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## Constraint Satisfaction: Algorithms and Complexity

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### *Series 2*

#### **Problem 1**

Prove that for a relational structure  $\Gamma$  the following is equivalent:

- Every relation  $R$  in  $\Gamma$  is *2-decomposable*, that is,  $R$  contains all  $n$ -tuples  $(t_1, \dots, t_n)$  such that for all  $i, j \in \{1, \dots, n\}$  there is a tuple  $s \in R$  such that  $t_i = s_i$  and  $t_j = s_j$ .
- Every relation that is primitive positive definable in  $\Gamma$  is definable by a conjunction of binary primitive positive definable relations in  $\Gamma$ .

#### **Problem 2**

For a tree  $T$  with a distinguished vertex  $v \in V(T)$ , consider the structure  $(V(T); <, E)$  where

- $E$  is the binary relation that contains all pairs  $(x, y)$  such that the distance between  $x$  and  $v$  is strictly smaller than the distance between  $y$  and  $v$ , and
- $<$  is a linear extension of  $E$ .

Find such a tree  $T$  so that the corresponding structure  $(V(T); <, E)$  can *not* be solved by arc consistency.