A Stanley-Wilf/Marcus-Tardos Type Result for Ordered Set Partitions

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July 1, 2013

Outline

1. A Walk Down Memory Lane
   - Perm Matrices and the F-H Conjecture
   - Proof that FHC $\Longrightarrow$ SWC
2. Ordered Set Partitions
3. A Marcus-Tardos Result for Ordered Set Partitions

Pattern Avoidance in Permutations

Definition

Let $p = p_1 p_2 \cdots p_n \in \mathcal{S}_n$*, the symmetric group on* n *elements, and* $q \in \mathcal{S}_k$*. We say that* p *contains* q *if there is a subsequence of* p*,* $p' = p_{i_1} p_{i_2} \cdots p_{i_k}$ *with* p' *order isomorphic to* q*. Otherwise we say that* p *avoids* q*.*

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Definition

Let $p \in \mathcal{S}_k$. Define $\mathcal{S}_n(p)$ to be the set of permutations of $[n]$ that avoid p.

The Stanley-Wilf Conjecture (Arratia Version [1])

In the early 1990's, Richard Stanley and Herbert Wilf conjectured that for each permutation p there is some $C \in [1, \infty)$ such that

$$\limsup_{n \to \infty} |\mathcal{S}_n(p)|^{1/n} = C.$$

That is, $|\mathcal{S}_n(p)|$ has exponential growth.

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Theorem (Marcus-Tardos 2004 [4])

For each permutation p there is a constant $C \in [1, \infty)$ such that

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Permutation Matrices

Definition

For any permutation $p = p_1 p_2 \cdots p_n \in \mathcal{S}_n$, *let* $M_p = [m_{ij}]$ *be the 0-1 matrix with* $m_{p_i i} = 1$ *for* $1 \leq i \leq n$ *and the remaining entries 0.*

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Example

The permutation 1342 *has associated matrix*

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

The Füredi-Hajnal Conjecture

Definition

Let A and P be 0-1 matrices. We say that A contains the $k \times \ell$ matrix $P = [p_{ij}]$ if there exists a $k \times \ell$ submatrix $B = [b_{ij}]$ of A with $b_{ij} = 1$ whenever $p_{ij} = 1$. Otherwise we say that A avoids P.

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Example

A copy of $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$ *in* $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}$.

The Füredi-Hajnal Conjecture

Lemma

If $p \in \mathcal{S}_n$ has matrix P and $q \in \mathcal{S}_k$ has matrix Q then p avoids q iff P avoids Q.

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Conjecture (F-H 1992, Proved by Marcus and Tardos in 2004)

Let P be a permutation matrix. Let $f(n,P)$ be the maximal number of 1's in an $n \times n$ 0-1 matrix avoiding P. Then

$$f(n,P) = O(n).$$

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Theorem (Klazar 2000 [3])

The Füredi-Hajnal Conjecture implies the Stanley-Wilf Conjecture.

Partitioning and Reducing a Matrix

Example

$n = 5$ *and* $k = 2$.

$$A = \left[\begin{array}{cc|cc|c} 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ \hline 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ \hline 1 & 1 & 1 & 0 & 1 \end{array}\right]$$

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Lemma

Let P be a permutation matrix. If A avoids P then so does B.

FHC $\Longrightarrow$ SWC

Observation: Let p be a permutation and P its associated matrix. Let $T_n(P)$ be the set of 0-1 matrices avoiding P. Then $|S_n(p)| \leq |T_n(P)|$.

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Proof that FHC $\implies$ SWC: Let P be a permutation matrix and suppose that $f(n, P) = O(n)$. We have that

$$|T_{2n}(P)| \leq |T_n(P)| \cdot 15^{f(n,P)}.$$

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- Let $A \in T_{2n}(P)$, and form B by partitioning A into 2×2 blocks and replacing blocks of all zeros by 0, and any other block by a 1.
- Each $B \in T_n(P)$ has at most $15^{f(n,P)}$ preimages.$\square$

Outline

Set Partitions

Definition

Let $[n] = \{1, 2, \ldots, n\}$ *a partition,* π*, of* $[n]$, $\pi \vdash [n]$ *is a family of sets* $B_1, B_2, \ldots, B_k$*, called blocks, such that* $B_1 \uplus B_2 \uplus \cdots \uplus B_k = [n]$*. We write*

$$\pi = B_1/B_2/\ldots/B_k,$$

where $\min B_1 < \min B_2 < \cdots < \min B_k$.

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Example

$\pi = 13/247/56 \vdash [7]$.

