Optimality for Tough Combinatorial Hydro Valley Problems

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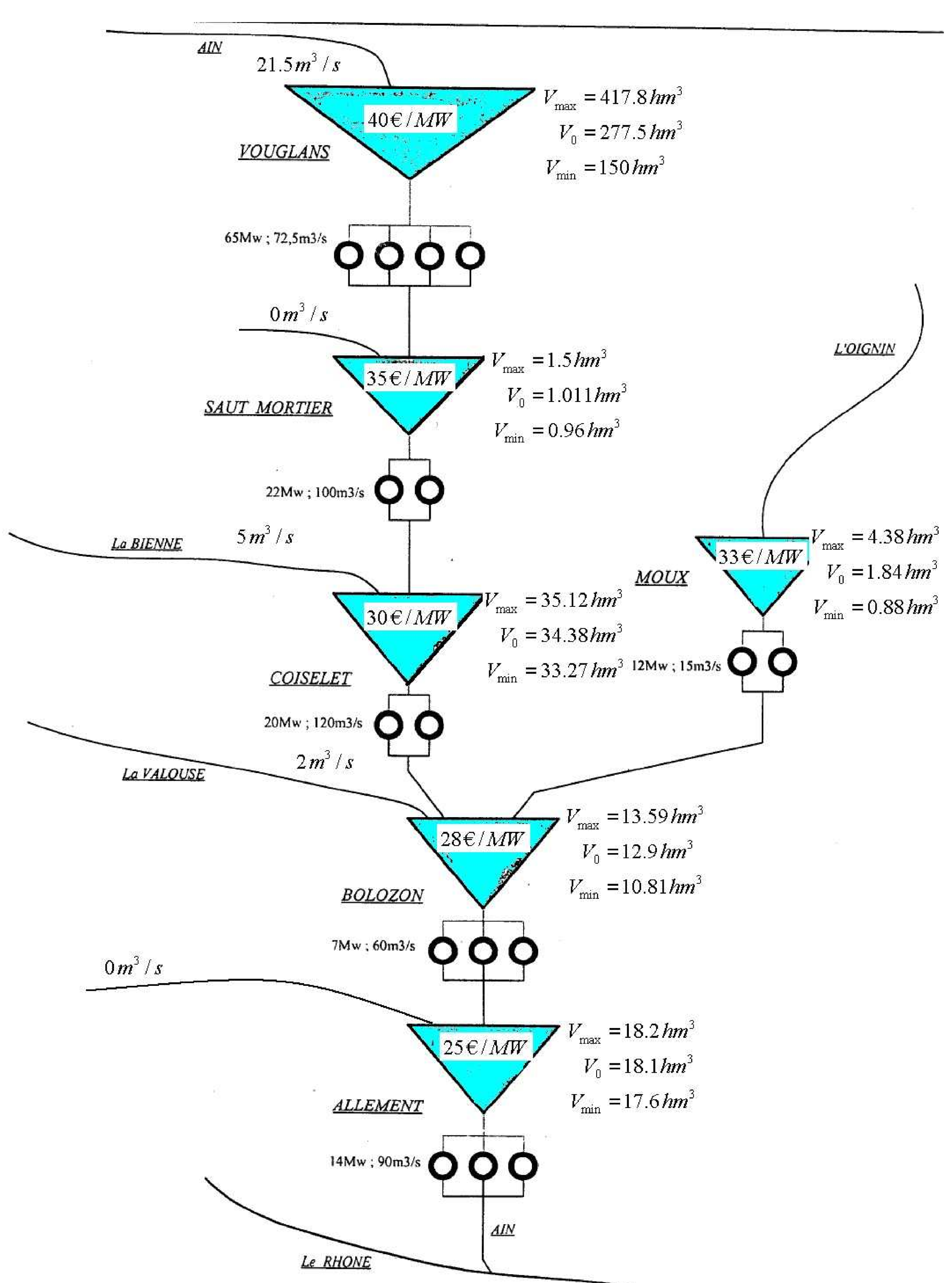
Introduction

The aim of the project is to study a crucial problem in energy management: the **Unit Commitment (UC)** sub-problem dedicated to **hydro valley management**. When continuous, such a problem is easily solved to optimality by any current LP solver. However, the introduction of combinatorial elements leads to far tougher hydro valley problems. This is especially true for some of the larger French Hydro valleys. This difficulty is amplified by the fact that such hydro valley problems are typically solved many times over the course of the iterations of some Decomposition method for the UC problem. One employs **optimization methods** to take decisions on the **production schedule** in the short/medium term for a hydro valley composed of different and connected reservoirs.

