

Rappel:  $0, \text{succ}, \pi_i, 0$

$$\text{PR} \begin{cases} f(0, \vec{y}) = g(\vec{y}) \\ f(\text{succ}(x), \vec{y}) = h(x, f, \vec{y}, \vec{y}) \end{cases}$$

$$\text{PRA} \begin{cases} f(0, y, \vec{z}) = g_1(y, \vec{z}) \\ f(\text{succ}(x), 0, \vec{z}) = g_2(x, \vec{z}) \\ f(\text{succ}(x), \text{succ}(y), \vec{z}) = f(x, y, \vec{z}) \end{cases}$$

$$\text{PR1} \begin{cases} f_1(0, \vec{y}) = g_1(\vec{y}) \\ f_1(\text{succ}(x), \vec{y}) = h_1(x, f_2(x, \vec{y}), \vec{y}) \\ f_2(0, \vec{y}) = g_2(\vec{y}) \\ f_2(\text{succ}(x), \vec{y}) = h_2(x, f_1(x, \vec{y}), \vec{y}) \end{cases}$$

$$\text{PRV} \begin{cases} f(0, y, \vec{z}) = g(y, \vec{z}) \\ f(\text{succ}(x), y, \vec{z}) = h(x, f(x, y, \vec{z}), y, \vec{z}) \end{cases}$$

C'est la classe des fonctions primitives récursives  $\mathcal{P}\mathcal{R}$

## Ackermann

$$\text{Ack}(0, y) = \text{succ}(y)$$

$$\text{Ack}(\text{succ}(x), 0) = \text{Ack}(x, \text{succ}(0))$$

$$\text{Ack}(\text{succ}(x), \text{succ}(y)) = \text{Ack}(x, \text{Ack}(\text{succ}(x), y))$$

$$\text{Ack}(0) = \lambda y. \text{succ}(y)$$

$$\text{Ack}(\text{succ}(x)) = G(x, \text{Ack}(x))$$

$$G(x, n)(0) = (n \text{ succ}(0))$$

$$G(x, n)(\text{succ}(y)) = (n \ G(x, n)(y))$$

$$\text{Rec}(0, \lambda y. \text{succ}(y), [x, n] \lambda y. \text{Rec}(y, (n \ \text{succ}(0)) [x', u] (n \ (u' x))))$$