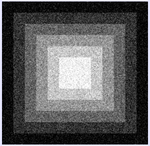
Regularization

Regularization (Tikhonov)

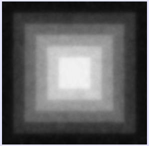
- **From** : Introduce by A. N. Tikhonov in 1963
- **Goal** : Consider restoration as find the minimum of

$$E(u) = \|u - v\|_{L^2}^2 + \beta \cdot \|\nabla u\|_{L^2}^2 \qquad \text{where} \qquad \|u\|_{L^p} = \left(\int_\Omega |u(x)|^p \, dx\right)^{\frac{1}{p}}$$

Problem



"Cubes" image $\sigma_b = 30$



Tikhonov restoration

Solution

- Regularize differently.
- Decrease the weight of big gradients.

BV Space

Definition

BV $\Rightarrow$ **Space of functions with bounded variations.**

$$BV(\Omega) = \{u \in L^1(\Omega) \mid \int_\Omega |\nabla u| < +\infty\}$$

Exact definition uses duality, because $|\nabla u|$ can be a measure.

with the semi-norm

$$|u|_{BV} = \int_\Omega |\nabla u| = TV(u) \qquad \Rightarrow \qquad \text{Total Variation}$$

Advantages

- Discontinuities are authorized along curves.
- Good space for geometric images.
- Existence and unicity of the solution.

Total Variation

Definition (co-area – continuous)

Let $u \in BV(\Omega)$. Total variation of u is

$$TV(u) = \int_{\Omega} |\nabla u| = \int_{\mathbb{R}} \int_{d\{u \leq \lambda\}} ds \, d\lambda,$$

where $\{u \leq \lambda\}$ is equivalent to $\{u(x) \in \Omega \mid u(x) \leq \lambda\}$.

Definition (co-area – discrete)

Let u be a discrete function. Total variation of u is

$$TV(u) = \sum_{\lambda=0}^{L-2} \sum_{\{p,q\} \in \mathcal{N}} w_{p,q} |u_p^\lambda - u_q^\lambda| \qquad \text{where} \qquad u_p^\lambda = \mathbf{1}_{\{u_p \geq \lambda\}}$$

Remarks

- (-) Details suppression (textures).
- (+) Allows sharp contours.

TV models

Definition

Let $v \in L^1(\Omega)$ the observed image. The TV model consist of finding

$$\underset{u \in BV(\Omega)}{\text{argmin}}\ TV(u) + \beta \|u - v\|_{L^\alpha}^{\alpha} \qquad \alpha \in \{1, 2\}$$

$TV + L_2$ Model / ROF (Rudin Osher Fatemi 92)

- (+) Strictly convex $\Rightarrow$ unicity.
- (-) Lost of contrast (iterative regularization).
- Gaussian noise.

$TV + L_1$ Model (Nikolova 2004)

- (-) Convex $\Rightarrow$ not unicity.
- (+) No contrast lost.
- Impulsive noise.

Level set approach

Principle

1. Decompose the image in order to solve a succession of quadratic binary optimization problems $\bar{u}^\lambda$ (MRF)
2. Solve each problem $\bar{u}^\lambda$ where the solution is a level set
3. Reconstruct $\bar{u}$ from $\bar{u}^\lambda$ (trivial)

Level set decomposition λ

- Upper-set $\rightarrow U^\lambda(u) = \{p \in \Omega \mid u_p \geq \lambda\}$
- Lower-set $\rightarrow L^\lambda(u) = \{p \in \Omega \mid u_p \leq \lambda\}$

Reconstruction

$$u_p = sup\{\lambda \in \mathcal{L} \mid p \in U_\lambda(u)\} \qquad \forall p \in \Omega$$

