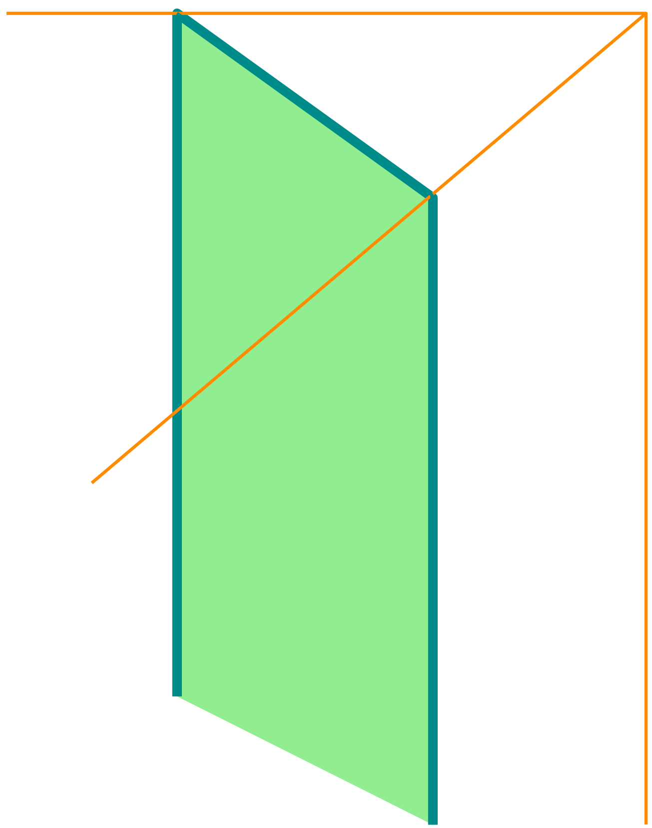
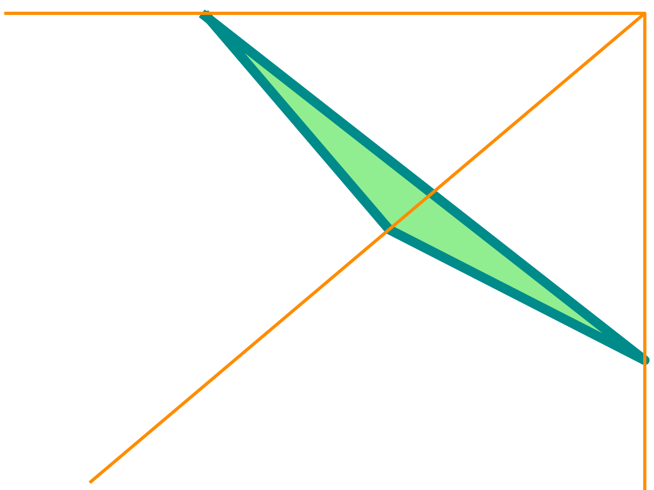
Making Octants Colorful