Example



Notation

Sets:

- $(\mathcal{N}, \mathcal{V})$ = directed graph representing the hydro valley.
- $\mathcal{T}$ = set of time steps.
- $\mathcal{U}$ = set of turbines.
- $\mathcal{P}$ = set of pumping stations.
- $\mathcal{S}$ = set of active power and spinning reserves.
- $\mathcal{F}_n = \{m \in \mathcal{N} | a_{mn} = 1\}$ $(n \in \mathcal{N})$.
- $\mathcal{D}_n = \{m \in \mathcal{N} | a_{nm} = 1\}$ $(n \in \mathcal{N})$.
- $\mathcal{X}_{ut}$ = set of operational levels for turbine u at time period t $(u \in \mathcal{U}, t \in \mathcal{T})$

Parameters:

- A_{nm} = connection matrix element (it is 1 when the water released from reservoir n flows into reservoir m, 0 otherwise).
- T = time step size [hours].
- D_v = the number of time steps the water takes to flow (get pumped) through arc v $(v \in \mathcal{V})$.
- I_{nt} = inflows at period t to reservoir n $(t \in \mathcal{T}, n \in \mathcal{N})$ [m³/h]
- G = gradient slopes [m³/h²]
- X_{uti} = i-th operational level for turbine u at time period t $(u \in \mathcal{U}, t \in \mathcal{T}, i \in \mathcal{X}_{ut})$ [m³/h]
- $\overline{X}_u$ = upper bound on the water flow passing through turbine u $(u \in \mathcal{U})$ [m³/h]
- $\overline{Y}_p$ = upper bound on the water pumped by pump p $(p \in \mathcal{P})$ [m³/h]
- $\mu_u = \{v \in \mathcal{V} |$ turbine u represents arc $v\}$
- $\mu'_p = \{v \in \mathcal{V} |$ pump p represents arc $v\}$
- $\rho_u(x, v)$ = efficiency function of turbine u [MW]
- $\theta_p(y)$ = efficiency function of pump p [MW]

Variables

x_{ut} = water flow passing through turbine u in period t $(u \in \mathcal{U}, t \in \mathcal{T})$ [m³/h].
y_{pt} = water pumped by pump p in period t $(p \in \mathcal{P}, t \in \mathcal{T})$ [m³/h].
v_{nt} = water volume in reservoir n in period t $(n \in \mathcal{N}, t \in \mathcal{T})$ [m³].
z_{uti} = auxiliary binary variable (1 when turbine u has operational level $\geq i$ in period t, 0 otherwise) $u \in \mathcal{U}, t \in \mathcal{T}$.

Constraints

- **Equilibrium constraint** $(\forall n \in \mathcal{N}, t \in \mathcal{T})$:

$$v_{nt} = v_{n(t-1)} + \sum_{m \in \mathcal{F}_n : D_{(m,n)} \leq t} \; \sum_{u \in \mathcal{U} : \mu_u = (m,n)} x_{u(t-D_{(m,n)})} T - \sum_{m \in \mathcal{F}_n} \; \sum_{u \in \mathcal{U} : \mu_u(n,m)} x_{ut} T$$
$$+ \sum_{m \in \mathcal{D}_n : D_{(m,n)} \leq t} \; \sum_{p \in \mathcal{P} : (n,m)} y_{p(t-D_{(m,n)})} T - \sum_{m \in \mathcal{D}_n} \; \sum_{p \in \mathcal{P} : \mu'_p = (m,n)} y_{pt} T + I_{nt} T$$

with v_{n0} = initial volume of reservoir n $(n \in \mathcal{N})$.