Level set approach

Reformulation - $TV + L_1$

$$\underset{u^\lambda \in \{0,1\}^N}{\text{argmin}} \; E_1^\lambda(u^\lambda) = TV(u^\lambda) + \beta \sum_{p \in \Omega} [(1 - y_p) u_p^\lambda + y_p (1 - u_p^\lambda)]$$

with

$$y_p = \mathbf{1}_{\{v_p \geq \lambda\}}$$

Reformulation - $TV + L_2$

$$\underset{u^\lambda \in \{0,1\}^N}{\text{argmin}} \; E_2^\lambda(u^\lambda) = TV(u^\lambda) + 2\beta \sum_{p \in \Omega} \Big((\lambda - 0.5) u_p^\lambda + v_p (1 - u_p^\lambda) \Big)$$

Level set approach

Reformulation

- Here, MRF are positive-negative quadratic pseudo-boolean functions, ie all the linear terms are positive and all the quadratic terms are negative (equivalent to submodular functions).
- Solve MRF is thus equivalent to find a maximal independant set in a bipartite graph, ie find a maximal flow – minimal cut in an associated graph.

Theorem

Minimizing E is equivalent to minimizing all the E^λ for each level.

Total energy $E(u) = \sum_{\lambda=0}^{L-2} E^\lambda(u^\lambda)$ can be minimized because $\{\bar{u^\lambda}\}_{\lambda=0\ldots L-2}$ is monotonous, ie:

$$\bar{u}^\lambda \leq \bar{u}^\mu \qquad \forall \lambda < \mu.$$

The optimal solution is given by

$$\forall p \in \Omega, \bar{u}_p = max\{\lambda, \bar{u}^\lambda = 1\}.$$

Minimal cut (graph cut) as energy minimization

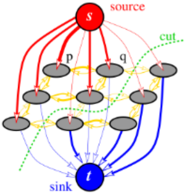
Notations

$\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is a directed weighted graph with two terminals s, t where

- $\mathcal{V} = \{1, \ldots, k\} \cup \{s\} \cup \{t\}$, $n = |\mathcal{V}|$
- $\mathcal{E} = \{(i, j) \mid 1 \leq i, j \leq n, i \neq j\}$, $m = |\mathcal{E}|$
- **Capacity** $\Rightarrow c : \mathcal{E} \to \mathbb{R}^+ \cup +\infty$
- **Flow** $\Rightarrow f : \mathcal{E} \to \mathbb{R}$

Vocabulary

Node s	$\longrightarrow$	source
Node t	$\longrightarrow$	sink
N-links	$\longrightarrow$	arcs (i, j)
T-links	$\longrightarrow$	arcs (s, i) and (i, t)



Example of a graph for a 3×3 image.

Definitions

Definition (flow)

Let $\mathcal{G}$ be a graph. $f(i,j)$ must verify

1) **Capacity constraints** $\quad f(i,j) \leq c(i,j) \quad \forall i,j \in \mathcal{V} \text{ et } \forall (i,j) \in \mathcal{E}$
2) **Flow symmetry** $\quad f(i,j) = -f(j,i) \quad \forall i,j \in \mathcal{V} \text{ et } \forall (i,j) \in \mathcal{E}$
3) **Kirchhoff law** $\quad \sum_{\substack{j \in \mathcal{V}-\{s,t\} \\ (i,j)\in\mathcal{E}}} f(i,j) = 0 \quad \forall i \in \mathcal{V} - \{s,t\}$

Definition (cut)

Cut is a partition $\mathcal{C} = (\mathcal{S}, \mathcal{T})$ of $\mathcal{V}$ such

$$s \in \mathcal{S},\ t \in \mathcal{T} \qquad \text{et} \qquad \mathcal{S} \cap \mathcal{T} = \emptyset,\ \mathcal{S} \cup \mathcal{T} = \mathcal{V}$$

Definition (Cut capacity)

The capacity of a cut $\mathcal{C}$ is

$$|\mathcal{C}| = \sum_{\substack{i \in \mathcal{S}, j \in \mathcal{T} \\ (i,j)\in\mathcal{E}}} c(i,j)$$

General principle

Theorem (Energy minimization (Greig Porteous Seheult 89))

Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be a directed weighted graph and E be an energy function. E can be minimized using a minimal cut in $\mathcal{G}$ for the image binary case.

