A Hybrid Linear Logic for Constrained Transition Systems

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To reason about state transition systems, we need a logic of state. Linear logic [8] is such a logic and has been successfully used to model such diverse systems as process calculi, references and concurrency in programming languages, security protocols, multi-set rewriting, and graph algorithms. Linear logic achieves this versatility by representing propositions as *resources* that are combined using $\otimes$, which can then be transformed using the linear implication ($\multimap$). However, linear implication is timeless: there is no way to correlate two concurrent transitions. If resources have lifetimes and state changes have temporal, probabilistic or stochastic *constraints*, then the logic will allow inferences that may not be realizable in the system being modelled. The need for formal reasoning in such constrained systems has led to the creation of specialized formalisms such as Continuous Stochastic Logic (*CSL*) [2] or Probabilistic *CTL* [9], that pay a considerable encoding overhead for the states and transitions in exchange for the constraint reasoning not provided by linear logic. A prominent alternative to the logical approach is to use a suitably enriched process algebra such as the stochastic and probabilistic π-calculi or the κ-calculus [6]. Processes are animated by means of simulation and then compared with the observations. Process calculi do not however completely fill the need for *formal logical reasoning for constrained transition systems.*

We propose a simple yet general method to add constraint reasoning to linear logic. It is an old idea—*labelled deduction* [13] with *hybrid* connectives [3]—applied to a new domain. Precisely, we parameterize ordinary logical truth on a *constraint domain*: $A@w$ stands for the truth of A under constraint w. Only a basic monoidal structure is assumed about the constraints from a proof-theoretic standpoint. We then use the hybrid connectives of *satisfaction* and *localization* to perform generic symbolic reasoning on the constraints at the propositional level. We call the result *hybrid linear logic* (*HyLL*). *HyLL* has a generic cut-free (but cut admitting) sequent calculus that can be strengthened with a focusing restriction [1] to obtain a normal form for proofs. Any instance of *HyLL* that gives a semantic interpretation to the constraints continues to enjoy these proof-theoretic properties.

Focusing allows us to treat *HyLL* as a *logical framework* for constrained transition systems. Logical frameworks with hybrid connectives have been considered before; hybrid *LF* (*HLF*), for example, is a generic mechanism to add many different kinds of resource-awareness, including linearity, to ordinary *LF* [12]. *HLF* follows the usual *LF* methodology by keeping the logic of the framework minimal: its proof objects are canonical (β-normal η-long) natural deduction terms, where canonicity is known to be brittle because of permutative equivalences [14]. With a focused sequent calculus, we have more direct access to a canonical representation of proofs, so we can enrich the framework with any connectives that obey the focusing discipline. The representational adequacy of an encoding in terms of (partial) focused sequent derivations tends to be more straightforward than in a natural deduction formulation. We illustrate this by encoding the synchronous stochastic π-calculus ($S\pi$) in *HyLL* using rate functions as constraints.

In addition to the novel stochastic component, our encoding of $S\pi$ is a conceptual improvement over other encodings of π-calculi in linear logic. In particular, we perform a full propositional reflection of processes as in [10], but our encoding is first-order and adequate as in [4]. *HyLL* does not itself prescribe an operational semantics for the encoding of processes; thus, bisimilarity in continuous time Markov chains (*CTMC*) is not the same as logical equivalence in stochastic *HyLL*, unlike in *CSL* [7]. This is not a deficiency; rather, the *combination* of focused *HyLL* proofs and a proof search strategy tailored to a particular encoding is necessary to produce faithful symbolic executions. This exactly mirrors *Sπ* where it is the simulation rather than the transitions in the process calculus that is shown to be faithful to the *CTMC* semantics [11].

This work has the following main contributions. First is the logic *HyLL* itself and its associated proof-theory, which has a clean judgemental pedigree in the Martin-Löf tradition. Second, we show how to obtain many different instances of *HyLL* for particular constraint domains because we only assume a basic monoidal structure for constraints. Third, we illustrate the use of focused sequent derivations to obtain adequate encodings by giving a novel adequate encoding of *Sπ*. Our encoding is, in fact, *fully adequate* – partial focused proofs are in bijection with traces. The ability to encode *Sπ* gives an indication of the versatility of *HyLL*. Eventually, we hope to use *HyLL* to encode the stochastic transition systems of formal molecular biology in a sense similar to the κ-calculus [6]. This is the focus of our efforts at the I3S.

The full version of this paper is available as a technical report [5].

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