Distance Constraint Satisfaction Problems

Manuel Bodirsky

CNRS/LIX,École Polytechnique

Joint work with Víctor Dalmau, Barnaby Martin, Michael Pinsker

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Constraint Satisfaction Problems

Informal description

Constraint Satisfaction Problem (CSP)

A computational problem:

- **Input:** a set of variables and a set of constraints imposed on these variables
- **Question:** is there an assignment of values to the variables such that all the constraints are satisfied?

Examples and Applications of CSPs in:

Artificial Intelligence, Type Systems for Programming Languages, Computational Linguistics, Database Theory, Computational Biology, Graph Theory, Finite Model Theory, Computational Real Geometry, Computer Algebra, Operations Research, Boolean Satisfiability, Complexity Theory, ...

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Will be interested in computational complexity of CSPs

Which CSPs can be solved in polynomial time?

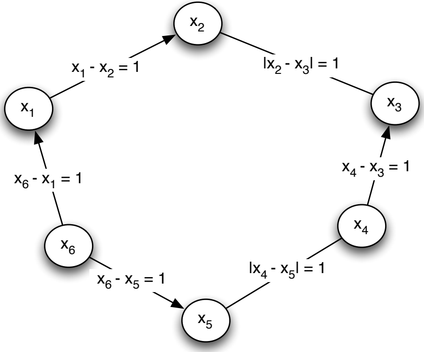
Which CSPs are NP-hard?

Examples of Constraint Satisfaction Problems

Problem 1.

Input: A finite set of variables $x_1, \ldots, x_n$, a finite set of constraints of the form $x_i - x_j = 1$ or of the form $|x_i - x_j| = 1$.

Question: Is there a mapping $s : \{x_1, \ldots, x_n\} \to \mathbb{Z}$ that satisfies all constraints?



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NP-hard: Instance satisfiable iff corresponding graph is 3-colorable.

Constraint Satisfaction Problems: Formal Definition

Let τ be a finite set of relation symbols.
Let $\Gamma = (D; R_1, R_2, \dots)$ be a τ-structure (also called template).

CSP(Γ)

Input: A primitive positive τ-sentence Φ,
i.e., a first-order sentence of the form

$$\exists x_1, \dots, x_n.\ \psi_1 \wedge \dots \wedge \psi_l$$

where ψ_i are atomic, i.e. of the form $R(x_{i_1}, \dots, x_{i_k})$ for $R \in \tau$.

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The Computational Complexity of CSPs

Fact (MB+Grohe'ICALP08): for every computational problem $\mathcal{P}$ there is a structure Γ such that $\mathcal{P}$ and CSP(Γ) are equivalent (under polynomial-time Turing reductions).

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Note:

- $(\mathbb{Z};\ x - y = 1, |x - y| = 1)$ and $(\mathbb{Z};\ x - y = 1, |x - y| \in \{1, 3\})$ do have a first-order definition in $(\mathbb{Z}; \{(x,y) \mid x - y = 1\})$.
- Allow any number of relations of any arity!

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From a model-theoretic perspective, the structure $\big(\mathbb{Z}; \{(x, y) \mid x - y = 1\}\big)$ is among the simplest structures that is not ω-categorical.

Main Result

Let Γ, Δ be τ-structures. A mapping $f : \Gamma \to \Delta$ is called a homomorphism if $(f(t_1), \ldots, f(t_n)) \in R^\Delta$ whenever $(t_1, \ldots, t_n) \in R^\Gamma$, for all tuples t and all $R \in \tau$. Γ and Δ are said to be homomorphically equivalent if there is a homomorphism from Γ to Δ and vice versa.

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  In this case, $\mathrm{CSP}(\Gamma)$ is equivalent to a finite domain CSP.
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  In this case, $\mathrm{CSP}(\Gamma)$ is NP-hard.
- Γ has a modular median polymorphism.
  In this case, $\mathrm{CSP}(\Gamma)$ can be solved in polynomial time.

Constraint Propagation and Majority Polymorphisms

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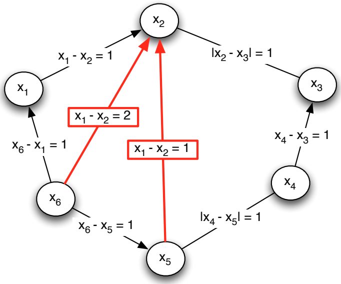
Idea: perform k-local inferences until a fixed point is reached, which is then called k-consistent.

Example for $k = 3$: Look at the constraints on three variables and add the binary constraints they imply.



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If Γ has a majority polymorphism and an instance Φ of CSP(Γ) is 3-consistent and does not contain *false*, then Φ is satisfiable.

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Theorem 3.

Let Γ be a finite degree structure with first-order definition in $(\mathbb{Z}; \{(x,y) \mid x - y = 1\})$ and a majority polymorphism. Then $\mathrm{CSP}(\Gamma)$ is in P.

The Modular Median Operation

The *d-modular median* is the operation $m_d : \mathbb{Z}^3 \to \mathbb{Z}$ defined as follows:

- If x, y, z are congruent modulo d, then $m_d(x, y, z)$ equals the median of x, y, z.
- If precisely two arguments from x, y, z are congruent modulo d, then $m_d(x, y, z)$ equals the first of those arguments in the ordered sequence (x, y, z).
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Let Γ be a first-order expansion of $(\mathbb{Z}; \{(x, y) \mid x - y = 1\})$. Then

- Γ is preserved by a modular median and $\text{CSP}(\Gamma)$ is in P, or
- $\text{CSP}(\Gamma)$ is NP-hard.

Automorphisms, Endomorphisms

An endomorphism of Γ is a homomorphism from Γ to itself.

Example: $x \mapsto (x \mod 2)$ is an endomorphism of

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Let Γ be a finite-degree structure with a first-order definition in $\big(\mathbb{Z}; \{(x,y) \mid x - y = 1\}\big)$. Then

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2. Either Γ has an endomorphism with finite range, or it has an endomorphism whose image induced in Γ a structure isomorphic to a structure Δ with a first-order definition in $\big(\mathbb{Z}; \{(x,y) \mid x-y=1\}\big)$ all of whose endomorphisms are automorphisms.

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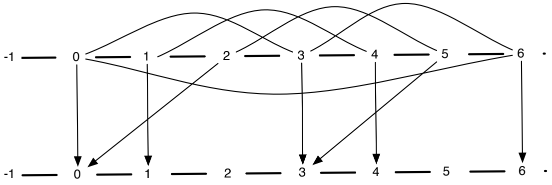
Example: $\Gamma = \big(\mathbb{Z}; \{(x,y) \mid |x-y| \in \{1,3,6\}, \{(x,y) \mid |x-y| = 3\}\}\big)$.



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The CSP of the expansion of Γ by a primitive positive definable relation reduces to $\mathrm{CSP}(\Gamma)$ in polynomial time.
Hence, may assume in the following that Γ contains the relation $\{(x,y) \mid x - y = 1\}$.

Concluding Remarks

Distance CSPs confirm the importance of constraint propagation:

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Important open problems:

- Classify the complexity of finite-domain CSPs with a transitive template.
- If a finite-domain CSP or a distance CSP has a majority polymorphism, can it be solved in linear time? (the algorithm presented here is cubic)