Principle

1. Construct a graph $\mathcal{G}$.
2. Compute a minimal cut $\mathcal{C} = (\mathcal{S}, \mathcal{T})$ in $\mathcal{G} \Rightarrow$ minimize E.
3. Assign a value to each u_p such that

$$\begin{cases} u_p = 0 & \text{if } p \in \mathcal{S} \\ u_p = 1 & \text{if } p \in \mathcal{T} \end{cases}$$

Maximum flow / minimal cut

Maximum flow algorithms

- **Augmenting paths**
  Principle : Find iteratively a non saturated path from s to t in $\mathcal{G}$.

  Algorithms :

Ford-Fulkerson	$\rightarrow$	$O(m \cdot f)$, where $f = \text{maximum flow}$
Edmons-Karp	$\rightarrow$	$O(nm^2)$
Dinic	$\rightarrow$	$O(n^2 m)$
Boykov-Kolmogorov	$\rightarrow$	$O(n^2 m \lvert \mathcal{C} \rvert)$

- **Push-relabel**
  Principe : Propagate an excess of flow repeatedly from s to t in $\mathcal{G}$.

  Algorithms :

General push flow relabel	$\rightarrow$	$O(n^2 m)$
Push flow relabel with dynamic trees	$\rightarrow$	$O(nmlog(n))$

Energy representation

Questions

- Which energies can be minimized via minimal cuts ?
- How construct the graph to minimize E ?

Definition (representation (Kolmogorov Zabih 02))

Let E be an energy function with n binary variables

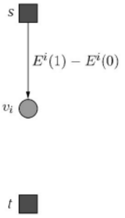
$$E(x_1, \ldots, x_n) = \sum_i E_i(x_i) + \sum_{i<j} E_{i,j}(x_i, x_j) \qquad \text{with} \qquad x_i \in \{0, 1\}.$$

- Every function with one variable can be represented by a graph.
- Every function with two variables can be represented by a graph iff

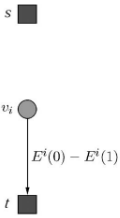
$$E_{i,j}(0,0) + E_{i,j}(1,1) \leq E_{i,j}(0,1) + E_{i,j}(1,0) \qquad \text{(submodular)}$$

Energy representation

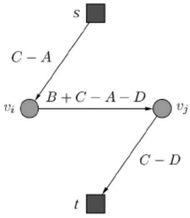
$$E^{i,j} = \begin{array}{|c|c|}\hline E^{i,j}(0,0) & E^{i,j}(0,1) \\ \hline E^{i,j}(1,0) & E^{i,j}(1,1) \\ \hline \end{array} = \begin{array}{|c|c|}\hline A & B \\ \hline C & D \\ \hline \end{array}$$



Energy E_i with $E_i(0) < E_i(1)$



Energy E_i with $E_i(0) > E_i(1)$



Energy E_{ij} with $C > A$ and $C > D$

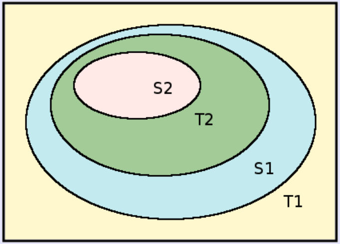
Minimization algorithms

Sequential algorithm

- **Proposed by** : Darbon, Chambolle, Zalesky.
- **Principle** : Do L independant optimizations.
- **Complexity** : $O(L \times F)$ with $O(F)$ the complexity to find the maximal flow – minimal cut.
- **Execution time** : < 1 min.

Dyadic algorithm

- **Proposed by** : Darbon, Chambolle, Hochbaum.
- **Principle** : Use the overlap between the level sets.
- **Complexity** : $O(log_2(L))$.



Results

Tests caracteristics

- Computer: AMD Athlon 64 X2 Dual Core 6000+, 2Go of RAM.
- Implementation under MegaWave2.
- Kolmogorov *et al.* library to compute the maximum flow.
- Images 256^2 and 512^2.
- Averages over 10 launchings.