and other cover-decomposability problems

Kolja Knauer
Université Montpellier 2





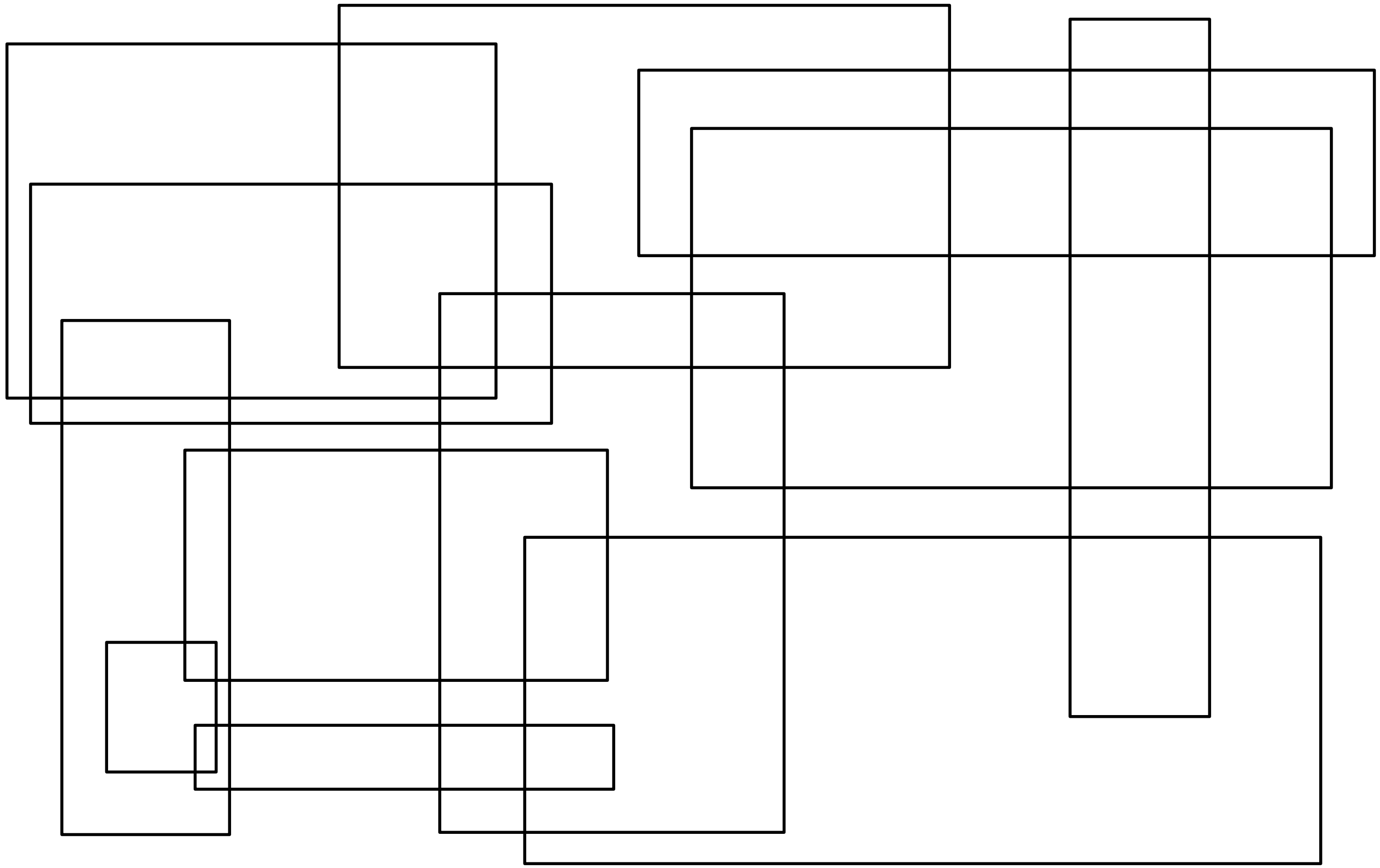
joint with Jean Cardinal, Piotr Micek, Torsten Ueckerdt

Jounées EGOS
LIX, École Polytechnique, October 29, 2013

The (primal) Problem [Pach 80]:

Decompose multiply covering covers into multiple covers...

