Foundations of Privacy

Lecture 8

Motivation

Can differential privacy be adapted to different privacy requirements?

Can we use differential privacy on secrets that are not databases?

Outline

- Generalization of differential privacy
- Privacy in the context of statistical databases
- Privacy in location-based systems

▶ Adjacency: $x \sim_h x'$ iff they differ in exactly one individual

$$x = \langle 32, 41, 27 \rangle$$

 $x' = \langle 32, 52, 27 \rangle$

• $K: \mathcal{X} \to \mathcal{P}(\mathcal{Z})$ satisfies ϵ -differential privacy iff

$$K(x)(Z) \le e^{\epsilon} K(x')(Z) \qquad \forall x \sim_h x'$$

 $ightharpoonup \epsilon$: distinguishability level between adjacent databases

► Hamming distance $d_h(x, x')$: # of elements in which x, x' differ

$$x = \langle 32, 41, 27 \rangle$$

 $x' = \langle 21, 52, 27 \rangle$ $d_h(x, x') = 2$

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- Expected properties:
 - $\epsilon(x,x)=0$

 - $\begin{vmatrix}
 \boldsymbol{\varepsilon}(x_1, x_2) \leq b \\
 \boldsymbol{\varepsilon}(x_3, x_2) \leq b
 \end{vmatrix} \Rightarrow \boldsymbol{\varepsilon}(x_1, x_3) \leq f(b)$

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 \end{vmatrix} \Rightarrow \boldsymbol{\varepsilon}(x_1, x_3) \leq f(b)$
- We take $\boldsymbol{\varepsilon}(x, x')$ to be a metric, denoted d_{χ}

$$d_{\mathcal{X}}(x_1, x_3) \le d_{\mathcal{X}}(x_1, x_2) + d_{\mathcal{X}}(x_3, x_2)$$

$$d_x$$
-privacy
 $K(x)(Z) \le e^{d_X(x,x')} K(x')(Z) \quad \forall x, x'$

- ▶ the less distinguishable two secrets are, the more similar the outcome should be
- ► There is no ϵ , but we can just rescale the metric in order to obtain the desired level of privacy: $d_{\chi} = \epsilon d_{\chi}'$
- ϵ -differential privacy = ϵd_h -privacy

$$d_{x}$$
-privacy $K(x)(Z) \le e^{d_{x}(x,x')} K(x')(Z) \quad \forall x, x'$

This notion of privacy protects the accuracy of the data

- Foundations
 - Compositionality
 - ► Implementation: Laplacian
 - Optimality results
- Applications
 - Statistical databases (normalized) Manhattan distance
 - Location privacy Geographical distance
 - In general, every domain equipped with a metric

Compositionality

If K, K' are d_{χ} and d_{χ}' differentially private, then the composition of the two mechanisms, (K, K'), is $d_{\chi} + d_{\chi}'$ differentially private

Answering queries

- ▶ Query $f: \mathcal{X} \to \mathcal{Y}$
- ▶ f is Δ -sensitivite wrt d_{χ} , d_{γ} iff:

$$\Delta = \max_{x,x'} \frac{d_{y}(f(x), f(x'))}{d_{x}(x, x')}$$

- ▶ If $H: \mathcal{Y} \to \mathcal{P}(\mathcal{Z})$ satisfies $d_{\mathcal{Y}}$ -privacy then $H \circ f$ satisfies $\Delta d_{\mathcal{X}}$ -privacy
- ► *H* can be implemented in the usual way as Laplacian noise:

$$H(y)(z) = c \cdot e^{\frac{-d_{y}(z,y)}{\Delta}\epsilon}$$

We can easily prove that H satisfies $\frac{dy}{\Delta}\epsilon$ -privacy , and consequently $H\circ f$ satisfies $d_x\epsilon$ -privacy

Outline

- ► Generalization of differential privacy
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- Privacy in location-based systems

► The Hamming distance is independent from the actual values

$$x_1 = \langle 32, 0, 27 \rangle$$

 $x_2 = \langle 32, 0.01, 27 \rangle$
 $x_3 = \langle 32, 10^6, 27 \rangle$ $d_h(x_1, x_2) = d_h(x_1, x_3) = 1$

