## SECOND INRIA-DFKI EUROPEAN SUMMER SCHOOL ON ARTIFICIAL INTELLIGENCE

# Trusted AI: Privacy and Fairness 

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## Content of the lectures

- Privacy
- Motivations
- Central Differential Privacy
- Local Differential Privacy
- Privacy vs Utility
- Fairness
- Motivations
- Some notions of fairness


# Content of the lectures 

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## Motivations

Privacy is not a new issue, but in our times the problem is exacerbated by the Big Data revolution: data are collected and stored in enormous amounts, and there are the computing resources, and in particular the power of machine learning, allowing to analyse them and extract all sort of sensitive
 information

Also, data are accumulated at an increasing speed. According to a research made recenly by IMB, $90 \%$ of the world data had been generated in the last 2 years!


## Risks about privacy breaches

Sensitive information can be used for fraudulent purposes.

- Credentials

Examples: credit card numbers, home access code, passwords, ...
Risks: Stealing personal property

- Information about the individual

Examples: medical status, intimate videos, religious beliefs, political opinions
Risks: discrimination, blackmailing, public shame

- Identification information, i.e., information that can uniquely identify an individual Examples: name, SSN, bank information, biometric data (such as fingerprint and DNA) Risks: Identity theft


## Privacy: stakeholders



## Issue I: Inference attacks

The problem of Privacy is complicated because sensitive information can be derived using side information, i.e., correlated information that is necessarily public or anyway available to the attacker (inference attacks).
Example: all voters vote for the same candidate

- The typical countermeasures used in security (e.g., encryption, access control) do not help against this kind of information leakage
- The side knowledge of the adversary can increase with time


## Issue 2:Trade off with utility

The methods to protect privacy should not destroy the utility of the data.

One of the main issues in the research about privacyprotection mechanisms is to find a good trade-off with utility

In general we consider two kinds of utility:

- the Quality of Service (QoS)
- the precision of the analysis, including the accuracy in case of machine learning models


## Issues concerning privacy



## Issue 3: Whom can we trust?

## Issues concerning privacy



## Issue 3: Whom can we trust?

I. Centralized model: we trust the server / data curator.

- The sanitization is done by the curator.
- Utility is the precision of analysis / accuracy of ML models.
- Two cases:
I. the (sanitized) micro data are made available, or

2. they are not available, we can only query the database
3. Local model: the server / curator may be corrupted or unable to protect the data.

- The sanitisation must be done at the user's side
- Both kinds of utility (QoS, precision/accuracy) should be taken into account
- The sanitised micro data are made publicly accessible.

The local model has become more popular recently since people tend to trust less and less the service providers and curators (also due to recent scandals). Some big companies (e.g., Google and Apple,Amazon) have developed their own LDP systems.

## Scenario 1.1:

## Global model

The micro data are made available

## First solution:

## anonymization

- This is the most obvious solution: remove the identity of individuals from the database, so that the sensitive information cannot be directly linked to the individual
- Example: assume that we have a medical database, where the sensitive information is disease that has been diagnosed

|  | Name | age | Disease |
| :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | Jon Snow | 30 | cold |
| 2 | Jamie Lannister | 39 | amputed hand |
| $\mathbf{3}$ | Arya Stark | 16 | stomac ache |
| 4 | Bran Stark | 14 | crippled |
| $\mathbf{5}$ | Sandor Clegane | 45 | ignifobia |
| 6 | Jorah Mormont | 48 | gleyscale |
| $\mathbf{7}$ | Eddad Stark | 32 | headache |
| $\mathbf{8}$ | Ramsay Bolton | 32 | psychopath |
| $\mathbf{0}$ | Daenerys Targaryen | 25 | mania of grandeur |