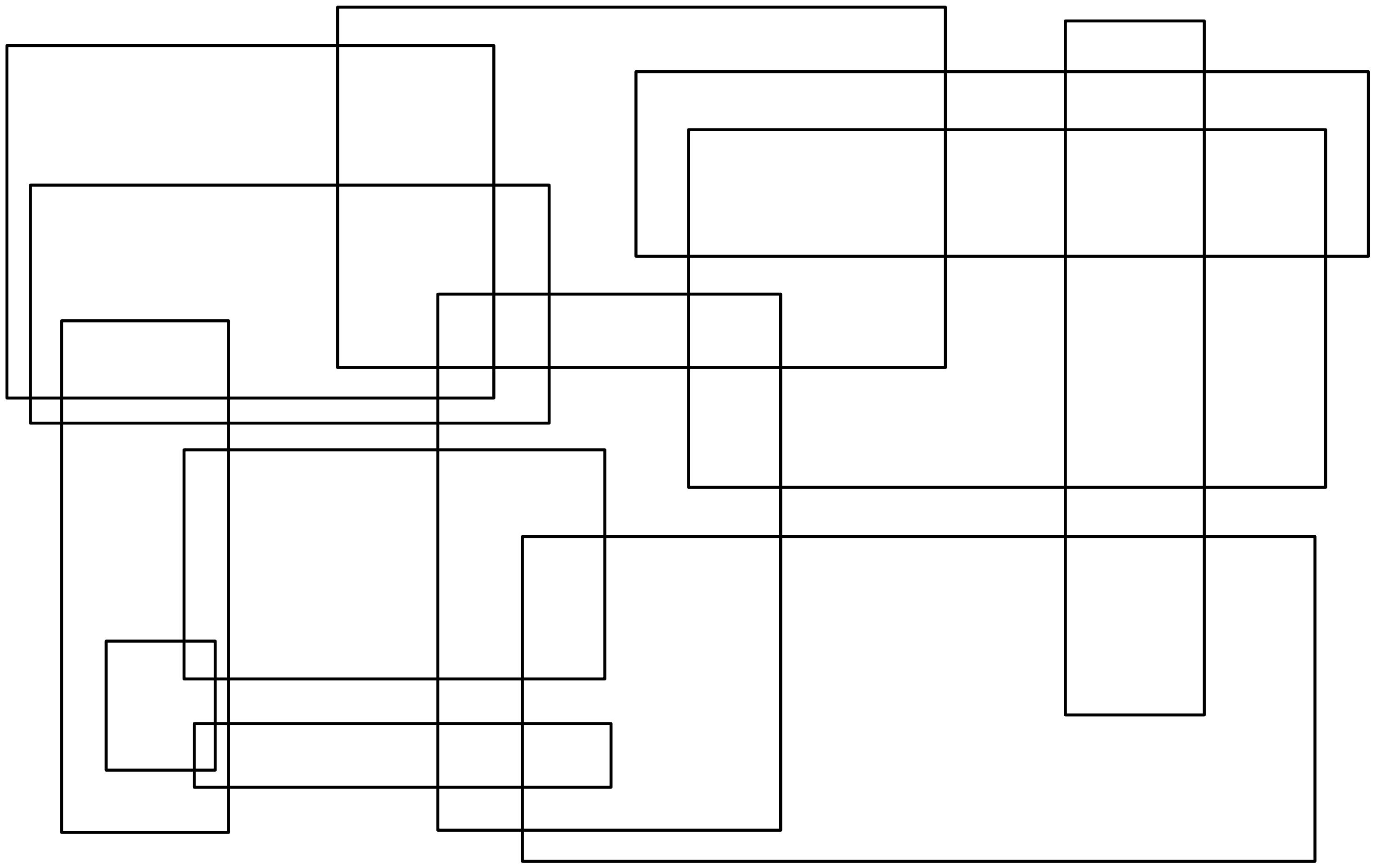
Given some geometric objects (ranges) 2-color them such that any point contained in many of them is contained in one of each color.



The (primal) Problem [Pach 80]:

Decompose multiply covering covers into multiple covers...

Given some **axix alligned rectangles**, 2-color them such that any point contained in **three** of them is contained in one of each color.



