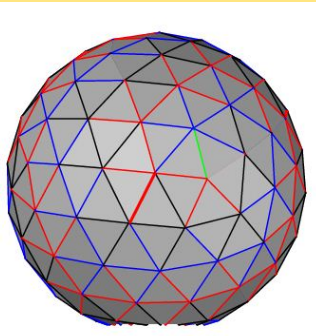
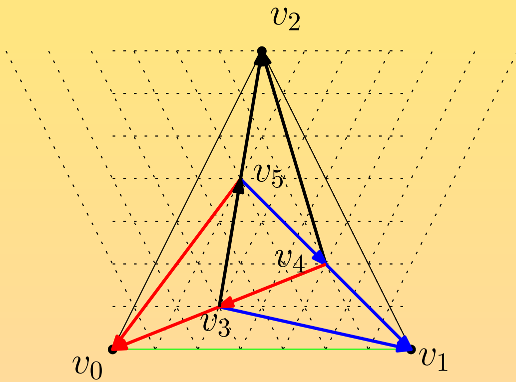
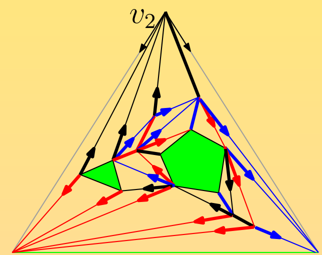
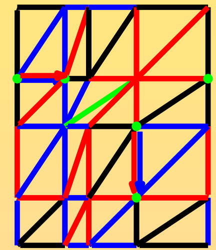
Array-based compact data structures for triangle meshes









ISAAC,Yokohama, 6-8 december 2011

Luca Castelli Aleardi

Olivier Devillers

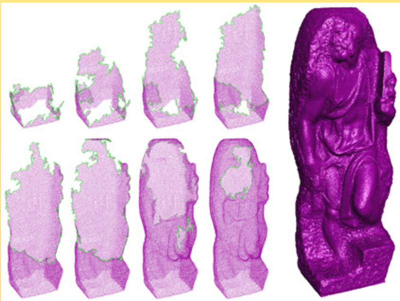
ÉCOLE POLYTECHNIQUE

INRIA SOPHIA ANTIPOLIS

Geometrica - INRIA Sophia

Compact Data Structures: motivation and goal

St. Matthew (Stanford's Digital Michelangelo Project, 2000)



6 Giga bytes (for storing on disk)
186 millions vertices

David statue (Stanford's Digital Michelangelo Project, 2000)

2 billions polygons

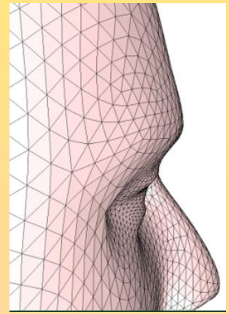
32 Giga bytes (without compression)

Connectivity information

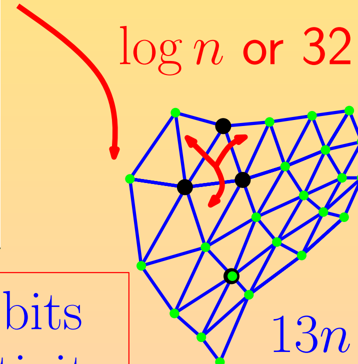
vertex — 1 reference to a triangle

triangle — 3 references to vertices

3 references to triangles



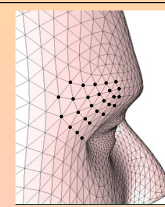
$\log n$ or 32 bits



$416n$ bits connectivity

$13n \log n$

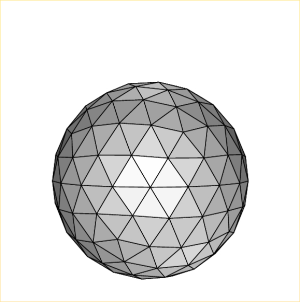
Geometric object



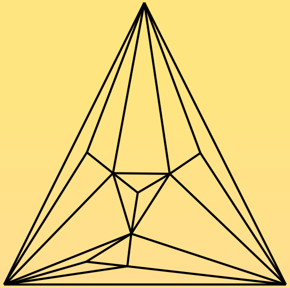
$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

between 30 et 96 bits/vertex

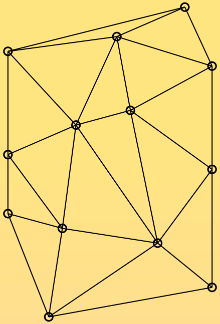
Today we deal with triangulations



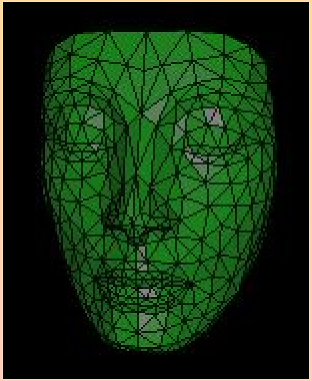
genus 0 triangle mesh

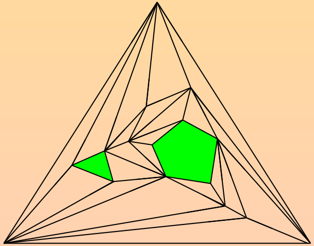


(random) planar triangulation



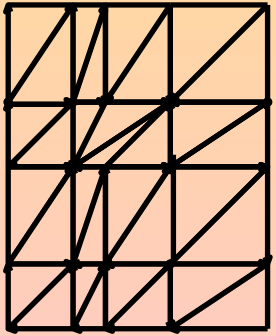
2D Delaunay triangulation





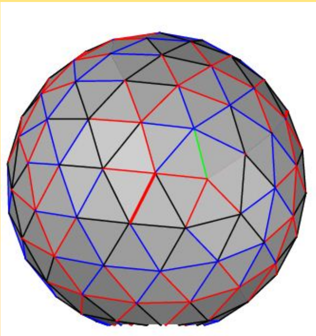
triangulations with boundaries

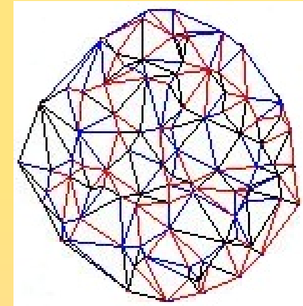
higher genus triangulations



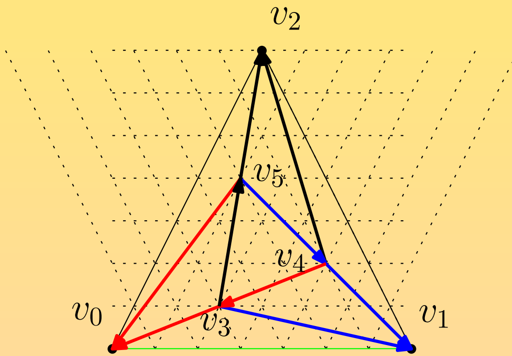
(planar) Triangle meshes

Surface triangle mesh endowed with a Schnyder wood

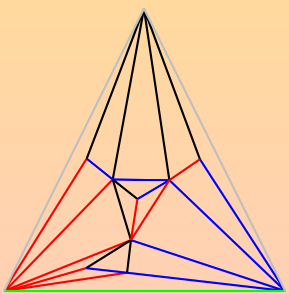




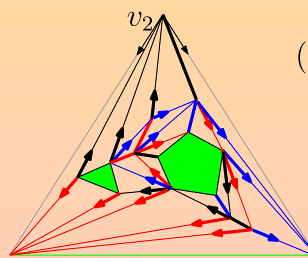
Delaunay triangulation of random points (endowed with a Schnyder wood)