## First solution: anonymization

- Anonymization removes the column of the name, so that, for instance, the grayscale disease cannot be directly linked to Jorah Mormont
- Hystorically the first method, still used nowadays
- However, this solution has been (already several years ago) shown to be ineffective, i.e., vulnerable to deanonymization attacks

|  | Name | age | Disease |
| :---: | :---: | :---: | :---: |
| 1 | - | 30 | cold |
| 2 | - | 39 | amputed hand |
| 3 | - | 16 | stomac ache |
| 4 | - | 14 | crippled |
| 5 | - | 45 | ignifobia |
| 6 | - | 32 | gleyscale |
| 7 | - | 32 | psychopath |
| 8 | - | 25 | mania of grandeur |
| 9 |  |  |  |

## De-anonymization attack (I). Sweeney'98



## De-anonymization attack (I). Sweeney'98



87 \% of US population is uniquely identifiable by 5-digit ZIP, gender, DOB

This attack has lead to the proposal of k-anonymity

## K-anonymity [Samarati \& Sweeney]

- Quasi-identifier: Set of attributes that can be linked with external data to uniquely identify individuals
- Make every record in the table indistinguishable from a least $\mathrm{k}-1$ other records with respect to quasi-identifiers. This can be done by:
- suppression of attributes, and/or
- generalization of attributes, and/or
- addition of dummy records
- Linking on quasi-identifiers yields at least $k$ records for each possible value of the quasi-identifier


## K-anonymity

Example: 4-anonymity w.r.t. the quasi-identifiers (nationality, ZIP, age)

- achieved by suppressing the nationality and generalizing ZIP and age


Figure 1. Inpatient Microdata

|  | Non-Sensitive |  |  | Sensitive |
| :---: | :---: | :---: | :---: | :---: |
|  | Zip Code | Age | Nationality | Condition |
| 1 | $130^{* *}$ | $<30$ | $*$ | Heart Disease |
| 2 | $130^{* *}$ | $<30$ | $*$ | Heart Disease |
| 3 | $130^{* *}$ | $<30$ | $*$ | Viral Infection |
| 4 | $130^{* *}$ | $<30$ | $*$ | Viral Infection |
| 5 | $1485^{*}$ | $\geq 40$ | $*$ | Cancer |
| 6 | $1485^{*}$ | $\geq 40$ | $*$ | Heart Disease |
| 7 | $1485^{*}$ | $\geq 40$ | $*$ | Viral Infection |
| 0 | $1405^{*}$ | $\geq 40$ |  | Vicol Infantion |
| 9 | $130^{* *}$ | $3 *$ | $*$ | Cancer |
| 10 | $130^{* *}$ | $3 *$ | $*$ | Cancer |
| 11 | $130^{* *}$ | $3 *$ | $*$ | Cancer |
| 12 | $130^{* *}$ | $3 *$ | $*$ | Cancer |

Figure 2. 4-anonymous Inpatient Microdata

## Problems with k-anonymity and similar methods

- Everything can turn out to be a quasiidentifier
- Especially in high-dimensional and sparse databases.
- Composition attacks
- Combination of knowledge coming from different sources
- Open world: Even if present data are protected, in the future there may be some new knowledge available

What if we adopt a more controlled setting ?

Scenario 1.2:

Centralized model

Micro data not accessible, we can only query the DB

We will see that, even in this setting, k -anonymity may fail to provide privacy

There is still the problem of composition attacks

## Example

- A medical database D1 containing correlation between a certain disease and age.
- Query:"what is the minimal age of a person with the disease"

| name | age | disease |
| :---: | :---: | :---: |
| Alice | 30 | no |
| Bob | 30 | no |
| Carl | 40 | no |
| Don | 40 | yes |
| Ellie | 50 | no |
| Frank | 50 | yes |

D1 is 2-anonymous with respect to the query. Namely, every possible answer partitions the records in groups of at least 2 elements

| Alice | Bob |
| :---: | :---: |
| Carl | Don |
| Ellie | Frank |

- A medical database D2 containing correlation between the disease and weight.
- Query: "what is the minimal weight of a person with the disease"