- **Gradient constraint** $(\forall u \in \mathcal{U}, t \in \mathcal{T})$:

$$-GT \leq x_{ut} - x_{u(t-1)} \leq GT$$

with x_{u0} is the initial flow in turbine u $(\forall u \in \mathcal{U})$.

- **Discrete operational levels constraints** $(u \in \mathcal{U}, t \in \mathcal{T})$:

$$x_{ut} = \sum_{i = \mathcal{X}_{ut}} z_{uti}(X_{uti} - X_{ut(i-1)})$$
$$z_{ut(i+1)} \leq z_{uti} \qquad (i \in \mathcal{X}_{ut})$$
$$-1 \leq z_{uti} - z_{u(t-1)i} - z_{u(t+1)(i+1)} \leq 0 \qquad (i \in \mathcal{X}_{ut})$$
$$0 \leq z_{ut1} + z'_{pt1} \leq 1 \qquad \forall u \in \mathcal{U}, p \in \mathcal{P} : \mathfrak{i}(u,p) = 1$$
$$0 \leq z_{ut1} + z'_{p(t+1)1} \leq 1 \qquad 1 \forall u \in \mathcal{U}, p \in \mathcal{P} : \mathfrak{i}(u,p) = 1$$
$$0 \leq z_{u(t+1)1} + z'_{pt1} \leq 1 \qquad \forall u \in \mathcal{U}, p \in \mathcal{P} : \mathfrak{i}(u,p) = 1$$

where $\mathfrak{i}(u,p) = 1$ when turbine u is reversable and acts also as pump p.

- **Bounds on discharged water due to active power and spinning reserves**:

$$0 \leq x_{ut} + \sum_{s \in \mathcal{S}} f_s(x_{ut}) \leq X_u \qquad (u \in \mathcal{U}, t \in \mathcal{T})$$

- **Simple bounds**:

$$0 \leq x_{ut} \leq X_u \qquad (u \in \mathcal{U}, t \in \mathcal{T})$$
$$0 \leq y_{pt} \leq Y_p \qquad (u \in \mathcal{U}, t \in \mathcal{T})$$
$$z_{uti} \in \{0, 1\} \qquad (u \in \mathcal{U}, t \in \mathcal{T}, i \in \mathcal{X})$$

Objective function

Minimizing:

- Cost incurred by pumping $\sum_{t \in \mathcal{T}} \lambda_t T \sum_{p \in \mathcal{P}} \theta_p(y_{pt})$
- Cost of using water expressed by the water-values [different modeling possibilities]

Minus

- Gain generated by pumping

$$\sum_{t \in \mathcal{T}} \lambda_t T \sum_{u \in \mathcal{U}} \rho_u(x_{ut}, v_{nt}) + \sum_{s \in \mathcal{S}} \sum_{t \in \mathcal{T}} \lambda_t T \sum_{u \in \mathcal{U}} (\rho_u(\sum_{s' \in \mathcal{S} : s' \leq s} f_S(x_{ut}), v_{mt}) + \rho_u(\sum_{s' \in \mathcal{S} : s' < s} f'_s(x_u t), v_{nt}))$$

where λ_t are price signals (see project "Consistent Dual Signals and Optimal Primal Solutions")

Solution Approaches

This problem is practically difficult to solve because of the **complicated constraints** involved, and the **large size of real instances**. Moreover, because of the presence of continuous and binary variables and general nonlinear constraints, the mathematical programming formulation describes a Mixed Integer (Non) Linear Programming (MI(N)LP) problem. This kind of problems are among the most difficult to solve in mathematical programming and can be attacked in different ways. Solving these large-scale, real-world problems with the required efficiency by straightforward application of standard MI(N)LP optimization tools is impossible. Therefore, specialized methods or are required to provide solutions with **provable high accuracy in a limited amount of time**. These require to simultaneously pursue two very definitely connected but different lines of research:

1. modeling and reformulations: formulation strengthening, cuts, decomposition methods, and approximations to efficiently provide effective lower bounds on the optimal value;
2. heuristics: matheuristics, possibly exploiting the formulations/decompositions/approximations of point 1, to efficiently provide good quality feasible solutions.

For more information...

Visit the project web site:
`http://www.lix.polytechnique.fr/~dambrosio/PGMO.php.`