Schnyder drawing of a planar triangulation

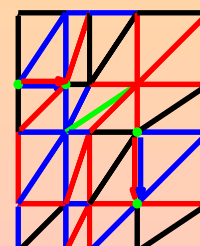


Tutte drawing of a random planar triangulation



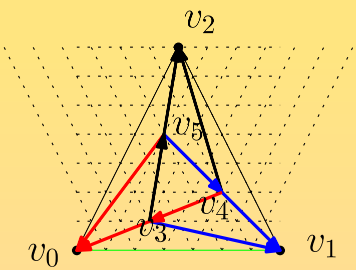
Schnyder woods for triangulations with multiple boundaries of arbitrary size (Castelli-Fusy-Lewiner,CCCG2010)

g-Schnyder woods, for genus g triangulations (Castelli-Fusy-Lewiner, SoCG08)



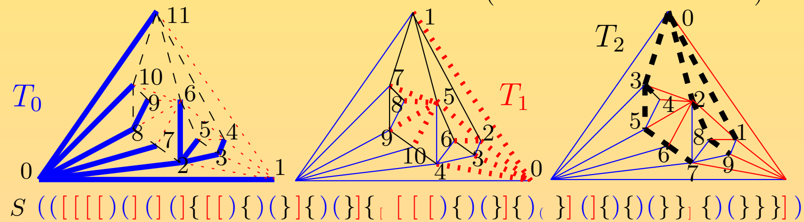
Schnyder woods: applications
(and related properties)

grid drawing



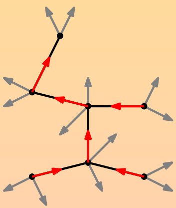
Graph encoding

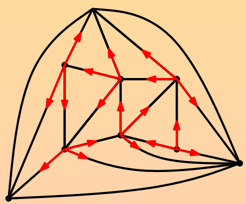
(Chuang-Garg-He-Kao-Lu Icalp '98)
(He-Kao-Lu '99)
(Chiang et al. Soda'01)
(Barbay-Castelli Aleardi-He-Munro Isaac'07)
(Castelli Aleardi-Fusy-Lewiner SoCG08)
(Castelli Aleardi-Fusy-Lewiner CCCG'10)
(Yamanaka-Nakano '08)



graph counting, random generation

(Poulalhon-Schaeffer, Icalp 03)

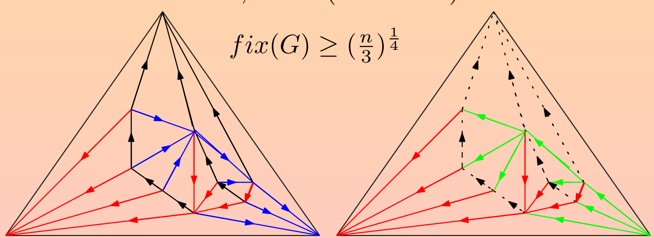




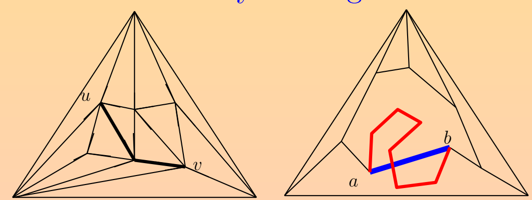
Untangling geometric graphs

Bose, Dujmovic, Hurtado, Langerman, Morin, Wood (DCG 2009)

$$fix(G) \geq (\frac{n}{3})^{\frac{1}{4}}$$



Greedy routing

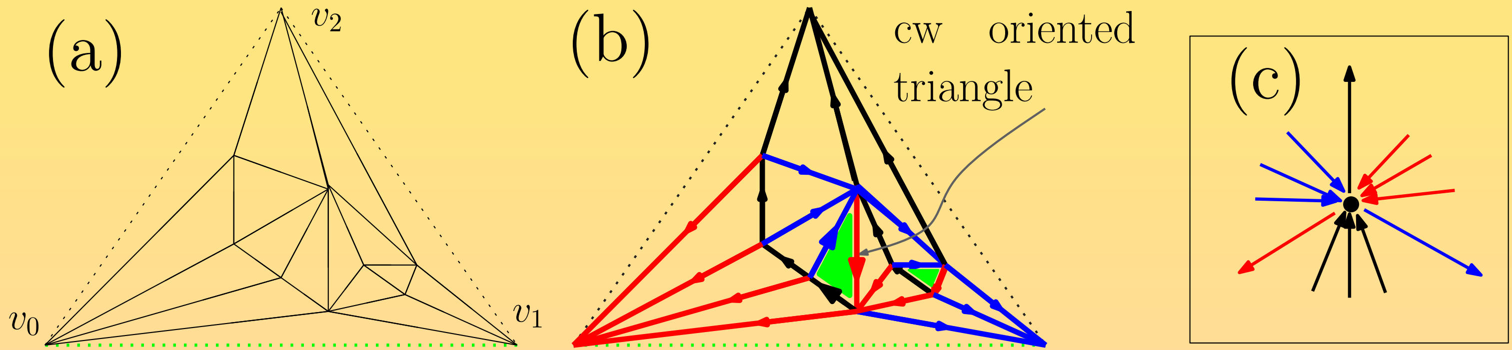


Every planar triangulation admits a *greedy drawing* (Dhandapani, Soda08)

Conjectured by Papadimitriou and Ratajczak for 3-connected planar graphs

Schnyder woods: the definition

v_0 v_1 v_2 outer face



i) edge are colored and oriented in such a way that each inner node has exactly one outgoing edge of each color

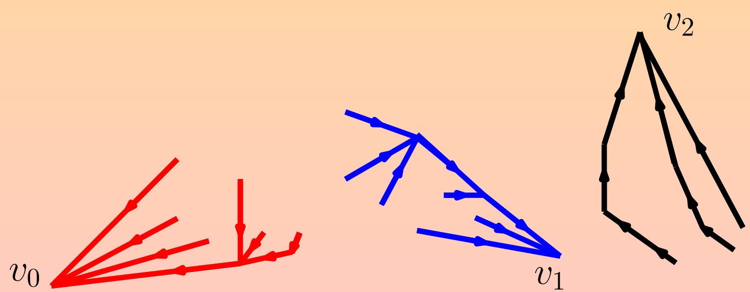
ii) colors and orientations around each inner node must respect the local Schnyder condition

Theorem

Every planar triangulation admits a Schnyder wood, which can be computed in linear time.