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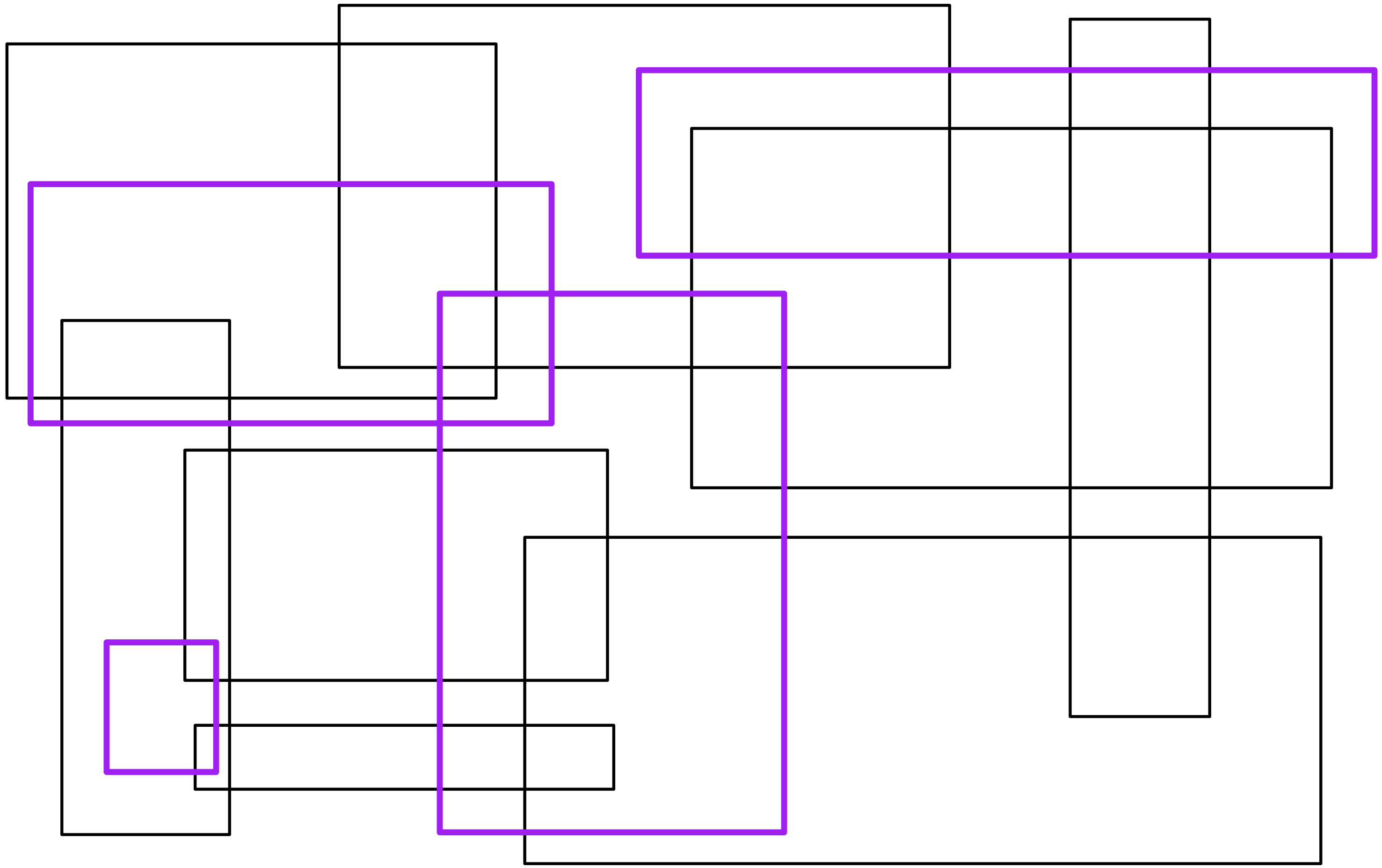
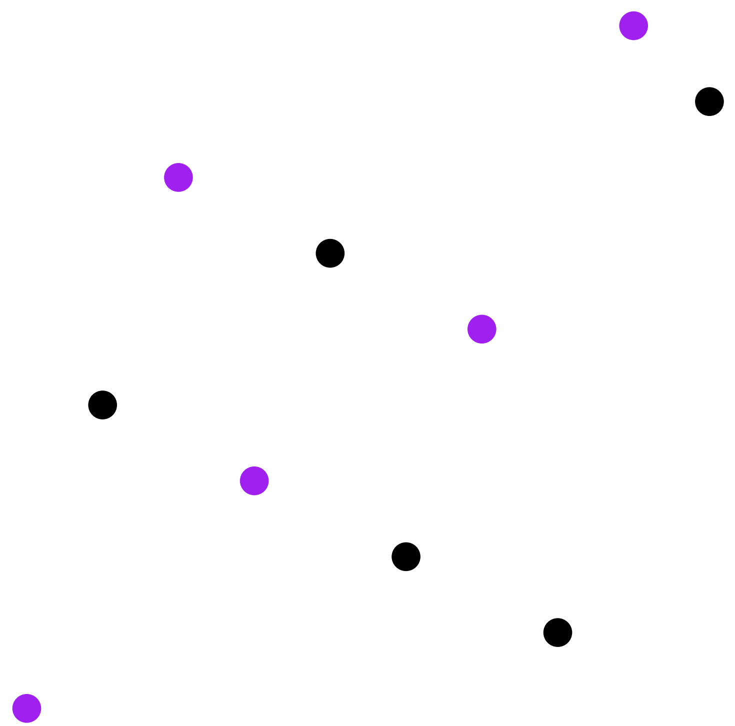


The *dual* Problem:

Making *ranges* colorful...

Given some **points**, 2-color them such that any axis alligned rectangle containing **three** of them is contains in one of each color.