Also D2 is 2-anonymous

| name | weight | disease |
| :---: | :---: | :---: |
| Alice | 60 | no |
| Bob | 90 | no |
| Carl | 90 | no |
| Don | 100 | yes |
| Ellie | 60 | no |
| Frank | 100 | yes |


| Alice | Bob |
| :---: | :---: |
| Carl | Don |
| Ellie | Frank |

k -anonymity is not compositional

Combine with the two queries: minimal weight and the minimal age of a person with the disease Answers: 40, I00. Unique!

| name | age | disease |
| :---: | :---: | :---: |
| Alice | 30 | no |
| Bob | 30 | no |
| Carl | 40 | no |
| Don | 40 | yes |
| Ellie | 50 | no |
| Frank | 50 | yes |


| name | weight | disease |
| :---: | :---: | :---: |
| Alice | 60 | no |
| Bob | 90 | no |
| Carl | 90 | no |
| Don | 100 | yes |
| Ellie | 60 | no |
| Frank | 100 | yes |


| Alice | Bob |
| :---: | :---: |
| Carl | Don |
| Ellie | Frank |

Composition attacks are a general problem of
Deterministic approaches: They are all based on the principle that one observation corresponds to many possible values of the secret (group anonymity)

Observables


Problem of the deterministic approaches: the combination of observations determines smaller and smaller intersections on the domain of the secrets, and eventually result in singletones

## Secrets

## Observations



Problem of the deterministic approaches: the combination of observations determines smaller and smaller intersections on the domain of the secrets, and eventually result in singletones

## Secrets

## Observations



## Too bad!!! What can we do?

## Use probabilistic approaches!

Most of the state-of-the-art techniques, and in particular differential privacy, are indeed based on randomization

## Probabilistic approaches

Every secret can generate any observable, according to a certain probability distribution.

Observables


## Probabilistic approaches

By the Bayes law
$p(s \mid o) \propto p(o \mid s)$
Secrets
Observables


## Probabilistic approaches



## Probabilistic approaches



# Randomized approach for DB sanitisation 

- Allow accessing the DB only by queries
- Introduce some probabilistic noise on the answer so to obfuscate the link with any particular individual


## Noisy answers

minimal age:
40 with probability I/2
30 with probability I/4
50 with probability I/4

| name | age | disease |
| :---: | :---: | :---: |
| Alice | 30 | no |
| Bob | 30 | no |
| Carl | 40 | no |
| Don | 40 | yes |
| Ellie | 50 | no |
| Frank | 50 | yes |


| Alice | Bob |
| :---: | :---: |
| Carl | Don |
| Ellie | Frank |

## Noisy answers

minimal weight:
100 with prob. 4/7
90 with prob. $2 / 7$
60 with prob. I/7

| name | weight | disease |
| :---: | :---: | :---: |
| Alice | 60 | no |
| Bob | 90 | no |
| Carl | 90 | no |
| Don | 100 | yes |
| Ellie | 60 | no |
| Frank | 100 | yes |


| Alice | Bob |
| :---: | :---: |
| Carl | Don |
| Ellie | Frank |

## Noisy answers

Even if he combines the answers, the adversary cannot tell for sure whether a certain person has the disease

| name | age | disease |
| :---: | :---: | :---: |
| Alice | 30 | no |
| Bob | 30 | no |
| Carl | 40 | no |
| Don | 40 | yes |
| Ellie | 50 | no |
| Frank | 50 | yes |


| name | weight | disease |
| :---: | :---: | :---: |
| Alice | 60 | no |
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| Don | 100 | yes |
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| Alice | Bob |
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We assume the following setting:

- Centralized model (i.e., the data curator is trusted)
- Micro-data are not publicly accessible. The information can only be accessed by querying the DB


In this context, the access to the DB is via an interface (mechanism) which receives the queries, computes the answers and sanitises them before reporting them