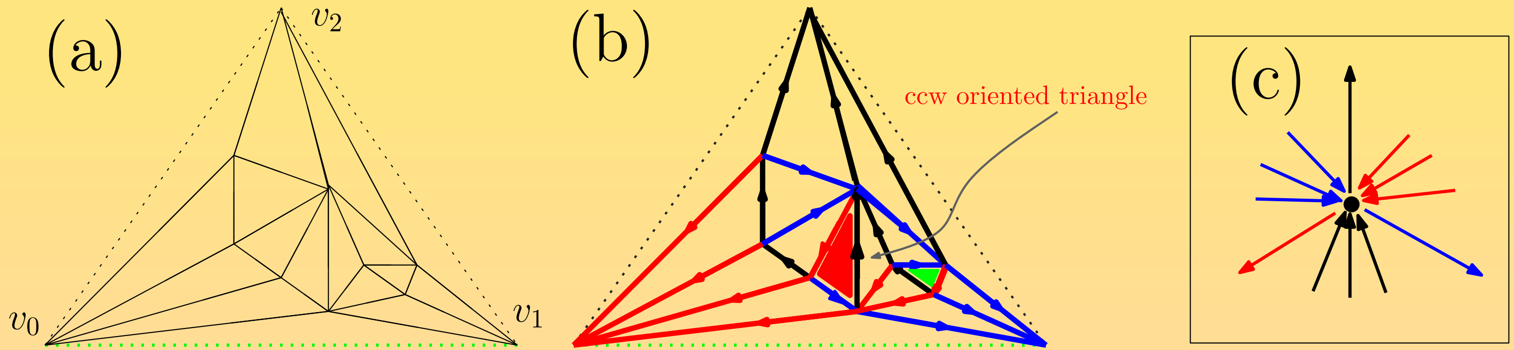
Theorem

The three set T_0, T_1, T_2 are spanning trees of (the inner nodes of) T:



minimal Schnyder woods: the definition (no ccw triangles)

v_0 v_1 v_2 outer face

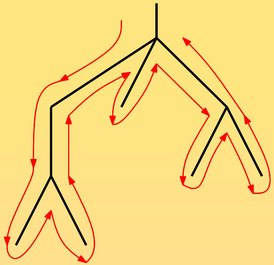


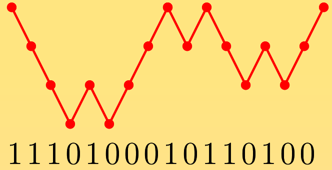
i) edge are colored and oriented in such a way that each inner node has exactly one outgoing edge of each color

ii) colors and orientations around each inner node must respect the local Schnyder condition

Several kinds of encodings: plane trees (with n nodes)

compression schemes



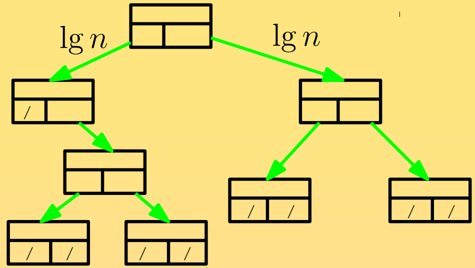


$2n$ bits

$$\|\mathcal{B}_n\| = \frac{1}{n+1}\binom{2n}{n} \approx 2^{2n} n^{-\frac{3}{2}}$$

$$log_2\|\mathcal{B}_n\| = 2n + O(\lg n)$$

(Explicit) data structures



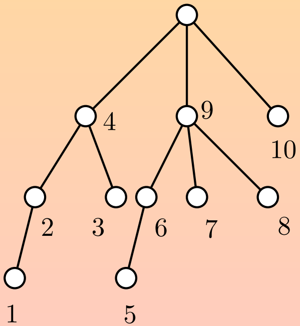
we need $2n$ references, or $\Theta(n \lg n)$ bits

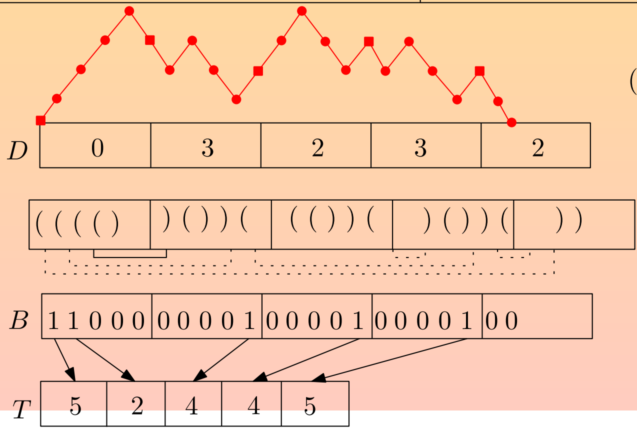
Explicit (pointer based) implementation

(array based)

Succinct representations

(Jacobson Focs89, Munro and Raman Focs97)





$2n + o(n)$ bits

$+$

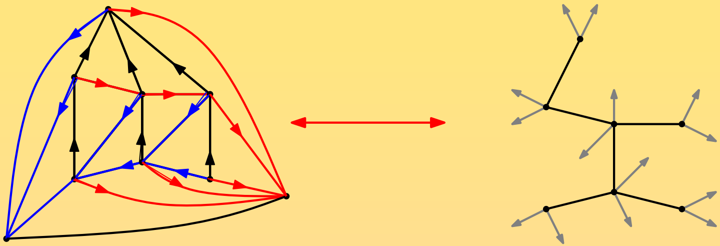
$O(1)$ time navigation

Word-RAM model

Several kinds of encodings: triangle meshes (n vertices)

Optimal compression scheme

(Poulalhon Schaeffer, Icalp03)

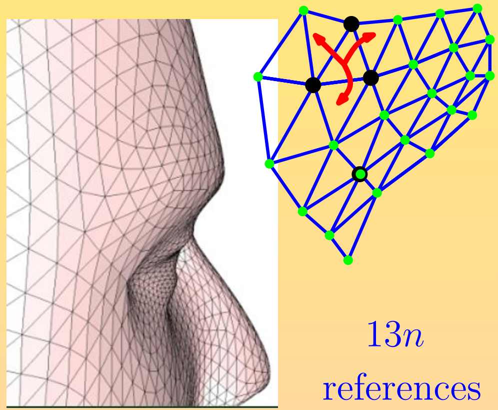


$$\Psi_n = \frac{2(4n+1)!}{(3n+2)!(n+1)!} \approx \frac{16}{27}\sqrt{\frac{3}{2\pi}}n^{-5/2}(\frac{256}{27})^n$$

$$\frac{1}{n}\log_2 \Psi_n \approx \log_2(\frac{256}{27}) \approx 3.2451 \text{ bits/vertex}$$

(Explicit) Geometric data structures

$\beta n + O(1)$ references (pointers)