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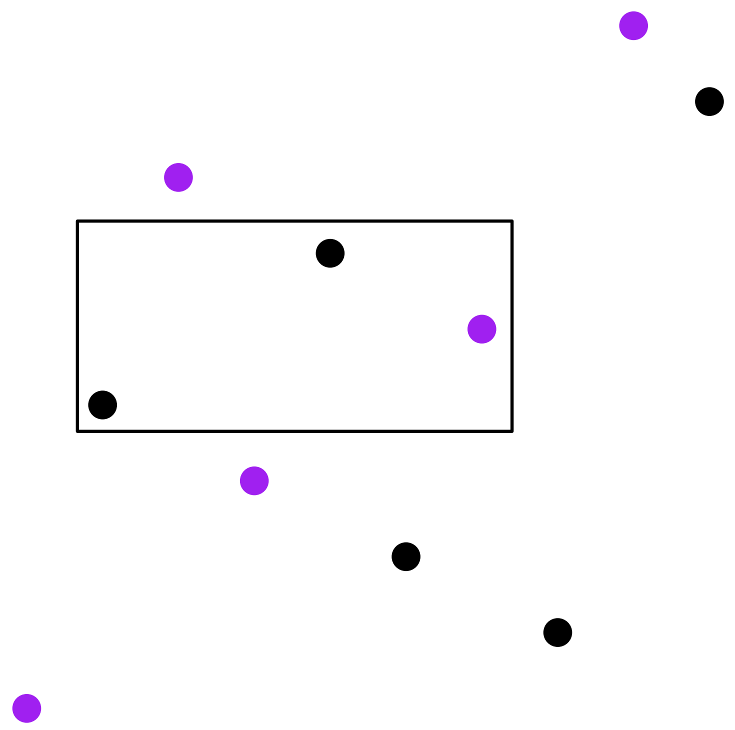

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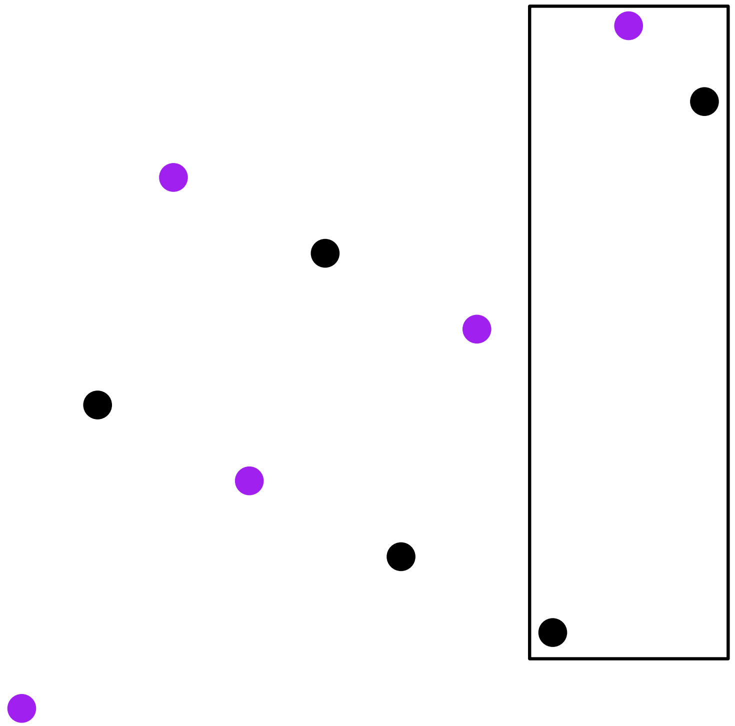