Images

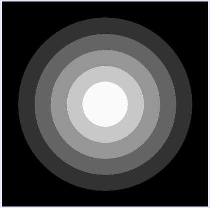


Image "Circles"


Image "Man"


Image "Elaine"

Results - $TV + L_1$

Neighborhood: 4-connexity

Image	Algorithm	$\beta = 0.25$	$\beta = 0.5$	$\beta = 1.0$	$\beta = 2.0$	$\beta = 4.0$
“Circles” 256^2	Sequential	0.07	0.07	0.06	0.06	0.06
	Dyadic	0.11	0.09	0.09	0.09	0.08
“Circles” 512^2	Sequential	0.29	0.27	0.27	0.27	0.27
	Dyadic	0.41	0.37	0.36	0.36	0.36
“Man” 256^2	Sequential	5.14	3.63	2.89	2.45	2.25
	Dyadic	0.46	0.35	0.23	0.13	0.10
“Man” 512^2	Sequential	19.65	14.18	11.73	10.30	9.67
	Dyadic	1.89	1.37	0.92	0.54	0.43
“Elaine” 256^2	Sequential	4.05	2.96	2.40	2.12	2.01
	Dyadic	0.43	0.32	0.23	0.13	0.10
“Elaine” 512^2	Sequential	15.85	11.67	9.97	9.14	8.71
	Dyadic	1.98	1.37	0.95	0.56	0.43

Neighborhood: 8-connexity

Image	Algorithm	$\beta = 0.25$	$\beta = 0.5$	$\beta = 1.0$	$\beta = 2.0$	$\beta = 4.0$
“Circles” 256^2	Sequential	0.23	0.18	0.16	0.16	0.16
	Dyadic	0.37	0.26	0.23	0.22	0.21
“Circles” 512^2	Sequential	0.79	0.67	0.64	0.64	0.63
	Dyadic	1.19	0.94	0.87	0.85	0.85
“Man” 256^2	Sequential	19.84	12.14	8.68	7.01	6.19
	Dyadic	1.61	0.97	0.69	0.48	0.27
“Man” 512^2	Sequential	74.19	44.80	35.66	27.97	25.01
	Dyadic	6.94	3.93	2.89	1.85	1.10
“Elaine” 256^2	Sequential	13.87	9.52	7.36	6.11	5.54
	Dyadic	1.49	0.87	0.66	0.47	0.27
“Elaine” 512^2	Sequential	56.59	36.39	27.48	24.17	22.57
	Dyadic	6.15	3.75	2.63	1.89	1.12

Results - $TV + L_2$

Neighborhood: 4-connexity

Image	Algorithm	$\beta = 0.01$	$\beta = 0.02$	$\beta = 0.04$	$\beta = 0.08$	$\beta = 0.16$
"Circles" 256^2	Sequential	3.46	2.76	2.46	2.32	2.26
	Dyadic	0.21	0.49	0.28	0.15	0.10
"Circles" 512^2	Sequential	14.41	11.74	10.26	9.68	9.57
	Dyadic	4.60	2.23	0.91	0.48	0.39
"Man" 256^2	Sequential	4.03	3.36	2.96	2.70	2.55
	Dyadic	0.55	0.39	0.30	0.24	0.20
"Man" 512^2	Sequential	17.27	14.16	12.40	11.26	10.64
	Dyadic	2.37	1.74	1.35	1.08	0.84
"Elaine" 256^2	Sequential	4.01	3.34	2.91	2.64	2.47
	Dyadic	0.53	0.42	0.33	0.26	0.20
"Elaine" 512^2	Sequential	17.25	14.38	12.29	11.08	10.48
	Dyadic	2.74	1.96	1.46	1.13	0.85