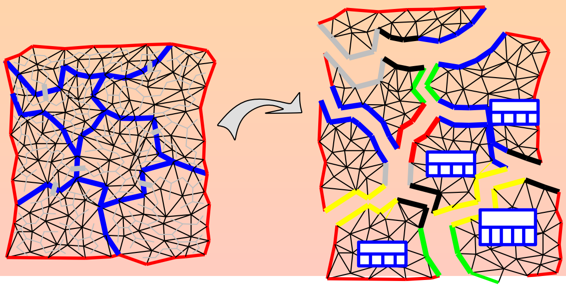
$2 \times n \times 6 \times \log n$

$+$

$n \times 1 \times \log n$

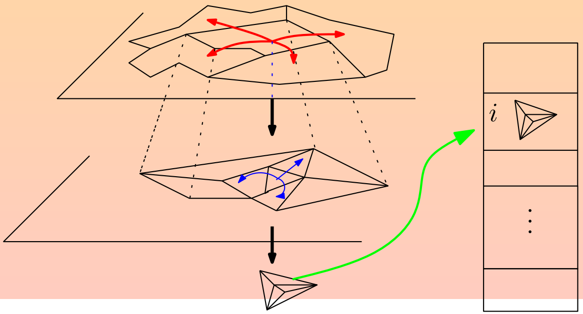
$13n$ references *or* $13n \log n$ bits

$$3.2451n + O(n^{\frac{n\log\log n}{\log n}}) = 3.2451n + o(n) \text{ bits}$$



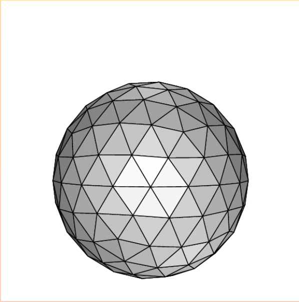
Succinct representations

(Castelli Aleardi- Devillers-Schaeffer, WADS05, SoCG06)

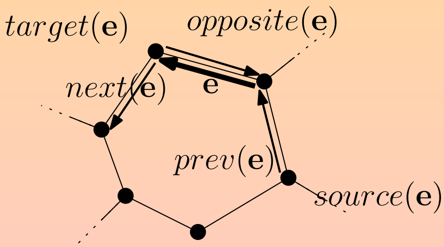


Popular (explicit) data structures for surface meshes

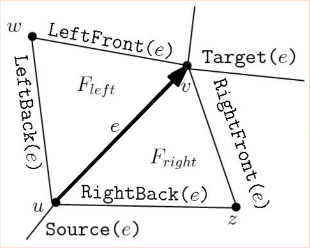
	Data Structure	size	navigation time	vertex access	dynamic
(non compact) popular data structures	Edge-based data structures	$18n + n$	$O(1)$	$O(1)$	*yes*
	Triangle based DS / Corner Table	$12n + n$	$O(1)$	$O(1)$	*yes*
existing compact data structures	Directed edge (Campagna et al. '99)	$12n + n$	$O(1)$	$O(1)$	*yes*
	2D Catalogs (Castelli Aleardi et al., '06)	$7.67n$	$O(1)$	$O(1)$	*yes*
	Star vertices (Kallmann et al. '02)	$7n$	$O(d)$	$O(1)$	*no*
	TRIPOD (Snoeyink, Speckmann, '99)	$6n$	$O(1)$	$O(d)$	*no*
	SOT (Gurung et al. 2010)	$6n$	$O(1)$	$O(d)$	*no*
Our results	Our Thm 2 (with no vertex permutation)	$5n$	$O(1)$	$O(d)$	*no*
	Our Thm 3 (with vertex permutation)	$4n$	$O(1)$	$O(d)$	*no*
	Our Cor 3 (with vertex permutation)	$6n$	$O(1)$	$O(1)$	*no*



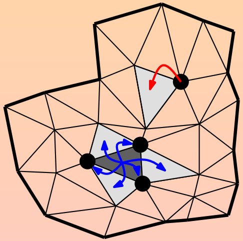
Half-edge



Winged-edge

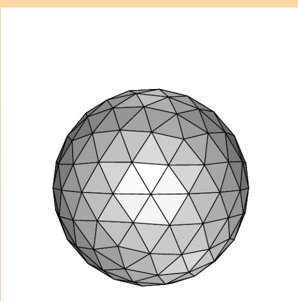


Triangle-based

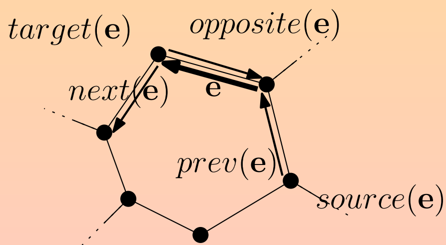


Popular (explicit) data structures for surface meshes

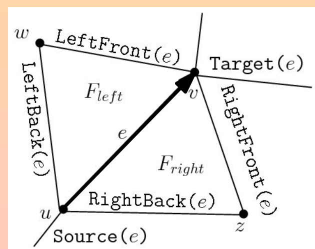
	Data Structure	size	navigation time	vertex access	dynamic
(non compact) popular data structures	Edge-based data structures	$18n+n$	$O(1)$	$O(1)$	*yes*
	Triangle based DS / Corner Table	$12n+n$	$O(1)$	$O(1)$	*yes*
existing compact data structures	Directed edge (Campagna et al. '99)	$12n+n$	$O(1)$	$O(1)$	*yes*
	2D Catalogs (Castelli Aleardi et al., '06)	$7.67n$	$O(1)$	$O(1)$	*yes*
	Star vertices (Kallmann et al. '02)	$7n$	$O(d)$	$O(1)$	*no*
	TRIPOD (Snoeyink, Speckmann, '99)	$6n$	$O(1)$	$O(d)$	*no*
	SOT (Gurung et al. 2010)	$6n$	$O(1)$	$O(d)$	*no*
ε between 0.09 and 0.3	SQUAD (Gurung et al. 2011)	$(4+\varepsilon)n$	$O(1)$	$O(d)$	*no*
Our results	Our Thm 2 (with no vertex permutation)	$5n$	$O(1)$	$O(d)$	*no*
	Our Thm 3 (with vertex permutation)	$4n$	$O(1)$	$O(d)$	*no*
	Our Cor 3 (with vertex permutation)	$6n$	$O(1)$	$O(1)$	*no*
ε about 0.8 and 0.3	LR (Gurung et al. 2011)	$(2+\varepsilon)n$	$O(1)$	$O(d)$	*no*