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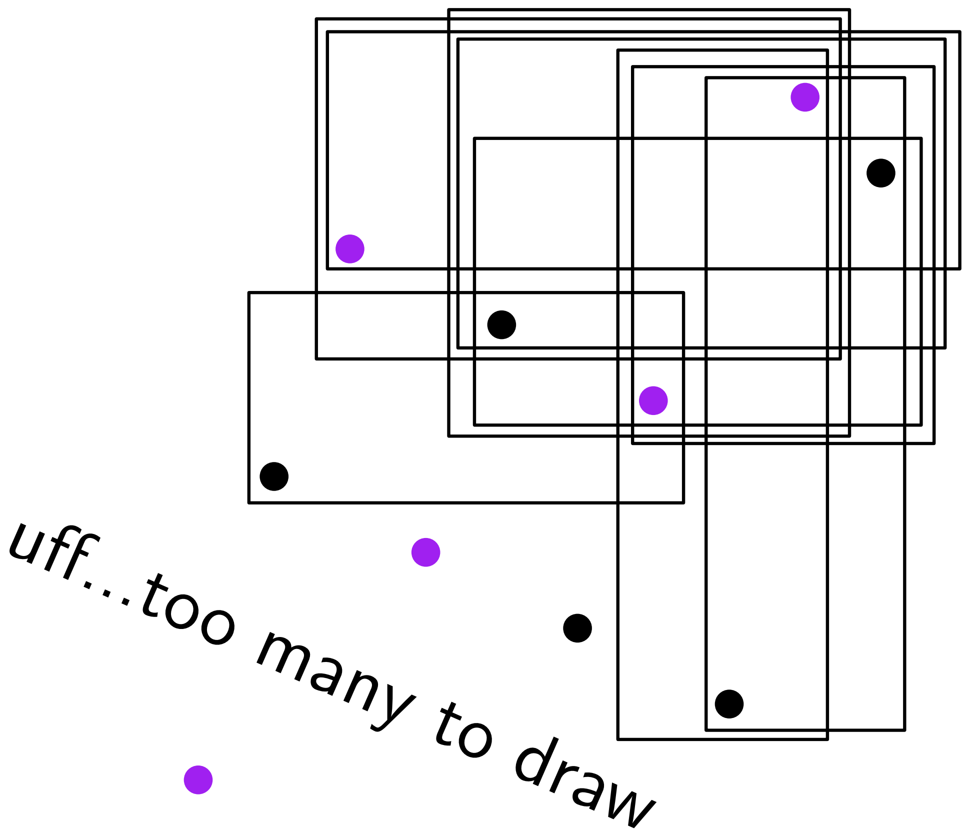

Why these examples were pedagogically bad:

Given some **axix alligned rectangles**, 2-color them such that any point contained in **three** of them is contained in one of each color.

Given some **points**, 2-color them such that any axis alligned rectangle containing **three** of them is contains in one of each color.

Thm[Pach, Tardos, Tóth '09]: For every c there is set R_c of axis alligned rectangles such that in every 2-coloring of R_c there is point contained in $\geq c$ rectangles but all of the same color.

Thm[Chen, Pach, Szegedy, Tardos '09]: For every p there is set P_p of points such that for every 2-coloring of P_p there is an axis alligned rectangle containing $\geq p$ points but all of the same color.

What we can do about it:

Given some **geometric ranges**, 2-color them such that any point contained in **three** of them is contained in one of each color.

Given some **points**, 2-color them such that any **geometric range** containing **three** of them is contains in one of each color.

What we can do about it:

Given some **geometric ranges**, $\mathbf{k}$ color them such that any point contained in $\mathbf{c(k)}$ of them is contained in one of each color.

Given some **points**, $\mathbf{k}$ color them such that any **geometric range** containing $\mathbf{p(k)}$ of them is contains in one of each color.

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	geometric ranges	$\mathbf{c(k)}$	$\mathbf{p(k)}$
	all halfplanes in $\mathbb{R}^2$	$\leq 4k-3$	$2k-1$
	all halfplanes in $\mathbb{R}^3$	?	∞
translates of	disk	?	
	convex polygon	$O(k)$	
	negative octant in $\mathbb{R}^3$	$\leq k^6$	
homothetic copies of	bottomless rectangles	$\leq k^6$	$1.6k \leq \cdot \leq 3k-2$
	triangle	$\leq k^6$	$\leq k^5$
	polygon	?	?
	disk	?	∞

