Algorithms and Combinatorics of Geometric Graphs (`Geomgraphs`) 2025-2026

TD7 (exercises)

Luca Castelli Aleardi

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Exercise 1 – Schnyder woods and coding of planar triangulations

Let us consider a triangulation $\mathcal{T}$ endowed with a Schnyder wood (T_0, T_1, T_2), having root face $f = (V_0, V_1, V_2)$. Let $\overline{T_0}$ denotes the tree obtained adding to T_0 the two edges (V_1, V_0) and (V_2, V_0) (oriented toward V_0). Consider the depth first traversal of $\overline{T_0}$ obtained by visiting in ccw order the edges starting from (V_0, V_1). Such a traversal defines an ordering $\pi = \{1, 2, \ldots, n\}$ on the vertices of $\mathcal{T}$ such that $\pi(V_0) = 1$, $\pi(V_1) = 2$ and $\pi(V_2) = n$, as illustrated in Figure 1.

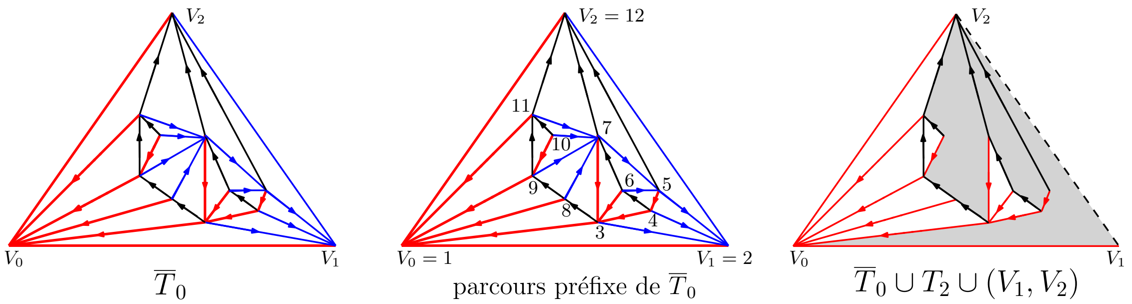

Figure 1: vertex labels correspond to a depth first traversal of the edges of the tree $\overline{T_0}$.

Question 1.1. *Show that if u is a descendant of v in the tree T_i, then u cannot be descendant or ancestor of v in the trees T_{i+1} and T_{i-1} (indices are computed modulo 3).*

Question 1.2. *Consider an edge (u, v) in the tree T_1 (blue) oriented toward v. Show that $\pi(u) > \pi(v)$. Similarly, let (w, z) an edge in the tree T_2 (black) oriented toward z. Show that $\pi(w) < \pi(z)$.*

The information of the edges in T_1 is redundant, as expressed in the question below.

Question 1.3. *Consider the map $G = \overline{T_0} \cup T_2 \cup (V_1, V_2)$ obtained by removing the edges of color 1 and adding the exterior edge (V_1, V_2). Show that from the map G we can recover the Schnyder wood (T_0, T_1, T_2) (without any knowledge of the starting triangulation $\mathcal{T}$).*

Question 1.4. *Using previous questions, show that a planar tringulation with n vertices (endowed with a Schnyder wood) can be encoded by two binary words S and S' whose total lenght satisfied $|S| + |S'| < 4n$.*