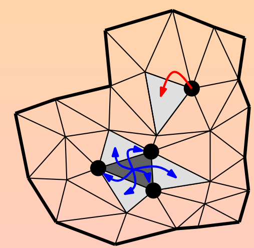
Half-edge



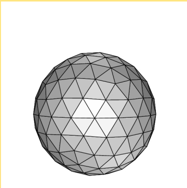
Winged-edge

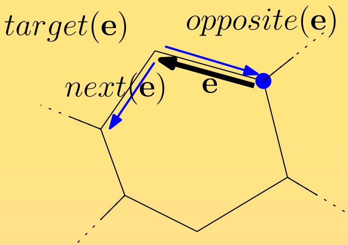


Triangle-based



Popular mesh data structures: space requirements





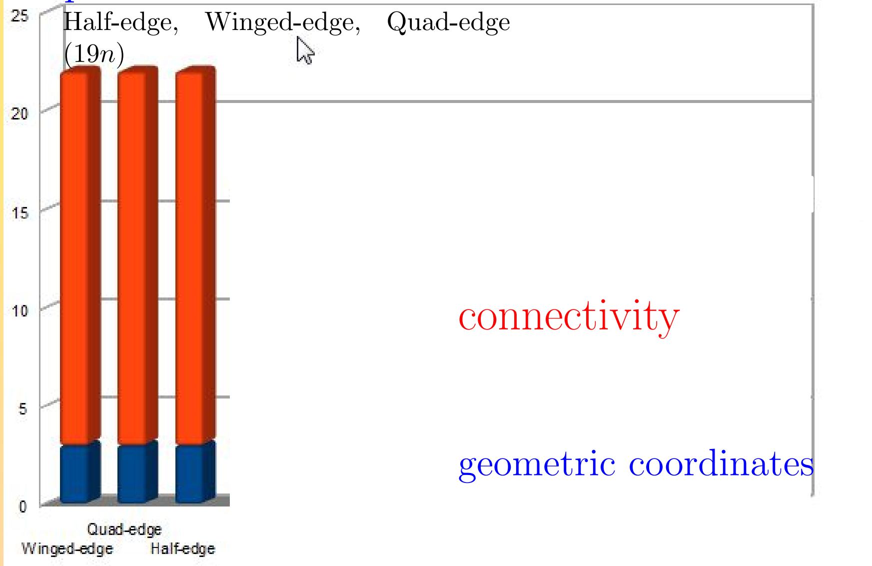
Half-edge

```
class Point{
  float x;
  float y;
  float z;
}
```

geometry

```
class Halfedge{
  Halfedge next, opposite;
  Vertex source;
}
 class Vertex{
  Halfedge  e;
  Point p;
}
```

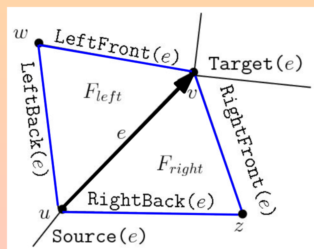
connectivity



Size (number of references)

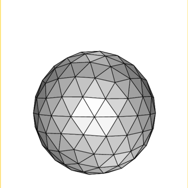
$$3 \times 2e + n = 18n + n$$

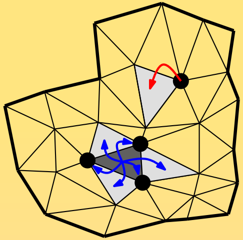
Winged-edge



$$(4 + 2) \times e + n = 18n + n$$

Popular mesh data structures: space requirements

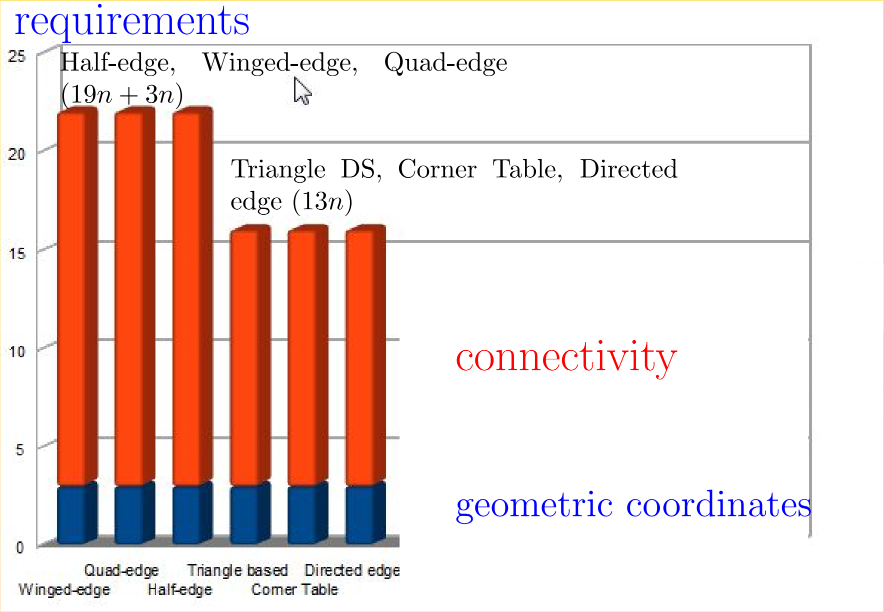




Triangle-based

```
class Triangle{
  Triangle t1, t2, t3;
  Vertex v1, v2, v3;
}
 class Vertex{
  Triangle  root;
  Point p;
}
```

```
class Point{
  float x;
  float y;
  float z;
}
```

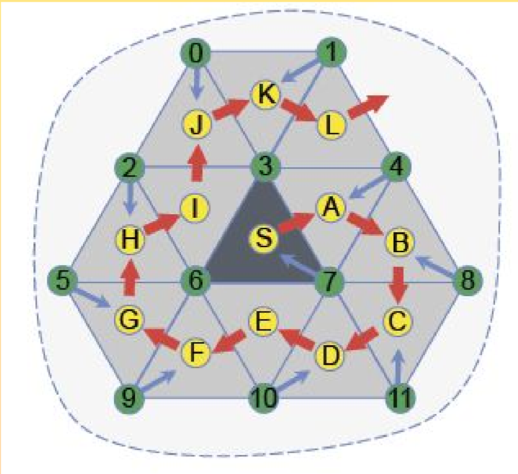


Size (number of references)

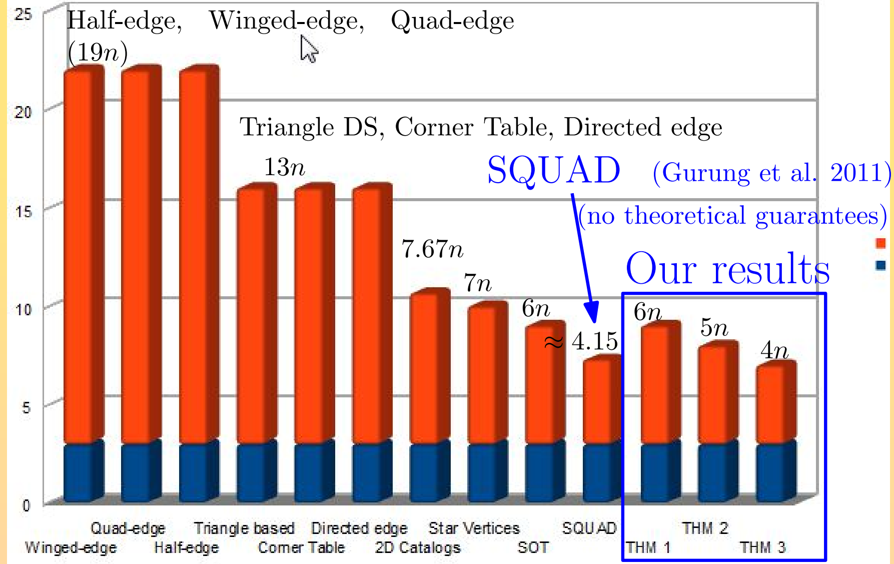
$$(3+3)\times f + n = 6\times 2n + n = 13n$$