## Adjacency

- Two databases $x_{1}, x_{2}$ are adjacent if they differ for exactly one record. We will indicate this property with the notation $x_{1} \sim x_{2}$
- $x_{1} \sim x_{2}$ represent the fact that $x_{1}$ and $x_{2}$ differ for the information relative to an individual. Either this individual has been added to $x_{2}$, or he has been removed from $x_{2}$


The adjacency relation is symmetric but not transitive

## Queries

- (The answer to) a query $f$ can be seen as a function from the set of databases $\mathcal{X}=V^{n}$ to a set of values $\mathcal{Y}$. Namely,

$$
f: \mathcal{X} \rightarrow \mathcal{Y}
$$

- $y=f(x)$ is the true answer of the query $f$ on the database $x$.
- For a given $f$, the distribution $\pi$ on $\mathcal{X}$ also induces a distribution on $\mathcal{Y}$. We will denote by $Y$ the random variable associated to the distribution on $\mathcal{Y}$.

Example: $\quad f=$ average of all values in the DB


$$
f(x)=(20+14+51+75) / 4=40
$$

## Randomized mechanisms

- A randomized mechanism for the query $f$ is any probabilistic function $\mathcal{K}$ from $\mathcal{X}$ to a set of values $\mathcal{Z}$. Namely,

$$
\mathcal{K}: \mathcal{X} \rightarrow \mathcal{D} \mathcal{Z}
$$

where $\mathcal{D} \mathcal{Z}$ represents the set of probability distributions on $\mathcal{Z}$.

- $\mathcal{Z}$ does not necessarily coincide with $\mathcal{Y}$.
- $z$ drawn from $\mathcal{K}(x)$ is a reported answer for the query on the $\mathrm{DB} x$.
- Note that $\pi$ and $\mathcal{K}$ induce a probability distribution also on $\mathcal{Z}$. We will denote by $Z$ the random variable associated to this probability distribution


## Differential Privacy

Definition (Differential Privacy) $\mathcal{K}$ is $\varepsilon$-differentially-private iff for every pair of databases $x_{1}, x_{2} \in \mathcal{X}$ s.t. $x_{1} \sim x_{2}$ and for every measurable $\mathcal{S} \subseteq \mathcal{Z}$ we have

$$
p\left(\mathcal{K}\left(x_{1}\right) \in \mathcal{S}\right) \leq e^{\varepsilon} p\left(\mathcal{K}\left(x_{2}\right) \in \mathcal{S}\right)
$$

where $p(\mathcal{K}(x) \in \mathcal{S})$ represents the probability that $\mathcal{K}$ applied to $x$ report an answer in $\mathcal{S}$

Note: $p(\mathcal{K}(x) \in \mathcal{S})$ represents a conditional probability. We will write it as $p(Z \in \mathcal{S} \mid X=x)$ when we need to make this fact more explicit.

## Meaning of Differential Privacy

Differential privacy essentially means that the presence or absence of an individual in a DB, does not make much difference for the information that the adversary acquires by querying the DB.


Hence an individual does not risks much by accepting that his data are collected in the DB

## Properties of differential privacy

- Two important properties that have made differential privacy so successful:
- Independence from the side knowledge of the adversary
- Compositionality


# Independence from the side knowledge of the adversary 

- The distribution $\pi$ on the databases is called prior, i.e., prior to the reported answer
- $\pi$ represents the knowledge that a potential adversary has about the database (before knowing the answer of $\mathcal{K}$ )
- We note that the definition of DP does not depend on $\pi$. This is a very good property, because it means that we can design mechanisms that satisfy DP without taking the knowledge of the adversary into account: the same mechanism will be good for all adversaries.


## Compositionality

- Differential privacy is compositional, namely: given two mechanisms $\mathcal{K}_{1}$ and $\mathcal{K}_{2}$ on $\mathcal{X}$ that are respectively $\varepsilon_{1}$ and $\varepsilon_{2}$-differentially private, their composition $\mathcal{K}_{1} \times \mathcal{K}_{2}$ is $\left(\varepsilon_{1}+\varepsilon_{2}\right)$-differentially private.