Ordered Set Partitions

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An ordered partition π *of* $[n]$ *is a partition of* $[n]$ *where we specify an order for the blocks.*

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Let $\mathcal{OP}_{n,k}$ *be the set of ordered partitions of* $[n]$ *with* k *blocks.*

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An ordered partition π of $[n]$ is a partition of $[n]$ where we specify an order for the blocks.

Definition

Let $\mathcal{OP}_{n,k}$ be the set of ordered partitions of $[n]$ with k blocks.

Notice that the set of permutations of $[n]$, $\mathcal{S}_n$, is in bijection with $\mathcal{OP}_{n,n}$.

Patterns in Ordered Partitions

Definition

An ordered partition $\sigma = B_1/B_2/\dots/B_k \in \mathcal{OP}_{n,k}$ *is said to contain a copy of a permutation* $p = p_1 p_2 \cdots p_m$ *if there is a sequence of elements* $a = a_{i_1} a_{i_2} \cdots a_{i_m}$ *with* $a_{i_j} \in B_{i_j}$ *for* $1 \leq j \leq m$ *with* $i_1 < i_2 < \cdots < i_m$ *such that* a *is order isomorphic to* p*. Otherwise we say* σ *avoids* p*.*

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Example

For example let $\pi = 56/247/13$*.* 523 *forms a copy of* 312*, and* 573 *forms a copy of* 231*.*

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Example

For example let $\pi = 56/247/13$*.* 523 *forms a copy of* 312*, and* 573 *forms a copy of* 231*. However,* π *avoids the permutations* 123 *and* 132*.*

Outline

A Marcus-Tardos Result for Ordered Partitions

Theorem (Godbole-G-Pudwell [2])

Let $p \in \mathcal{S}_m$. Then there is some constant $C \in [1, \infty)$ such that

$$\lim_{n \to \infty} |\mathcal{OP}_{n,k}(p)|^{1/n} = C.$$

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- $|\mathcal{OP}_{n,n}(123)| = C_n \sim \frac{4^n}{n^{3/2}\sqrt{\pi}}$
- $|\mathcal{OP}_{n,n-1}(123)| \sim K \cdot \frac{4^n}{\sqrt{n}}$

Holey Ordered Partitions Batman!

Definition

We will say a holey ordered partition of $[n]$ is an ordered partition of $[n]$, where some of the blocks are allowed to remain empty. Let $\mathcal{OP}^*_{n,k}$ be the set of holey ordered partitions of $[n]$ with k blocks.

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Example

$\pi = \emptyset/26/\emptyset/\emptyset/137/4/\emptyset/5$ *is a holey ordered partition of $[7]$ with 8 blocks.*

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$\pi = \emptyset/26/\emptyset/\emptyset/137/4/\emptyset/5$ *is a holey ordered partition of* $[7]$ *with 8 blocks.*

Definition

Let $p \in \mathcal{S}_m$*. We let* $\mathcal{OP}^*_{n,k}(p)$ *be the set of holey ordered partitions avoiding* p*.*

A Marcus-Tardos Result for Holey Ordered Partitions

Theorem

For each $p \in \mathcal{S}_m$, there is some constant $C \in [1, \infty)$ such that

$$\lim_{n\to\infty} |\mathcal{OP}^*_{n,k}(p)|^{1/n} = C.$$

A Marcus-Tardos Result for Holey Ordered Partitions

Proof: Let $p \in \mathcal{S}_m$, we define an injection

$$f : \mathcal{OP}^*_{a+b,k}(p) \to \mathcal{OP}^*_{a,k}(p) \times \mathcal{OP}^*_{b,k}(p).$$

Example

$$2/18/\emptyset/357/4/6$$
$$\downarrow$$
$$(2/1/\emptyset/35/4/\emptyset, \emptyset/8/\emptyset/7/\emptyset/6)$$
$$\downarrow$$
$$(2/1/\emptyset/35/4/\emptyset, \emptyset/3/\emptyset/2/\emptyset/1)$$

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Example

$$\begin{array}{c} 2/18/\emptyset/357/4/6 \\ \downarrow \\ (2/1/\emptyset/35/4/\emptyset, \emptyset/8/\emptyset/7/\emptyset/6) \\ \downarrow \\ (2/1/\emptyset/35/4/\emptyset, \emptyset/3/\emptyset/2/\emptyset/1) \end{array}$$

So

$$|\mathcal{OP}^*_{a+b,k}(p)| \leq |\mathcal{OP}^*_{a,k}(p)| \cdot |\mathcal{OP}^*_{b,k}(p)|.$$

A Marcus-Tardos Result for Holey Ordered Partitions

This gives us that $\log(|\mathcal{OP}^*_{a+b,k}(p)|) \leq \log(|\mathcal{OP}^*_{a,k}(p)|) + \log(|\mathcal{OP}^*_{b,k}(p)|)$, so $\log(|\mathcal{OP}^*_{n,k}(p)|)$ is a subadditive function.