Non compact vs. compact mesh data structures

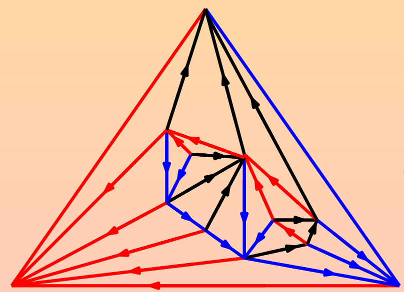
perform face reordering
perform face reordering



SOT data structure (Gurung et al. 2010)



TRIPOD data structure (Snoeyink and Speckmann, 1999)



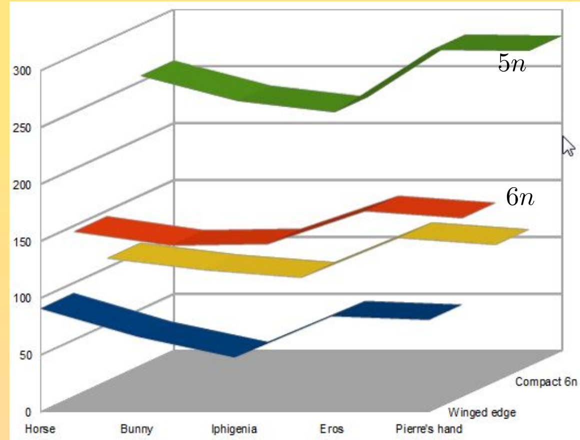
use Schnyder woods (store 3 edges per vertex)

Experimental comparison

Tested on 3D models and random planar triangulations

(timings are expressed in nanoseconds/vertex)

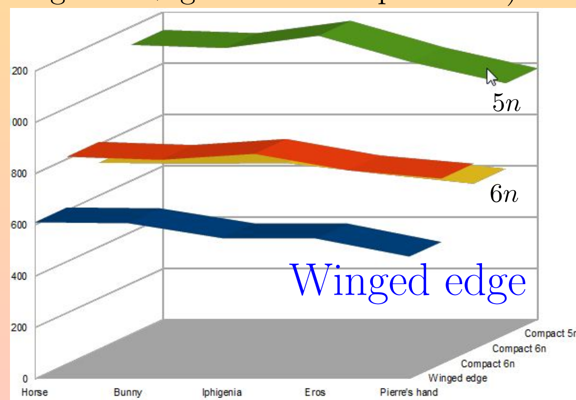
vertex degree (only topological navigation)



1.2 - 1.9 times slower

Winged edge vs. Our Compact DS

vertex normals (navigation + geometric computations)



1.19 - 1.35 times slower

First simple Compact DS (size $6n$)

$e := (u, v)$

$0 \le v \le n-1$

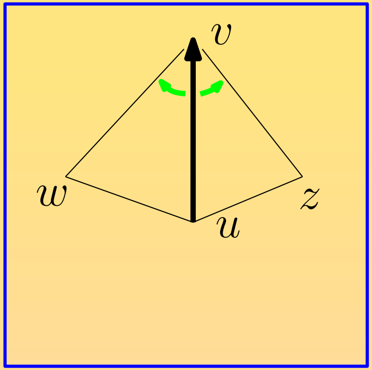
$0 \le e \le 3n$

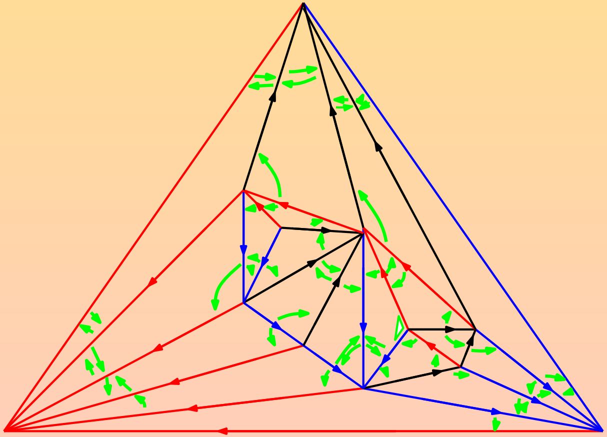
array based implementation of

a variation of TRIPOD

(J. Snoeyink and B. Speckmann, 1999)

+ reordering of edges according to the (original) vertex numbering



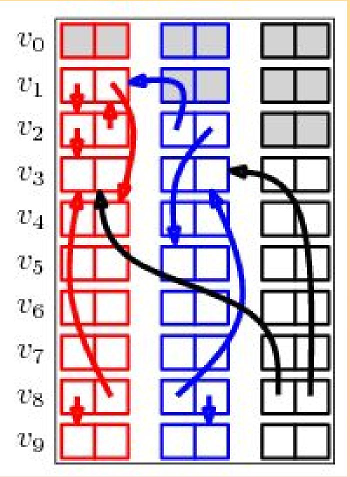


n lines

3 columns

2 references per edge

T



First simple Compact DS (size $6n$)

$e := (u, v)$ $\quad 0 \leq v \leq n-1$

$0 \leq e \leq 3n$

$$(w, u) := \text{retrieve} \begin{cases} (e+1)\%3 & \text{case 1} \\ (T[e]+2)\%3 & \text{case 2} \\ T[T[e]] & \text{case 3} \end{cases}$$

retrieving (z, u) is similar

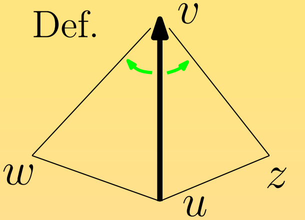
$u := \texttt{source}(e) = e/3$

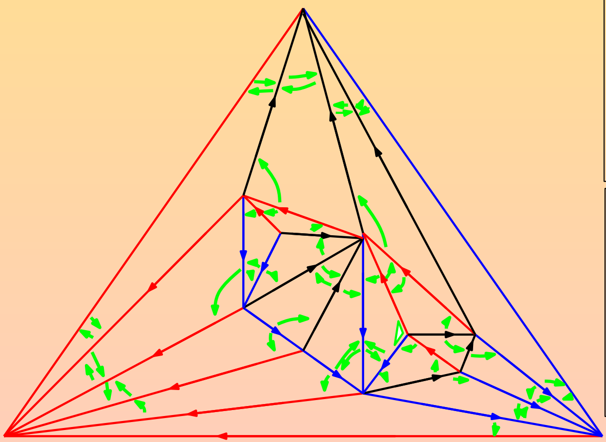
(w,v):=T[2e]