Note: $\mathcal{K}_{1} \times \mathcal{K}_{2}$ is defined by the following property: if $\mathcal{K}_{1}(x)$ reports $z_{1}$ and $\mathcal{K}_{2}(x)$ reports $z_{2}$, then $\left(\mathcal{K}_{1} \times \mathcal{K}_{2}\right)(x)$ reports $\left(z_{1}, z_{2}\right)$.

Proof: exercise

- Privacy budget: There is an initial budget $\alpha$ associated to the DB. Each time a user asks a query, answered by $\varepsilon$-differentially private mechanism, the budget is decreased by $\varepsilon$. When the budget is exhausted, users are not allowed to ask queries anymore.
Note that the budget is per DB and not per user because users may be colluded.


## Some "real" DP mechanisms

## Oblivious Mechanisms

- Given $f: \mathcal{X} \rightarrow \mathcal{Y}$ and $\mathcal{K}: \mathcal{X} \rightarrow \mathcal{Z}$, we say that $\mathcal{K}$ is oblivious if it depends only on $\mathcal{Y}$ (not on $\mathcal{X}$ )
- If $\mathcal{K}$ is oblivious, it can be seen as the composition of $f$ and a randomized mechanism $\mathcal{H}$ (noise) defined on the exact answers $\mathcal{K}=\mathcal{H} \circ f$

- Privacy concerns the information flow between the databases and the reported answers, while utility concerns the information flow between the correct answer and the reported answer


## A typical oblivious DP mechanism: Laplace noise

- Randomized mechanism for a query $f: \mathcal{X} \rightarrow \mathcal{Y}$.
- A typical randomized method: add Laplace noise to $y=f(x)$. Namely, report $z$ with a probability density function defined as:

$$
d P_{y}(z)=c e^{-\frac{|z-y|}{\Delta f} \varepsilon}
$$

where $\Delta f$ is the sensitivity of $f$ :

$$
\Delta f=\max _{x \sim x^{\prime} \in \mathcal{X}}\left|f(x)-f\left(x^{\prime}\right)\right|
$$

( $x \sim x^{\prime}$ means $x$ and $x^{\prime}$ are adjacent, i.e., they differ only for one record) and $c$ is a normalization factor:

$$
c=\frac{\varepsilon}{2 \Delta f}
$$



## The geometric mechanism

- The Laplacian noise is typically used in the case that $\mathcal{Y}$ (the set of true answers of the query) is a continuous numerical set, like the Reals.
- If $\mathcal{Y}$ is a discrete numerical set, like the Integers, then the typical mechanism used in this case is the geometric mechanism, which is a sort of discrete Laplacian.
- In the geometric mechanism, the probability distribution of the noise is:

$$
p(z \mid y)=c e^{-\frac{|z-y|}{\Delta f} \varepsilon}
$$

- In this expression, c is a normalization factor, defined so to obtain a probability distribution,
- $\Delta f$ is the sensitivity of query $f$


## Gaussian noise

The formula for gaussian noise is

$$
c e^{-\frac{(y-z)^{2}}{\sigma} \varepsilon}
$$

where $\boldsymbol{c}$ is a normalization factor and $\sigma$ is a suitable constant.

The gaussian mechanism does not satisfy differential privacy.
However it satisfies a more relaxed form of privacy called $(\varepsilon, \delta)$-DP

## Utility

There are various notions of utility.
We will focus on one of the most common ones, namely the utility as expected loss.

## Utility as expected loss

Assume:

- $\pi$ is the prior on $\mathcal{Y}$ (the true answers)
- $p_{\mathcal{K}}$ is the probability associated to the mechanism.
- $\ell$ is a loss function, that measures how much wee "loose" in reporting a noisy answer. Namely, $\ell(y, z)$ is the loss of precision when the true answer is $y$ but the mechanism reports $z$.

