Clustering to Given Connectivities

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Input: A graph and an integer k

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(known as CORRELATION CLUSTERING)

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FPT-algorithm: $1.62^{k} + n^{3}$ [S. Böcker, A golden ratio parameterized algorithm for Cluster Editing, Journal of Discrete Algorithms Volume 16, October 2012, Pages 79-89]

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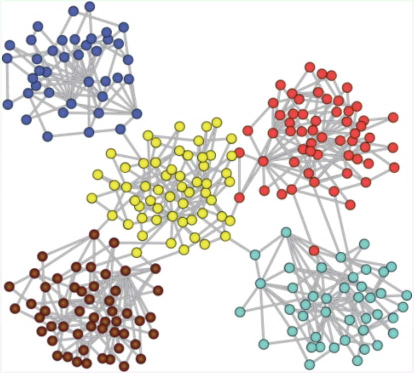

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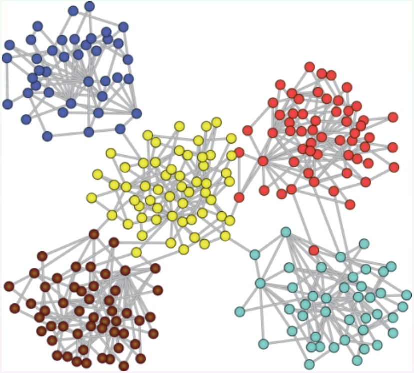

A partition of the vertices into vertex sets, called clusters, so that each cluster enjoys some desirable characteristics of "density" or "good interconnectivity", while having few edges between the clusters

FPT-algorithms and relaxations of clique density:

γ-quasi cliques: [P. Heggernes, D. Lokshtanov, J. Nederlof, C. Paul, and J. A. Telle, Generalized graph clustering: Recognizing (p, q)-cluster graphs, WG 2010]

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HIGHLY CONNECTED DELETION: each cluster C has edge-connectivity bigger than $|C|/2$

[F. Hüffner, C. Komusiewicz, A. Liebtrau, and R. Niedermeier, Partitioning biological networks into highly connected clusters with maximum edge coverage, IEEE/ACM Trans. Comput. Biology Bioinform., 11 (2014)]

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Moreover:

Clusters are of diameter at most s (s-clubs)

[S. Shahinpour and S. Butenko, Distance-based clique relaxations in networks: s-clique and s-club, in Models, Algorithms, and Technologies for Network Analysis, 2013]

Every vertex of a cluster should have an edge to all but at most $s-1$ other vertices of it (s-plexes)

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# of clusters to be obtained is exactly p

[F. V. Fomin, S. Kratsch, M. Pilipczuk, M. Pilipczuk, and Y. Villanger, Tight bounds for parameterized complexity of cluster editing with a small number of clusters, J. Comput. Syst. Sci., 80 (2014]

t-CUT problem:

Input: A graph G and an integer k

Question: is there a partition of a graph into exactly t nonempty components such that the total number of edges between the components is at most k

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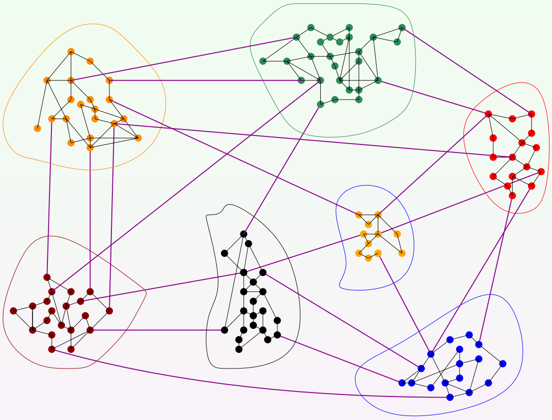

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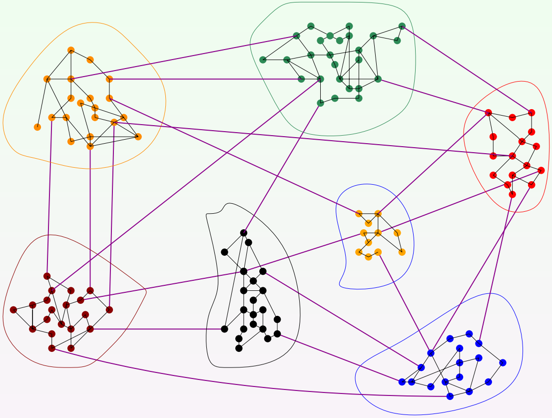
Clustering to Given Connectivities

Input: An n-vertex graph G, a t-tuple $\Lambda = \langle \lambda_1, \ldots, \lambda_t \rangle$ of positive integers, $\lambda_1 \leq \ldots \leq \lambda_t$, and a nonnegative integer k.

Task: Decide whether there is a set $F \subseteq E(G)$ with $|F| \leq k$ such that $G - F$ has t connected components $G_1, \ldots, G_t$ where each G_i is edge λ_i-connected for $i \in \{1, \ldots, t\}$.



a partition into exactly t nonempty components such that the total number of edges between the components is at most k and the connectivities of the clusters are $\lambda_1, \ldots, \lambda_t$

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Question: Is there a low parameterized-dependence algorithm?

Merci!