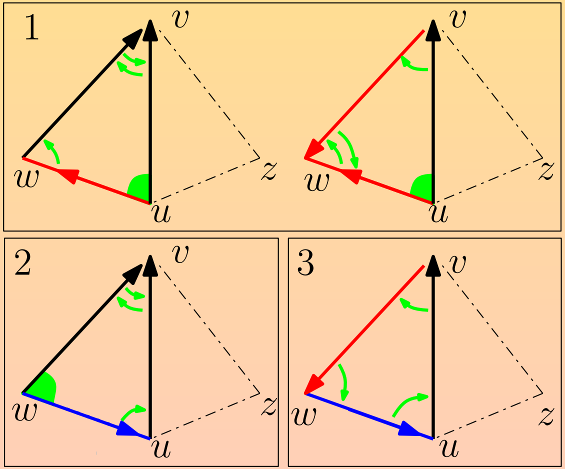
$(z, v) := T[2e+1]$

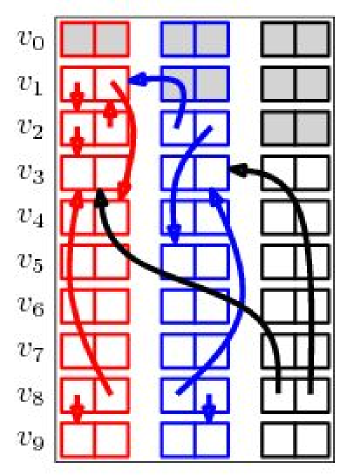
$\texttt{color}(e) = e\%3$

Def.



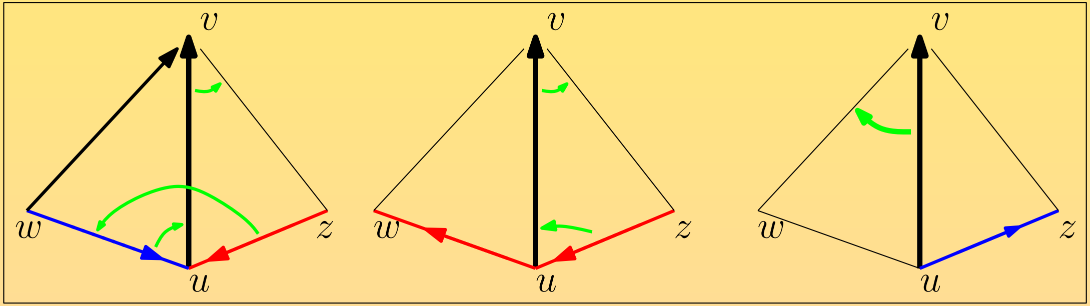




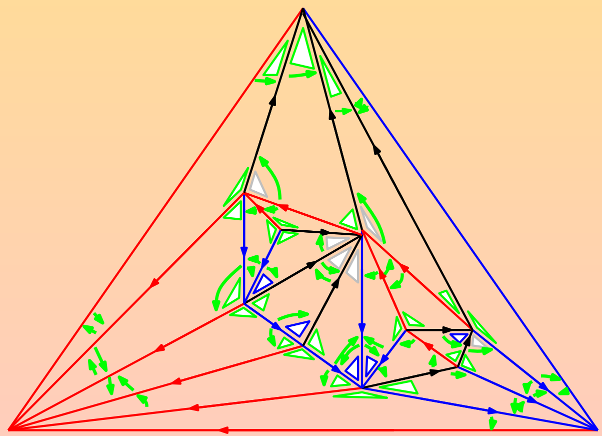


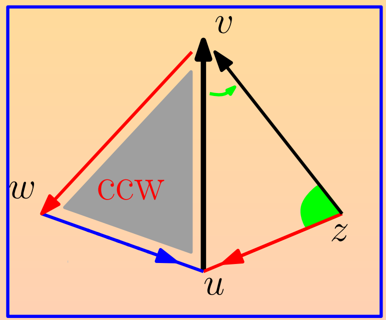
More compact DS (size $5n$): use minimal Schnyder woods

(less redundant and "slightly more difficult to implement")

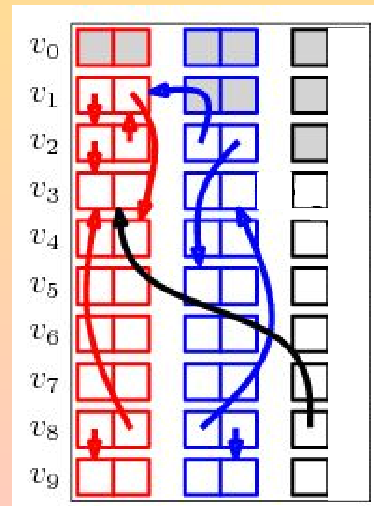


only 1 reference for black edges





forbidden configuration

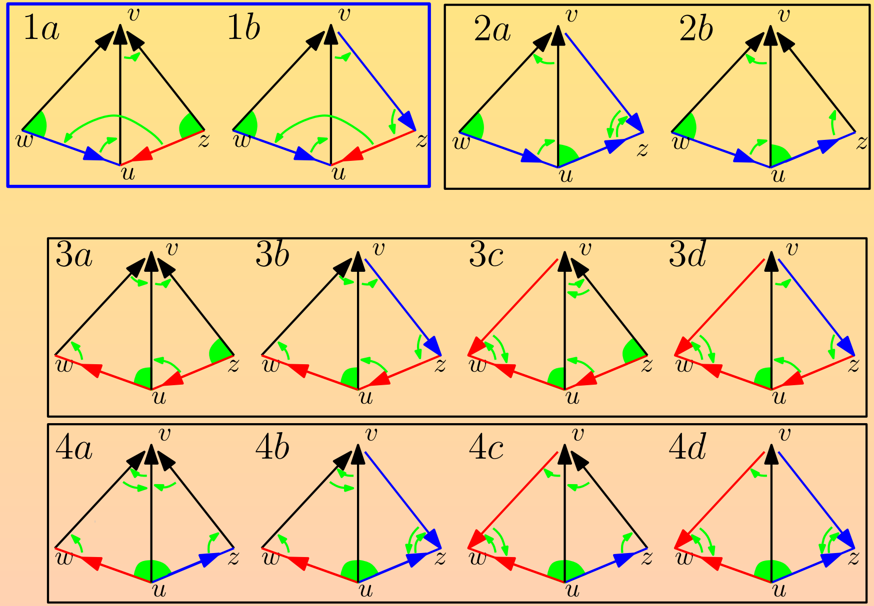


More compact DS (size $5n$): use minimal Schnyder woods

(less redundant and "slightly more difficult to implement")