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- Many queries are insensitive to minor changes in values

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- ▶ the disting. level between x_1 , x_2 and x_2 , x_3 is the same
- Many queries are insensitive to minor changes in values
- ▶ If ϵ is "weak", we might require higher protection for x_1, x_2

Manhattan metric:

$$d_1(x, x') = \sum_{i=1}^n d_v(x[i], x'[i])$$

Normalized Manhattan metric:

$$\widetilde{d}_1(x,x') = \frac{d_1(x,x')}{d_{\mathcal{V}}(\mathcal{V})}$$

where $d_{\mathcal{V}}(\mathcal{V})$ is the maximum distance among the values

▶ Stronger that Hamming: $\widetilde{d}_1(x, x') \leq d_h(x, x')$

$$x_1 = \langle 32, 0, 27 \rangle$$

 $x_2 = \langle 32, 0.01, 27 \rangle$ $\widetilde{d}_1(x_1, x_2) = 10^{-8}$
 $x_3 = \langle 32, 10^6, 27 \rangle$ $\widetilde{d}_1(x_1, x_3) = 1$

Advantages of the normalized Manhattan metric

Sensitivity:

- ▶ For a family of queries (sum, average, percentile, ...), the sensitivity wrt \widetilde{d}_1 , $d_{\mathbb{R}}$ and d_h , $d_{\mathbb{R}}$ coincide
- ▶ In general, \tilde{d}_1 is smaller than d_h
- ▶ hence we get stronger privacy with the same noise

Optimality:

- ▶ If the set of values is discrete, then sum, average and percentile queries induce a graph structure which is a straight line
- As a consequence, the Geometric mechanism is universally optimal for sum, average and percentile queries wrt \widetilde{d}_1
- ► In contrast, we saw that only counting queries have universally optimal mechanisms wrt *d_h*

The Manhattan metric

▶ We can use the Manhattan metric without normalization:

$$d_1(x, x') = \sum_{i=1}^n d_{\nu}(x[i], x'[i])$$

▶ d_1 can be much higher that Hamming, but Δ will be proportionally smaller than the usual sensitivity, so the protection, with respect to the introduced noise, is comparable.

Example:

$$x_1 = \langle 32, 0, 27 \rangle$$

 $x_2 = \langle 32, 0.01, 27 \rangle$ $\widetilde{d}_1(x_1, x_2) = 10^{-2}$
 $x_3 = \langle 32, 10^6, 27 \rangle$ $\widetilde{d}_1(x_1, x_3) = 10^6$

The Manhattan metric

- ► The Manhattan metric be useful when we need to prevent the attacker from getting very precise data (for instance because they can be used to identify an individual),
- Trade-off between privacy and utility
- ▶ Optimality results similar to \widetilde{d}_1

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Motivation

Geographical information is becoming essential for a variety of services: LBS, advertising, social networks, data mining, . . .



Privacy: location data are often sensitive and need protection

Location-Based Systems

A **location-based system** is a system that uses geographical information in order to provide a service.

- ▶ Retrieval of Points of Interest (POIs).
- ▶ Mapping Applications.
- ▶ Deals and discounts applications.
- ▶ Location-Aware Social Networks.



Location-Based Systems

- ▶ Location information is sensitive. (it can be linked to home, work, religion, political views, etc).
- ▶ Ideally: we want to **hide our true location**.
- ▶ Reality: we need to disclose some information.

Motivating example



Locate a restaurant close to my location

Motivating example

Goal:

- ► Hide the user's location (not identity) from the service provider
- Formal privacy guarantee

Constraints:

- ▶ Implementable in real-time on a smartphone
- ► No trusted party
- Optimally: no peer-to-peer communication



Existing privacy notions

k-anonymity (or *I*-diversity)

Hide the user's location among k points

- ▶ Include k-1 randomly generated points in the query
- ▶ Use a cloaking region including k points of interest



Problem: depends on the attacker's side information

Existing privacy notions

Differential Privacy

Changes in a single user's value should have negligible effect on the reported value

- Useful for publishing aggregate information about a large number of users
- ▶ Has been used in the context of geo-location
- ▶ Inadequate for our motivating example

Towards a Definition

- Secrets are locations.
- ▶ Attacker's goal: distinguish location x from x'.
- The closer two locations are, the more indistinguishable they should be.