Then: the expected utility $\operatorname{loss} \mathcal{U}\left(\pi, p_{\mathcal{K}}, \ell\right)$ is defined as:

$$
\begin{aligned}
\mathcal{U}\left(\pi, p_{\mathcal{K}}, \ell\right) & =\mathbb{E}_{\pi, p_{\mathcal{K}}} \ell(y, z) \\
& =\sum_{y, z} \pi(y) p_{\mathcal{K}}(z \mid y) \ell(y, z)
\end{aligned}
$$

## Optimal mechanisms

- Given a prior $\pi$, and a privacy level $\varepsilon$, an $\varepsilon$-differentially private mechanism $K$ is called optimal if it provides the best utility among all those which provide $\varepsilon$-differential privacy
- A mechanism is universally optimal if it is optimal for all priors $\pi$


## Counting Queries

- Counting queries are typical examples of discrete queries. They are of the form: How many individuals in the database satisfy the property $\mathcal{P}$ ?
- Examples:
- How many individuals in the DB are affected by diabetes?
- How many diabetic people are obese?


# Privacy vs utility: two fundamental results 

I. [Ghosh et al., STOC 2009]

The geometric mechanism is universally optimal for counting queries and any monotonic loss function

## Privacy vs utility: two fundamental results

2. [Brenner and Nissim, STOC 20I0] The counting queries are the only kind of queries for which a universally optimal mechanism exists

- This means that for other kind of queries one the optimal mechanism is relative to a specific user.
- $\quad$ The precise characterization is given in terms of the graph $(\mathcal{Y}, \sim)$ induced by ( $\mathcal{X}, \sim$ )

ok

not ok

not ok


## Local Differential Privacy

## DP in the Global Model



Collected dataset

## Local Differential Privacy



## Local Differential Privacy

## [ Jordan \&Wainwright 'I3]

Definition Let $\mathcal{X}$ be a set of possible values and $\mathcal{Y}$ the set of noisy values. A mechanism $\mathcal{K}$ is $\varepsilon$-locally differentially private ( $\varepsilon$-LDP) if for all $x_{1}, x_{2} \in \mathcal{X}$ and for all $y \in \mathcal{Y}$

$$
P[\mathcal{K}(x)=y] \leq e^{\varepsilon} P\left[\mathcal{K}\left(x^{\prime}\right)=y\right]
$$

or equivalently, using the conditional probability notation:

$$
p(y \mid x) \leq e^{\varepsilon} p\left(y \mid x^{\prime}\right)
$$

For instance, the Randomized Response protocol is $(\log 3)$-LPD


## The $k-R R$ mechanism (aka flat mechanism)

 [ Kairouz et al,'16]The flat mechanism is the simplest way to implement LPD. It is defined as follows:

$$
p(y \mid x)= \begin{cases}c e^{\varepsilon} & \text { if } x=y \\ c & \text { otherwise }\end{cases}
$$

where $c$ is a normalization constant.
namely $c=\frac{1}{k-1+e^{\varepsilon}}$ where $k$ is the size of the domain


Privacy Properties:

- Compositionality
- Independence from the side knowledge of the adversary


## d-privacy: a generalization of DP and LDP [Chatzikokolakis et al., 'I3]

## $d$-privacy

On a generic domain $\mathcal{X}$ provided with a distance $d$ :

$$
\forall x, x^{\prime} \in \mathcal{X}, \forall z \quad \frac{p(z \mid x)}{p\left(z \mid x^{\prime}\right)} \leq e^{\varepsilon d\left(x, x^{\prime}\right)}
$$


generalizes


Differential Privacy

- $x, x^{\prime}$ are databases
- $d$ is the Hamming distance


## Properties

- Like LDP, it can be applied at the user side
- Like DP and LDP, it is compositional


## Typical d-private mechanisms

Laplace, Geometric, and their higher-dimension versions

Planar Laplace

$$
d p_{x}(z)=\frac{\epsilon^{2}}{2 \pi} e^{\epsilon d(x, z)}
$$



Used especially for location privacy, where d-privacy is called geo-indistinguishability

## Statistical Utility

This notion is particularly important for local DP. The goal is to estimate as precisely as possible the true distribution on data from the reported answers.