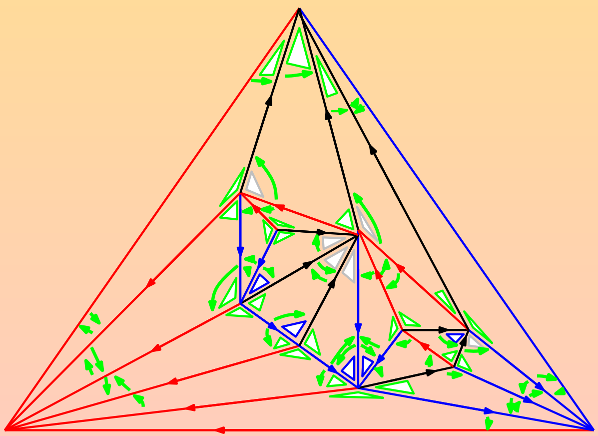
implementation: many cases to distinguish

$$\text{retrieve } (w,u) := \begin{cases} T[T[5u]-3] & \text{case } 1a \\ T[T[T[5u]]] & \text{case } 1b \\ \ldots & \ldots \end{cases}$$



case analysis for black edges

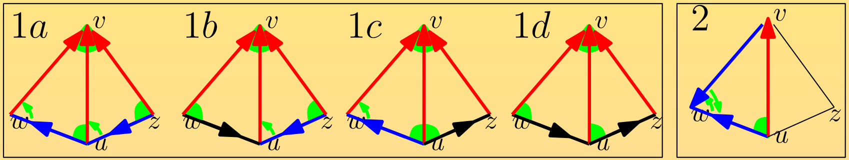
(similar case analysis for the other two colors)



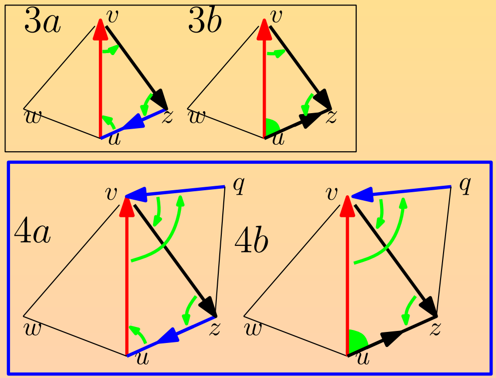
Most compact DS (4n references)

(still more compact using vertex reordering)

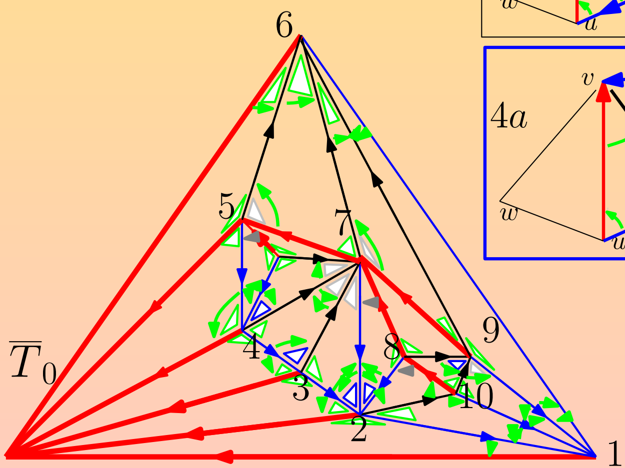
use the *DFUDS* (Depth First Unary Degree Sequence) order on $\overline{T}_0$

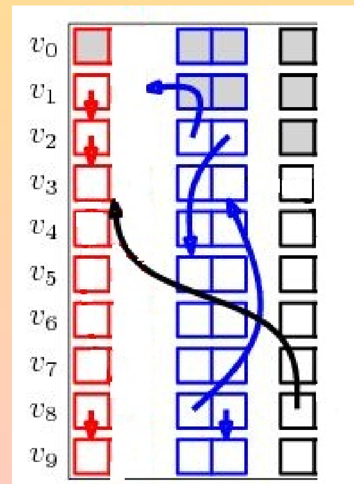


case analysis for red edges



only 1 reference for black and red edges

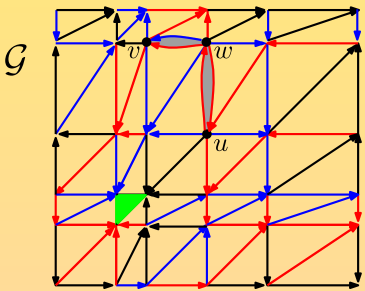




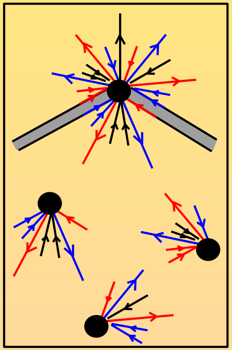
Compact DS for higher genus meshes (5n references)

combine previous ideas with the use of *genus g Schnyder woods*

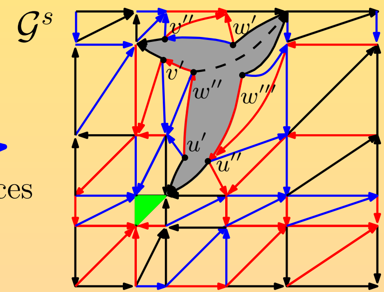
(Castelli Aleardi, Fusy, Lewiner SoCG'08)



$\mathcal{E}^s = \{(u,w),(v,w)\}$



split multiple vertices



$n + 4g$ vertices

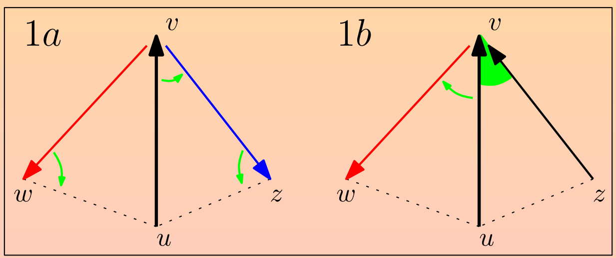
$2n + 2g$ faces

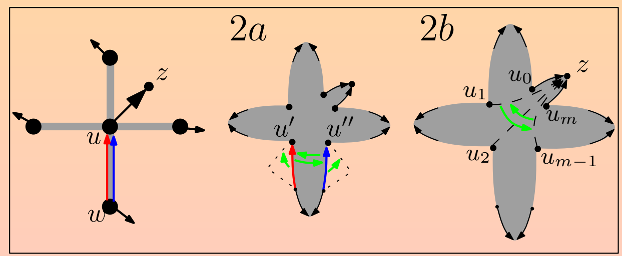
toroidal triangulation endowed with a g-Schnyder wood

almost all vertices have outgoing degree 3

all vertices have outgoing degree at most 3

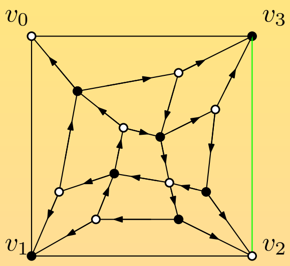
more cases to consider

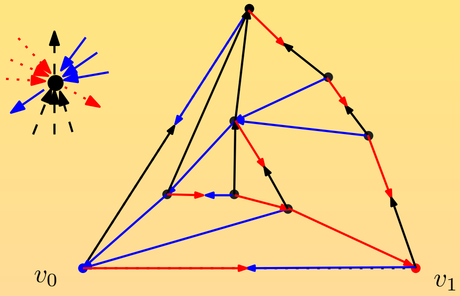




Conclusion

• possible extension to more general surface meshes (planar quadrangulations and 3-connected planar maps)





• is it possible to further reduce the space requirements: from $4n$ to $3n$ or less?