

# Probabilistic Methods in Concurrency

## Lecture 5

Basics of Measure Theory and Probability Theory  
Probabilistic Automata

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# Basics of Measure Theory and Continuous Probability Theory

- Continuous probabilistic spaces: the need for Measure Theory
- Some introductory examples:
  - Infinite sequence of coin tossing,
  - Intersection, complementation, union and countable union
  - Why countable
  - Concept of cone
- Measurable space,  $\sigma$ -field
- Base,  $\sigma$ -field generated by a base
- Examples
- Probability measure
- Monotonicity and continuity

Prakash Panangaden, Stochastic techniques in Concurrency. Lect notes.  
Section 2.1. and 2.4 (till page 16)

# Probabilistic Automata

- Nondeterministic choice and probabilistic choice
- Definition of probabilistic automata
- Concept of adversary
- Concept of execution
- The measurable space and the probability measure associated to the executions
- Some examples
  
- Roberto Segala. *Modeling and Verification of Randomized Distributed Real Time Systems*. PhD thesis, Laboratory for Computer Science, Massachusetts Institute of Technology, June 1995. Available as Technical Report MIT/LCS/TR-676.
  
- Roberto Segala and Nancy Lynch. Probabilistic simulations for probabilistic processes. *Nordic Journal of Computing*, 2(2):250--273, 1995. An extended abstract appeared in the Proceedings of CONCUR '94, LNCS 836: 22--25.