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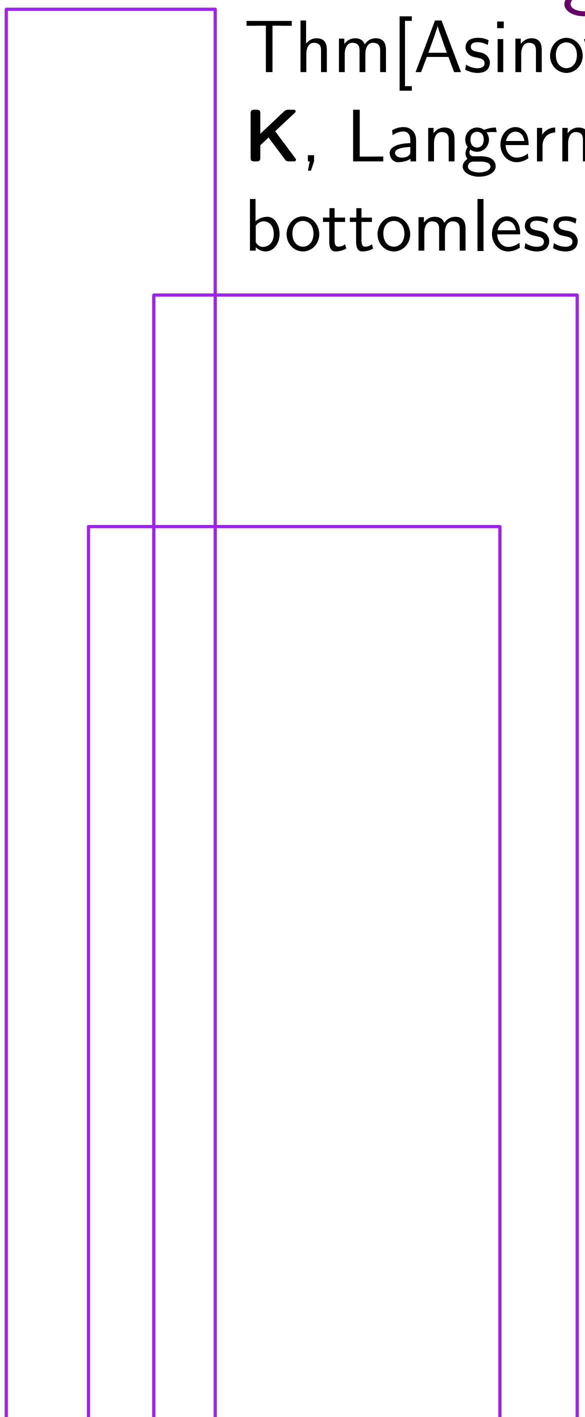
translates of disk	?	
translates of convex polygon	$O(k)$	
translates of negative octant in $\mathbb{R}^3$	$\leq k^6$	
homothetic copies of bottomless rectangles	$\leq k^6$	$1.6k \leq \cdot \leq 3k-2$
homothetic copies of triangle	$\leq k^6$	$\leq k^5$
homothetic copies of polygon	?	?
homothetic copies of disk	?	∞

Making bottomless rectangles colorful

Thm[Asinowski, **C**, Cohen, Collette, Hackl, Hoffmann, **K**, Langerman, Lasoń, **M**, Rote, **U** '13]: For bottomless rectangles we have $1.6k \leq p(k) \leq 3k - 2$.



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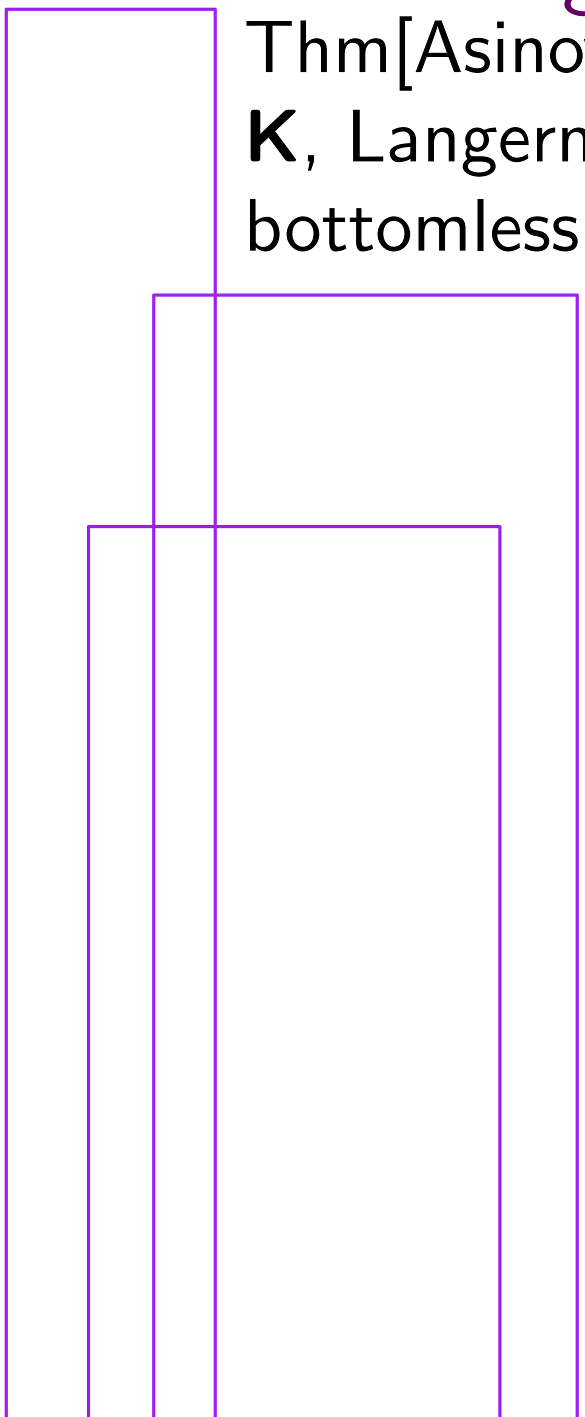