Neighborhood: 8-connexity

Image	Algorithm	$\beta = 0.01$	$\beta = 0.02$	$\beta = 0.04$	$\beta = 0.08$	$\beta = 0.16$
"Circles" 256^2	Sequential	10.34	8.13	6.91	6.36	6.03
	Dyadic	0.66	0.38	0.92	0.60	0.31
"Circles" 512^2	Sequential	43.84	32.68	27.79	24.90	23.99
	Dyadic	2.12	7.02	4.05	1.66	1.04
"Man" 256^2	Sequential	11.24	9.24	7.98	7.23	6.75
	Dyadic	1.00	0.90	0.73	0.57	0.48
"Man" 512^2	Sequential	46.66	37.18	31.91	28.78	26.83
	Dyadic	5.29	4.40	3.08	2.49	2.03
"Elaine" 256^2	Sequential	11.06	9.25	8.02	7.30	6.77
	Dyadic	1.09	0.89	0.77	0.62	0.51
"Elaine" 512^2	Sequential	48.13	38.25	33.07	29.49	27.39
	Dyadic	5.87	4.88	3.44	2.70	2.12

More results

Tests caracteristics

- Impulsive noise: $d_b = 20\%$ and $d_b = 40\%$.
- Gaussian noise: $\sigma_b = 15$ and $\sigma_b = 30$.
- Images 512^2.
- Connexity 8.

Images

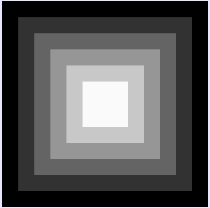
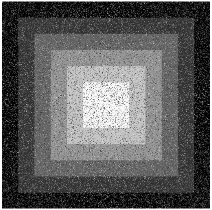


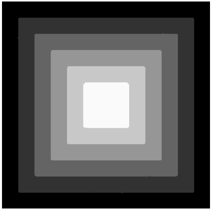
Image “Cubes”

Image “Man”