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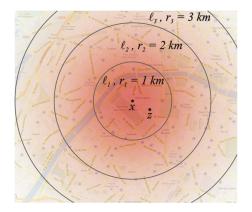
▶ What kind of privacy does the user expect to have?



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- Privacy depends on the accuracy of detecting x

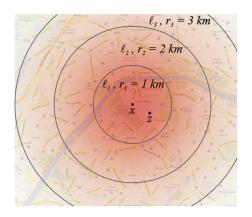


- ► What kind of privacy does the user expect to have?
- Privacy depends on the accuracy of detecting x
- ► A different privacy level / for each radius r



ϵ -geo-indistinguishability

Require privacy for any radius r with a proportional level $I(r) = \epsilon \cdot r$

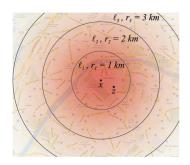


First approach for defining this notion

Intuitively we would like to require:

$$\frac{P(x|z)}{P(x'|z)} \le e^{\epsilon r} \qquad \forall r \, \forall x, x' : d_2(x, x') \le r$$

but this might fail because of the prior knowledge P(x)

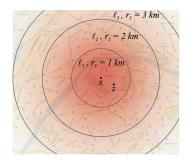


First approach for defining this notion

So we have to take it into account:

$$\frac{P(x|z)}{P(x'|z)} \le e^{\epsilon r} \frac{P(x)}{P(x')} \qquad \forall r \ \forall x, x' : d_2(x, x') \le r$$

are require this to hold for any prior P(x)

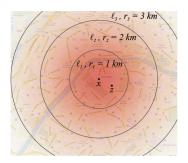


Second approach for defining this notion

Ideally we'd like the attacker's knowledge to be unaffected by z:

$$\frac{P(x|z)}{P(x)} \le e^{\epsilon r} \qquad \forall r, x$$

but z does provide information (i.e. that the user is in Paris)

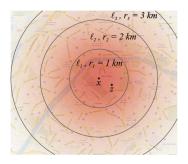


Second approach for defining this notion

So we restrict the increase in knowledge within the radius *r*:

$$\frac{P(x|z, B_r(x))}{P(x|B_r(x))} \le e^{\epsilon r} \qquad \forall r, x$$

again, this should hold for any prior P(x)

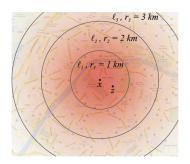


Third approach for defining this notion

Nearby points should produce similar observations:

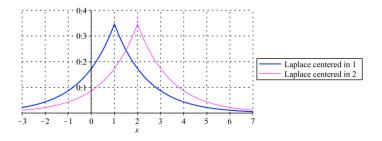
$$\frac{K(x)(z)}{K(x')(z)} \le e^{\epsilon r} \qquad \forall r \ \forall x, x' : d_2(x, x') \le r$$

which is the same as ϵd_2 -privacy.



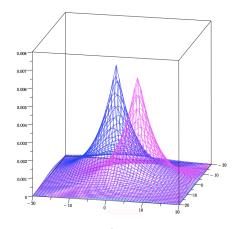
All three formulations are equivalent

The case of one dimension:



pdf:
$$\frac{\epsilon}{2} e^{-\epsilon|z-x|}$$

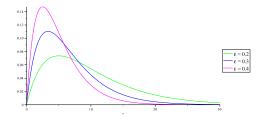
Similarly in two dimensions:



pdf:
$$\frac{\epsilon^2}{2\pi} e^{-\epsilon d_2(\vec{x}, \vec{z})}$$

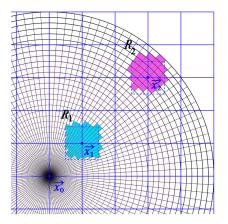
Drawing from this distribution:

- use polar coordinates
- ightharpoonup draw an angle θ uniformly
- draw a radius r from a gamma distribution