## Statistical utility: The matrix inversion method [ Kairouz et al, 'I6]

- Let $C$ be the stochastic matrix associated to the mechanism
- Let $q$ be the empirical distribution (derived from the noisy data).
- Compute the approximation of the true distribution as $r=q C^{-1}$

Example Assume $q(Y e s)=\frac{6}{10}$ and $q(N o)=\frac{4}{10}$. Then:

$$
\begin{aligned}
& \frac{3}{4} p(Y e s)+\frac{1}{4} p(N o)=\frac{6}{10} \\
& \frac{1}{4} p(Y e s)+\frac{3}{4} p(N o)=\frac{4}{10}
\end{aligned}
$$

From which we derive $p(Y e s)=\frac{7}{10}$ and $p(N o)=\frac{3}{10}$

| $y$ |  |  |  |
| :---: | :---: | :---: | :---: |
|  | yes | no |  |
|  |  |  |  |
|  | yes | $3 / 4$ |  |
|  | no | $1 / 4$ |  |
|  |  | $1 / 4$ |  |

## Statistical utility: The matrix inversion method

Problem 1: $C$ must be invertible

Problem 2: Assume $q(Y e s)=\frac{4}{5}$ and $q(N o)=\frac{1}{5}$. Then:

$$
\begin{aligned}
& \frac{3}{4} p(Y e s)+\frac{1}{4} p(N o)=\frac{4}{5} \\
& \frac{1}{4} p(Y e s)+\frac{3}{4} p(N o)=\frac{1}{5}
\end{aligned}
$$



From which we derive $p(Y e s)=\frac{11}{10}$ and $p(N o)=-\frac{1}{10}$

## Statistical utility: The matrix inversion method

$r=q C^{-1}$ may not be a distribution because it may contain negative elements. In order to try to obtain the true distribution $\pi$ we can either:

- set to 0 all the negative elements, and renormalize, or
- project $r$ on the simplex.

The resulting distribution however usually is not the best approximation of the original distribution.

## A more general and principled approach: Iterative Bayesian Update



The IBU:

- is based on the Maximization-Expectation method
- produces a Maximum Likelihood Estimator p of the true distribution $\pi$
- If $C$ is invertible, the MLE is unique and as the number of samples grows it converges to $\pi$


## The Iterative Bayesian Update

- Define $p^{(0)}=$ any distribution (for ex. the uniform distribution)
- Repeat: Define $p^{(n+1)}$ as the Bayesian update of $p^{(n)}$ weighted on the corresponding element of $q$, namely:

$$
p_{x}^{(n+1)}=\sum_{y} q_{y} \frac{p_{x}^{(n)} C_{x y}}{\sum_{z} p_{z}^{(n)} C_{z y}}
$$



- Note that $p^{(n+1)}=T\left(p^{(n)}\right)$
- When $C$ is invertible, $T$ has unique fix point (the MLE)
- Open problem: in some cases (with few samples) the MLE may not be the best estimation of the true distribution. We are trying to devise corrective methods.


## Comparison between LPD and d-privacy Experiments on the Gowalla dataset

- Gowalla is a dataset of geographical checkins in several cities in the world
- We have used it to compare the statistical utility of $k R R$ and Planar Laplace with the respective $\varepsilon$ calibrated so to satisfy the same privacy constraint: same level of privacy within about $1 \mathrm{Km}^{2}$



Gowalla checkins in an area of $3 \times 3 \mathrm{~km}^{2}$ in San Francisco downtown (about 10K checkins)


The real distribution
The noisy distribution and the result of the IBU (300 iterations)

## The kRR mechanism



The real distribution


The noisy distribution and the result of the IBU (500 iterations)

## Evaluation: San Francisco



## Evaluation: Paris



## Privacy preserving machine learning

In machine learning, the typical privacy issue is the membership inference in the training set. Namely the attacker is trying to infer whether a certain sample is part of the training set or not.