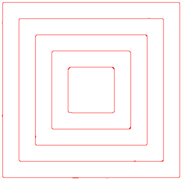
Results - $TV + L_1$ - $d_b = 20\%$



Noise - $SNR = 3.44$



Result - $\beta = 0.65$

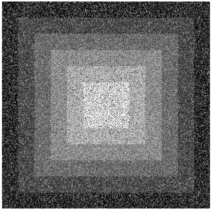


Level lines

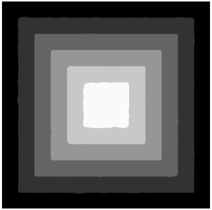
Bruit - $SNR = 2.11$

Result - $\beta = 3.8$

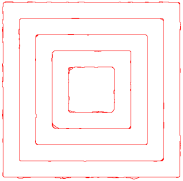
Results - $TV + L_1$ - $d_b = 40\%$



Noise - $SNR = 0.47$



Result - $\beta = 0.65$

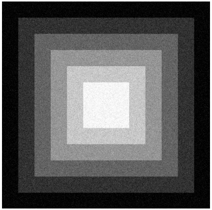


Level lines

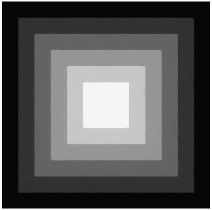
Noise - $SNR = -0.88$

Result - $\beta = 2.8$

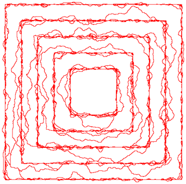
Results - $TV + L_2$ - $\sigma_b = 15$



Noise - $SNR = 14.70$



Result - $\beta = 0.04$

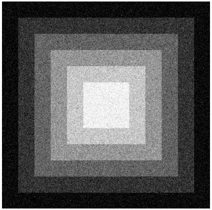


Level lines

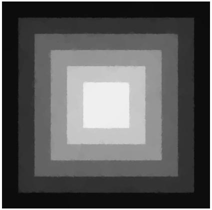
Noise - $SNR = 12.1$

Result - $\beta = 0.1$

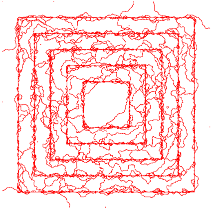
Results - $TV + L_2$ - $\sigma_b = 30$



Noise - $SNR = 8.82$



Result - $\beta = 0.03$



Level lines

Noise - $SNR = 6.25$

Result - $\beta = 0.06$

3D Images - Results - $TV + L^1$

Image	Algorithm	$\beta = 0.25$	$\beta = 0.5$	$\beta = 1.0$	$\beta = 2.0$	$\beta = 4.0$
SPHERE-40	Sequential	0.10	0.05	0.04	0.04	0.04
	Dyadic	0.60	0.17	0.18	0.15	0.15
SPHERE-40+d_b	Sequential	17.98	11.56	6.92	5.95	6.12
	Dyadic	0.72	0.22	0.20	0.20	0.21
SPHERE-80	Sequential	0.75	0.49	0.44	0.43	0.42
	Dyadic	2.09	1.82	1.77	1.75	1.75
SPHERE-80+d_b	Sequential	234.65	90.49	69.89	63.65	65.88
	Dyadic	2.96	2.38	2.24	2.19	2.33
FACTORIES-40	Sequential	10.93	8.62	6.72	5.91	5.04
	Dyadic	1.93	0.84	0.77	0.33	0.19
FACTORIES-40+d_b	Sequential	10.41	9.21	7.31	6.43	5.59
	Dyadic	0.95	0.96	0.79	0.37	0.19
FACTORIES-80	Sequential	154.71	96.00	73.69	62.15	55.72
	Dyadic	19.42	12.38	5.60	3.18	1.96
FACTORIES-80+d_b	Sequential	166.67	108.72	80.44	67.14	60.26
	Dyadic	20.01	10.37	6.90	3.59	2.01

Table: Computation times (seconds) for $TV + L^1$ with 6 connexity. 3D images: 40^3 and 80^3.

3D Images - Results - $TV + L^1$

Image	Algorithm	$\beta = 0.25$	$\beta = 0.5$	$\beta = 1.0$	$\beta = 2.0$	$\beta = 4.0$
SPHERE-40	Sequential	0.51	0.49	0.46	0.22	0.16
	Dyadic	3.53	3.40	3.14	0.68	0.62
SPHERE-40+d_b	Sequential	110.38	110.06	106.39	56.08	28.82
	Dyadic	4.73	4.48	3.78	0.97	0.84
SPHERE-80	Sequential	8.01	7.39	2.52	1.52	1.31
	Dyadic	59.35	54.78	6.69	5.72	5.51
SPHERE-80+d_b	Sequential	1802.33	1643.56	873.04	299.68	218.54
	Dyadic	73.46	63.76	9.33	7.31	7.16
FACTORIES-40	Sequential	44.16	43.96	52.74	43.61	29.97
	Dyadic	5.48	4.91	4.84	4.25	3.07
FACTORIES-40+d_b	Sequential	42.76	42.42	46.45	45.33	32.96
	Dyadic	5.48	5.12	7.77	5.48	3.72
FACTORIES-80	Sequential	587.27	1027.61	783.87	410.16	259.07
	Dyadic	78.69	119.34	129.47	95.27	23.31
FACTORIES-80+d_b	Sequential	530.08	720.27	819.68	477.99	286.81
	Dyadic	73.00	168.33	141.93	65.86	29.39

Table: Computation times (seconds) for $TV + L^1$ with 26 connexity. 3D images: 40^3 and 80^3.

