Introduction

Main document

Motivation We decided to have two chapters about our motivation.

Thus, Chapter 1 presents dynamical systems, with a mathematical point of view, rather classical, with well-known examples from literature.

The examples from Chapter 2 distinguish from those of previous chapter by the fact that they all involve at some point some concurrency between agents.

These two chapters do not aim to produce new results, but to put in a kind of perspective several systems, with some relations, in links with this work.

Chapter 1 The first chapter aims at outlining several basic facts.

The first is the richness, and the subtlety of possible behaviours of continuous dynamical systems. This point of view is rather classical, and is widely inspired from the excellent monograph [Hirsch et al., 2003], from which we take most of the examples.

The second is that several devices are intrinsically continuous, and can be used as such to do some computations.

Finally, we want to insist on the modelling power of a class of dynamical systems that we name, in relation with [Graça, 2006], the polynomial Cauchy problems.

We also discuss, with the very pedagogical article [Krivine et al., 2006], several problems that arise when one tries do discretize continuous systems. This aims at pointing out that continuous abstraction is often the good way to do.

The idea that reasoning on the continuum is an elegant way to reason about continuous or discrete systems is initiated in Chapter 1, but is essentially developed in Chapter 2.

Chapter 2 We present in Chapter 2, several models, sometimes intrinsically discrete, whose good level of abstraction, and understanding is the continuum.

Some of these models come from bioinformatics, for the modelling of genetic networks, some are used in population models in biology, some are from virology, and some are from computer virology models. We then present game theory, and its models, focusing on some of its models for dynamism.

We try to argue that these continuous models for dynamism become natural to talk about distributed algorithmic, in particular as soon as one faces with huge populations, for which one does not control interactions or topology, except maybe by probabilistic or statistical arguments.

This point of view, less classical, is recent. We discuss several recent models in distributed algorithmic that already integrate these considerations.

The examples of these two chapters show that this is very important to understand the power and the richness of all these models, for the cited applications, but also for distributed algorithmic and today's and (we believe it) future's computer science.

Chapter 3 This chapter is a survey on the computation theory of continuous time models.

The power of discrete space and time models is rather well understood thanks to Church-Turing thesis: indeed, it postulates that all reasonable and sufficiently powerful models have the same power, the one of Turing machines.

However, one can consider models that work with a continuous space. This is for example the case of the Blum Shub and Smale model over the reals [Blum et al., 1989] (this model is considered in Chapter 5). Models from recursive analysis also fall in this framework. These models are discrete time.

But one can also consider models with a continuous time. The chapters 1 and 2 mention some of them, some realistic, some rather not, and some rather futuristic in links with distributed algorithmic. But other important classes of models have been considered. We recall them in this chapter, by presenting a review of all what is known about their computational properties.

Chapter 4 This chapter present an overview of some of our results about the comparison of the power of several continuous time models, in relations with the PhD thesis of Emmanuel Hainry.

The GPAC (General Purpose Analog Computer) was introduced in 1941 by Shannon [Shannon, 1941] as a model of an analog computer of that time: the Differential Analyser.

Shannon proposed an exact characterization of functions that can be generated by this model. The results from Shannon have long been interpreted as the proof that the GPAC is a two weak model, and at least a model less powerful than recursive analysis.

In collaboration with Manuel Campagnolo, Daniel Graça, and Emmanuel Hainry, we proved that this is not.

This result takes all its perspectives if one understands that functions computed by GPAC correspond to polynomial Cauchy problems, and that we argue in chapter 1, that almost all the continuous dynamical systems considered in physics, biology, chemistry are of this type.

Among the continuous time models, there is also the class of $\mathbb{R}$-recursive functions introduced by Cris Moore in [Moore, 1996]. The paper from Moore presents some very interesting and original ideas to understand computations over the reals, that can be presented in the following way: since there does not exist a universally accepted notion of machine in the continuous world, why not try to circumvent the problem by starting from characterizations in classical computability of the complexity and computability classes that are machine independent, and in particular from algebraic characterizations.

Manuel Campagnolo, in his PhD [Campagnolo, 2001], supervised by Félix Costa and Cris Moore, proposes the very interesting idea of restricting to primitive recursive classes, that is to say without minimization operators, and proves that the replacement of the integration operator from Moore into a linear integration operator yields to a class of functions that can be related naturally to elementary computable functions over the integers.

We showed that this is indeed possible to go further. Elementary Computable and computable functions in the sense of recursive analysis can be characterized algebraically.

We present an overview of our main results on these topics on Chapter 4.

Chapter 5 In this chapter, we review some of our results about logical characterizations of complexity classes in the Blum Shub and Smale models, in relations with Paulin Jacobé de Naurois's PhD.

The Blum Shub and Smale model [Blum et al., 1989] is a computational model with a discrete time and a continuous space.

The model, initially defined in order to discuss algebraic complexity of problems over the field of reals, or more generally over a ring, has latter on been extended by Poizat in [Poizat, 1995], [Goode, 1994] into a computational model over an arbitrary logical structure.

Through the PhD of Paulin Jacobé de Naurois, co-supervised by Felipe Cucker in Hong Kong, and Jean-Yves Marion in Nancy, we tried to understand if this is possible to characterize complexity classes in a syntactic way in this model over an arbitrary structure.

The Chapter 5 presents an overview of all our results, in the framework of the characterizations of the so-called implicit complexity. All presented results in this chapter are in the spirit of Bellantoni and Cook's characterization of polynomial time in [Bellantoni and Cook, 1992].

Chapter 6 Finally, last chapter is a conclusion. We outline main questions.

Appendix

There seems to be a kind of tradition to put in appendix some publications.

Appendix A This appendix is article [Bournez, 2006].

The question of the existence of devices able to realize hypercomputations, that is to say to compute things that can not computed by any Turing machine, still rise to many discussions and controversies. The situation is far from being so clear as it appears.

We have been invited to express our point of view in a special issue on the subject, in the article [Bournez, 2006].

Following [Copeland, 2002], we recall some recurrent bad understandings of what the Church-Turing thesis says, and we present an overview of several mathematical models, with an exact characterization of their computational power.

The article [Bournez, 2006] contains several new results, and can be considered as an upgrade, with our today's understanding, of the results that existed when we started our PhD [Bournez, 1999].

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