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By Fekete's Lemma [5] we have $\lim_{n\to\infty} \dfrac{\log(|\mathcal{OP}^*_{n,k}(p)|)}{n} \in [0,\infty)$.

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By Fekete's Lemma [5] we have $\lim_{n\to\infty} \dfrac{\log(|\mathcal{OP}^*_{n,k}(p)|)}{n} \in [0, \infty)$.

Thus,

$$\lim_{n\to\infty} |\mathcal{OP}^*_{n,k}(p)|^{1/n} \in [1, \infty).\Box$$

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Theorem (Godbole-G-Pudwell [2])

Let $p \in \mathcal{S}_m$. Then there is some constant $C \in [1, \infty)$ such that

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We will show that

$$\frac{|\mathcal{OP}^*_{n,k}(p)|}{(k+1)^{2k}} \leq |\mathcal{OP}_{n,k}(p)| \leq |\mathcal{OP}^*_{n,k}(p)|.$$

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$$\frac{|\mathcal{OP}^*_{n,k}(p)|}{(k+1)^{2k}} \leq |\mathcal{OP}_{n,k}(p)| \leq |\mathcal{OP}^*_{n,k}(p)|.$$

For $n \geq k$, we will describe an injection

$$\psi : \mathcal{OP}^*_{n,k}(p) \to \mathcal{OP}_{n,k}(p) \times \{0, 1, \ldots, k\}^{2k}.$$

A Marcus-Tardos Result for Ordered Partitions

Suppose $p \in S_m$ and p does not end in m, $\sigma \in \mathcal{OP}^*_{n,k}(p)$ with $n \geq k$.

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Suppose $p \in S_m$ and p does not end in m, $\sigma \in \mathcal{OP}^*_{n,k}(p)$ with $n \geq k$.

We will describe an algorithm that gives $\psi(\sigma) = (\pi, w)$, where $\pi \in \mathcal{OP}_{n,k}(p)$ and $w \in \{0, 1, \ldots, k\}^{2k}$ that satisfies:

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- At each step the partition is p-avoiding.

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- At each step the partition is p-avoiding.
- The final partition has no holes.

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- At each step the partition is p-avoiding.
- The final partition has no holes.
- The process is invertible due to the information recorded in w.
- $\frac{|\mathcal{OP}^*_{n,k}(p)|}{(k+1)^{2k}} \leq |\mathcal{OP}_{n,k}(p)| \leq |\mathcal{OP}^*_{n,k}(p)|$.

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We will describe an algorithm that gives $\psi(\sigma) = (\pi, w)$, where $\pi \in \mathcal{OP}_{n,k}(p)$ and $w \in \{0, 1, \ldots, k\}^{2k}$ that satisfies:

- At each step the partition is p-avoiding.
- The final partition has no holes.
- The process is invertible due to the information recorded in w.
- $\frac{|\mathcal{OP}^*_{n,k}(p)|}{(k+1)^{2k}} \leq |\mathcal{OP}_{n,k}(p)| \leq |\mathcal{OP}^*_{n,k}(p)|$.
- Since k is fixed $\lim_{n\to\infty} |\mathcal{OP}_{n,k}(p)|^{1/n} = C$ for some $C \in [1, \infty)$.

A Marcus-Tardos Result for Ordered Partitions

Let $p = 132$.

	Partition	w
1	$\emptyset/8/345/\emptyset/12/6/7$	$\emptyset$

A Marcus-Tardos Result for Ordered Partitions

Let $p = 132$.

	Partition	w
1	$\emptyset/8/345/\emptyset/12/6/7$	$\emptyset$
2	$8/345/12/6/7/\emptyset/\emptyset$	2356700

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3	$\emptyset/345/12/6/\emptyset/7/8$	2356700

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4	$6/34/12/\emptyset/5/7/8$	2356700

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5	$6/3/12/4/5/7/8$	2356700

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5	$6/3/12/4/5/7/8$	2356700
6	$6/3/12/4/5/7/8$	23567004202251

Thank You

Merci!

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