3D Images - Results - $TV + L^2$

Image	Algorithm	$\beta = 0.01$	$\beta = 0.02$	$\beta = 0.04$	$\beta = 0.08$	$\beta = 0.16$
SPHERE-40	Sequential	7.52	6.19	5.55	5.34	5.05
	Dyadic	0.15	0.15	0.15	0.15	0.15
SPHERE-40+d_b	Sequential	7.29	6.18	5.74	5.46	5.36
	Dyadic	0.45	0.40	0.35	0.38	0.32
SPHERE-80	Sequential	81.86	67.72	63.29	57.48	56.52
	Dyadic	1.74	1.74	1.74	1.74	1.74
SPHERE-80+d_b	Sequential	78.92	68.82	63.94	60.42	59.70
	Dyadic	4.32	3.80	3.86	3.65	3.79
FACTORIES-40	Sequential	9.63	7.50	6.52	5.90	5.54
	Dyadic	0.99	0.94	0.56	0.41	0.36
FACTORIES-40+d_b	Sequential	9.60	7.64	6.69	6.04	5.62
	Dyadic	0.99	0.88	0.55	0.40	0.35
FACTORIES-80	Sequential	110.99	83.62	71.53	64.97	60.73
	Dyadic	19.58	11.02	6.34	5.14	4.50
FACTORIES-80+d_b	Sequential	112.19	84.65	72.29	65.26	61.34
	Dyadic	18.28	10.40	6.23	4.36	3.65
CELLULES-40	Sequential	6.33	6.72	6.00	5.46	5.19
	Dyadic	0.93	1.18	0.89	0.55	0.54
CELLULES-80	Sequential	107.74	84.60	68.65	61.70	58.07
	Dyadic	29.35	33.07	12.75	7.58	4.94

Table: Computation times (seconds) for $TV + L^2$ with 6 connexity. 3D images: 40^3 and 80^3.

3D Images - Results - $TV + L^2$

Image	Algorithme	$\beta = 0.01$	$\beta = 0.02$	$\beta = 0.04$	$\beta = 0.08$	$\beta = 0.16$
SPHERE-40	Sequential	66.48	42.82	30.55	24.54	21.71
	Dyadic	0.65	0.61	0.60	0.61	0.60
SPHERE-40+d_b	Sequential	64.45	41.20	29.81	25.01	22.64
	Dyadic	2.32	2.19	1.85	1.66	1.56
SPHERE-80	Sequential	598.28	378.50	274.36	219.81	209.11
	Dyadic	5.87	5.54	5.50	5.62	5.65
SPHERE-80+d_b	Sequential	550.09	347.81	260.82	219.12	196.73
	Dyadic	27.87	20.94	17.62	15.52	14.38
FACTORIES-40	Sequential	66.24	54.15	39.77	30.34	25.40
	Dyadic	8.90	5.69	3.75	3.55	2.12
FACTORIES-40+d_b	Sequential	65.75	54.01	39.59	30.29	25.75
	Dyadic	8.59	5.68	3.86	3.22	2.15
FACTORIES-80	Sequential	1099.51	673.16	391.58	281.06	227.92
	Dyadic	173.10	75.40	52.07	38.89	21.55
FACTORIES-80+d_b	Sequential	1086.86	662.72	387.27	281.60	231.48
	Dyadic	108.21	67.50	49.87	33.01	20.38
CELLULES-40	Sequential	23.09	23.98	24.71	26.91	24.28
	Dyadic	3.05	3.43	3.76	5.24	3.03
CELLULES-80	Sequential	262.14	277.04	373.07	294.75	239.85
	Dyadic	50.98	61.38	126.01	67.98	48.13

Table: Computation times (seconds) for $TV + L^2$ with 26 connexity. 3D images: 40^3 and 80^3.

Conclusion

Conclusion

TV minimization

- (+) Exact solutions.
- (+) Quick results.
- (-) Restricted energy classes.
- (-) Over-smoothing along the discontinuities.

Perspectives

Parametric flow

Objectif : re-use the flow value.

Conditions :

- Arcs $(s, i) \longrightarrow$ non-increasing capacities.
- Arcs $(i, t) \longrightarrow$ non-decreasing capacities.
- Arcs $(i, j) \longrightarrow$ constant capacities.

Results : Less improvements than for the dyadic technique (Darbon Chambolle 08).

Applications : interactive segmentation, video segmentation.

Perspectives

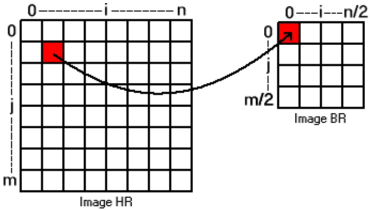
Extension to multiway cut

Multi-labelling

Extension to other operators

Goal : generalize restoration to other operators H (convolution, sampling).

Applications : confocal microscopy, IRM.



Extension to other energy minimization models

Potts model