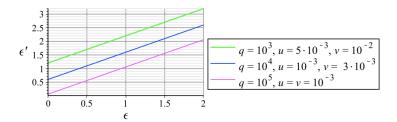
pdf: $\epsilon^2 r e^{-\epsilon r}$

- ▶ In practice locations are discretized
- \blacktriangleright (discretely) draw r, θ , map to the closest point on the grid
- ► Points correspond to differently shaped areas, leading to a vioation of geo-indistinguishability



Solution: adjust ϵ to compensate for these differences

$$\epsilon' = \epsilon + \frac{1}{u} \ln \frac{q - 2 + 3 e^{\epsilon v \sqrt{2}}}{q - 5}$$



Case study: Location-Based Services

Retrieve location-dependent information

- Restaurants
- Friends
- Gas stations
- Weather



Case study: Location-Based Services

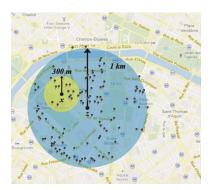
Solution:

- ▶ Add noise to the location x to obtain z
- ▶ Use *z* to query the provider
- ► Some services are insensitive to "small" perturbations (eg. weather, gas stations)
- ▶ In this case the quality of the results will not be affected

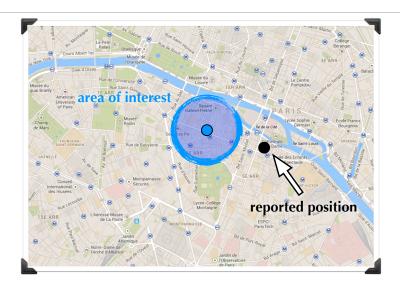


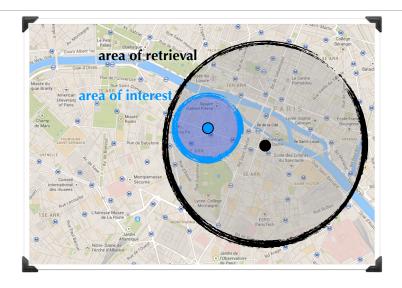
Case study: Location-Based Services

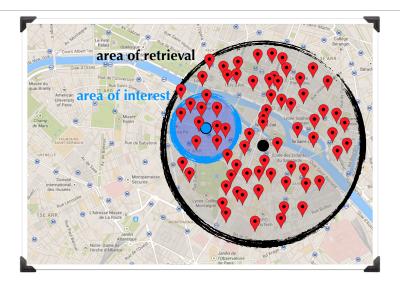
- ▶ Many LBS depend on the accuracy of the location eg. find restaurants within 300m from x
- ► In this case the query needs to be extended to a larger area eg. get restaurants within 1km from z
- ▶ Important: the area needs to be independent from z













- 9x9 = 81 "points".
- We compare 4 mechanisms.
- Configured to the same utility.
- Optimal mechanism by [Shroki et al., S&P 2012] for the corresponding prior. Obtained by linear optimization techniques.
- Three prior independent:

Planar Laplacian (discretized).

Optimal under uniform prior.

Simple cloaking.

1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18
19	20	21	22	23	24	25	26	27
28	29	30	31	32	33	34	35	36
37	38	39	40	41	42	43	44	45
46	47	48	49	50	51	52	53	54
55	56	57	58	59	60	61	62	63
64	65	66	67	68	69	70	71	72
73	74	75	76	77	78	79	80	81

- We fix the utility and measured the privacy.
- Utility loss measured as the expected distance between the true location and the reported one [Shroki et al., S&P 2012]
- Privacy measured as the expected error of the attacker (using prior information) [Shroki et al., S&P 2012]
- Priors: uniform over colored regions

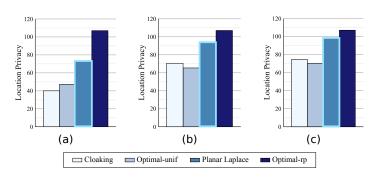
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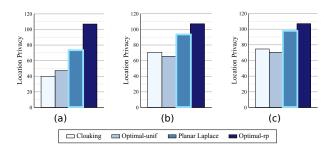
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The four mechanisms:

- Cloaking,
- Optimal by [Shroki et al. S&P 2012] for the uniform prior
- Ours (Planar Laplacian)
- Optimal by [Shroki et al. S&P 2012] for the given prior





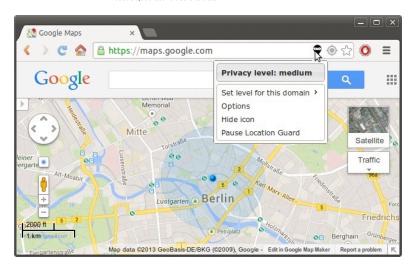
With respect to the privacy measures proposed by [Shokri et al, S&P 2012], our mechanism performs better than the other mechanisms proposed in the literature which are independent from the prior (and therefore from the adversary)

The only mechanism that outperforms ours is the optimal by [Shokri et al, S&P 2012] for the given prior, but that mechanism is adversary-dependent

Tool: "Location Guard"

http://www.lix.polytechnique.fr/~kostas/software.html

About 50,000 active users to date



Location Guard: goals

Provide a simple solution, for sporadic, real-time LBS access

Can we make it simple enough so that people actually use it?

Understandable, configurable by human beings

Low-level, application-agnostic solution

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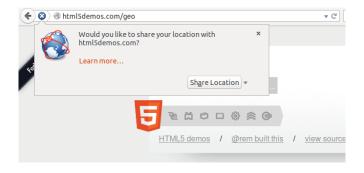
Understandable, configurable by human beings

Low-level, application-agnostic solution

OS-level on smartphones (problem: rooting the phone)

Browser level (desktop & mobile)

HTML5 geo-location API



Asking the browser for the user's location

Location Guard: adding noise

```
navigator.geolocation.getCurrentPosition(function(pos)
    alert(
        "Latitude: " + pos.coords.latitude +
        "Longitude:" + pos.coords.longitude
    );
);
```

Location Guard: adding noise

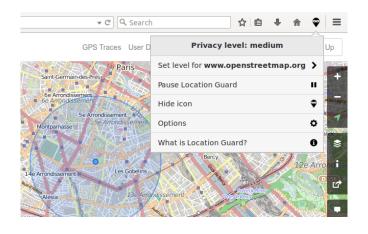
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Intercept the javascript call

Content-script, running in separate javascript environment Inject code in the page, replace navigator.geolocation

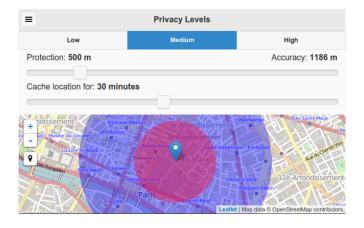
Transparent to the user

User interfaces are hard



No initial setup, user configuration if needed

User interfaces are hard



No initial setup, user configuration if needed

Mobile support



Mobile support



User adoption

Timeline

Nov 2013: Chrome Jul 2014: Firefox

Feb 2015: Firefox Mobile

Feb 2015: Opera

User adoption

Timeline

Nov 2013: Chrome Jul 2014: Firefox

Feb 2015: Firefox Mobile

Feb 2015: Opera

Current users

Chrome: 6224 active Firefox: 4642 active

Firefox Mobile: 370 active

Opera: 2134 downloads

How do users discover Location Guard?

No publicity

Huge number of extensions (1367 in Firefox privacy category alone!)

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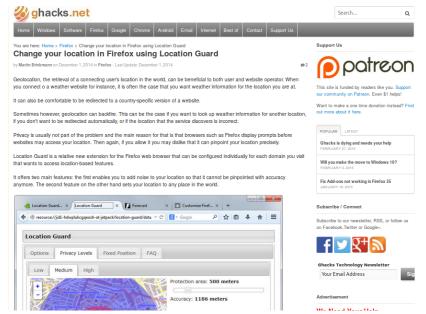
Occasional promotion by Google/Mozilla

Mostly by searching (users care about privacy!)

"location": position 1-2

"privacy": position 35-40

ghacks.net article



ghacks.net article



Chrome: linear growth

