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Applications of Reformulations in Mathematical Programming

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This is a summary of the author's PhD thesis supervised by Pierre Hansen, Leo Liberti, and Ider Tseveendorj and defended on September 18, 2012 at École Polytechnique, Palaiseau, France. The thesis is written in English and is available from the author upon request at `costa@lix.polytechnique.fr` and from `http://pastel.archives-ouvertes.fr/pastel-00746083`. This work concerns the study of some Mathematical Programming problems and the analysis of the impact given by the application of some reformulation techniques for solving such problems. We considered the following classification of reformulations: *exact reformulations* (also called *opt-reformulations*), *narrowings, relaxations* (see Liberti, "Reformulations in Mathematical Programming: Definitions and Systematics", *RAIRO-OR*, 43(1):55-86, 2009). For each kind of reformulation a different problem is considered, and it is shown that the reformulation was crucial to obtain a good solution.

The first part of the thesis concerns exact reformulations, where the set of global and local optima are the same as in the original formulation. The main problem studied is that of clustering by means of modularity maximization. Some exact methods and several heuristics are proposed in the literature to solve this problem. We focus on one of these heuristics, namely a hierarchical divisive approach that iteratively divides a cluster in two new clusters in an optimal way, by using a $0-1$ Mixed Integer Quadratic Programming (MIQP) model solved by CPLEX, until the modularity increases. Some reformulations of this model are proposed, leading to a reduction of the computational time by an order of magnitude. We then adapt the divisive heuristic to bipartite graphs, where the objective function to maximize is bipartite modularity. In this case the problem cannot be described by using a 0-1 MIQP model, thus different reformulations are employed. Tests showed the high impact of reformulation techniques on computational time, and also a good quality of the results in term of bipartite modularity value if compared with those obtained by other heuristics. After that, another contribution related to clustering is presented. In fact, one can find clusters by specifying some rules that each cluster must respect. We considered one of these criteria, called *strong* (see Radicchi et al.,"Defining and identifying communities in networks", *PNAS*, 101(9):2658-2663, 2004), and we modified it obtaining a new criterion that is called *almost-strong*. We then devised an algorithm to find clusters in the strong and almost-strong sense. A comparison of the results showed that the almost-strong criterion provided more informative partitions with respect to the strong one.

The second part of the thesis concerns narrowing reformulations, where the reformulated problem can have fewer global optima than the original one. This is useful for problems presenting several symmetric optima. We consider the problem of packing equal circles in a square as it involves a high degree of symmetry, i.e., given a solution, an equivalent one can be obtained by swapping a circle with another one, and also by swapping the x and y axes. Some Symmetry Breaking Constraints (SBCs) are proposed to make some of these symmetric optima infeasible, leading to narrowing reformulations. We use the Couenne solver, implementing a spatial Branch-and-Bound (sBB) algorithm, for our tests. The effect of the SBCs is that the size of the sBB tree is reduced, as well as the time to obtain the optimal solution. Moreover, by using some SBCs, good solutions are found already at the root node of the sBB tree. We then proposed other constraints that help tighten the formulation, and a conjecture about the tightening of the bounds of some of the problem variables, whose effect is to improve the quality of the bounds provided by Couenne at the beginning of the solution process.

The last part of the thesis is devoted to relaxations, where the optimal solution is a lower (upper) bound with respect to the original problem in case of minimization (maximization). We studied problems involving multilinear terms, and we compared two different relaxations which provide the same optimal solution. The first, called *primal*, is obtained by replacing each multilinear term with a new variable and by adjoining some linear constraints to the model. Such constraints are known explicitly only for bilinear terms (McCormick inequalities), trilinear terms (Meyer-Floudas inequalities), and partly for quadrilinear terms. On the other hand, we considered the *dual* relaxation, i.e., the convex combination (using dual variables) of the extreme points of the convex hull associated to the multilinear term. A comparison of the computational time needed to solve some random instances involving multilinear terms showed that the dual relaxation outperforms the primal when the number of variable increases, and that this behavior is even more remarkable when considering problems involving integer variables.