## Models of attacks:

- Black box: The attacker can only see the answers of the model on a test set
- White box: The attacker has access also to the internals of the model (architecture, weights of the nodes, etc.)
- Grey box: A combination of the above

The effectiveness of these attacks has been widely documented. Of course the white box attacks are more effective than the black box ones, but also in the black box, surpringly, the attacker can guess the presence or absence of the target sample with high probability of being correct.

## Counter measures

Central differential privacy has been successfully applied in "standard" machine learning. The method typically consists in adding Laplace or Gaussian noise to the weights updates during the gradient descent.

In ML, the typical metric for utility is the accuracy of the model

## Federated learning

A recent topic of research: (white box) attacks and counter measures in federated learning.

- In federated learning, we assume that there are various indviduals or organizations, each with its own training data, that they do not want to share
- The gradient descent is done in a distributed way: each organization computes the update based on its onw data, then sends it to the central coordinator, which combines them
- In federated learning makes sense to consider a very powerful white box attacker that can access the individual updates each time.

In the last part of this lecture Sayan Biswas will show a demo of the federated learning attacks, and how to mitigate them using d-privacy

# Content of the lectures 

- Privacy
- Motivations
- Central Differential Privacy
- Local Differential Privacy
- Privacy vs Utility
- Fairness
- Motivations
- Some notions of fairness


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## Motivation: risk of unfair decision in ML

The risk of unfair decisions is amplified by Machine Learning. Possible causes are:

- ML is based on correlation and not on causality (risk of bias-inducing correlations)
- Data used for training may be already biased.

Example: ML used to predict recidivity in the USA


Non recidivating black people twice as likely to be labelled high risk than non recidivating white people
https://www.propublica.org/article/machine-bias-risk-assessments-in-criminal-sentencing

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## Notation and basic notions

## Data model and predictor

$X$ Legitimate attributes
A Sensitive attribute (binary)
$Y$ Decision (binary)
$\widehat{Y}$ Prediction of the classifier (binary)

## Example

|  | $X$ | employment, salary (income) |
| :--- | :--- | :--- |
| $A$ | race |  |
| $Y$ | $Y$ | loan decision |
|  | $\widehat{Y}$ | prediction |



## Some fairness notions

## Statistical parity SP

$$
\mathbb{P}[\widehat{Y}=\hat{y} \mid A=0]=\mathbb{P}[\widehat{Y}=\hat{y} \mid A=1] \quad \widehat{Y} \perp A
$$

Statistical parity is usually too strong:

## Example

In the example of the loan, if the income status is unbalanced between the races, in order to satisfy SP the predictor should grant loans also to some of
 the people with insufficient income

## Conditional statistical parity CSP

$$
\mathbb{P}[\hat{Y}=\hat{y} \mid X=x, A=0]=\mathbb{P}[\widehat{Y}=\hat{y} \mid X=x, A=1] \quad \widehat{Y} \perp A \mid X
$$

## Example

The predictor can grant loans less frequently to a certain race, as long as this disparity is justified by the legitimate attributes (insufficient income)


## Some fairness notions

Previous notions usually have a negative impact on the accuracy of a classifier. In order to avoid this problem, Hardt, Price and Srebro [NIPS'16] introduced the following notion:

## Equalized odds EOdds

$$
\mathbb{P}[\widehat{Y}=y \mid Y=y, A=0]=\mathbb{P}[\widehat{Y}=y \mid Y=y, A=1] \quad \widehat{Y} \perp A \mid Y
$$

EOdds assumes implicitly that $Y$ is unbiased. If the training data do not respect this assumption, we should correct them.

## Example

The probability that the predictor takes the "right" decision does not depend on the race


# Thanks for the attention! 

## Questions?