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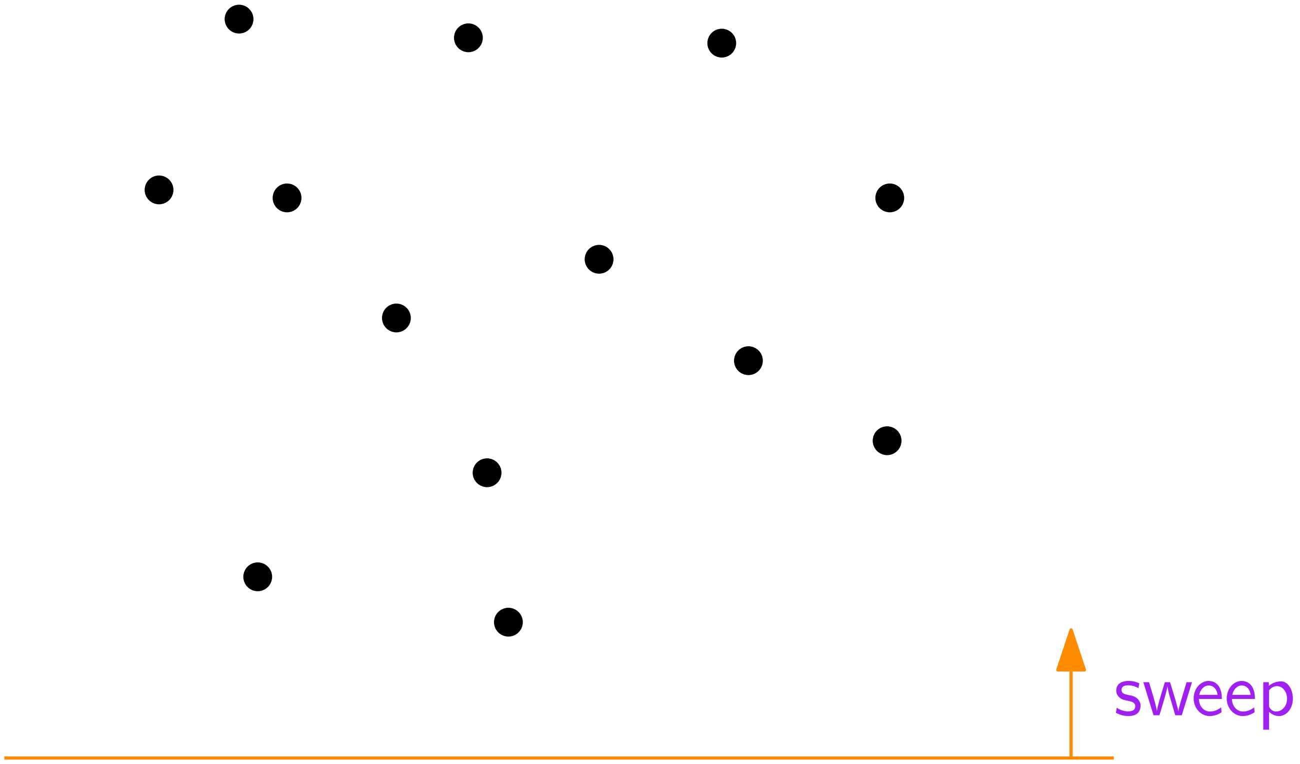

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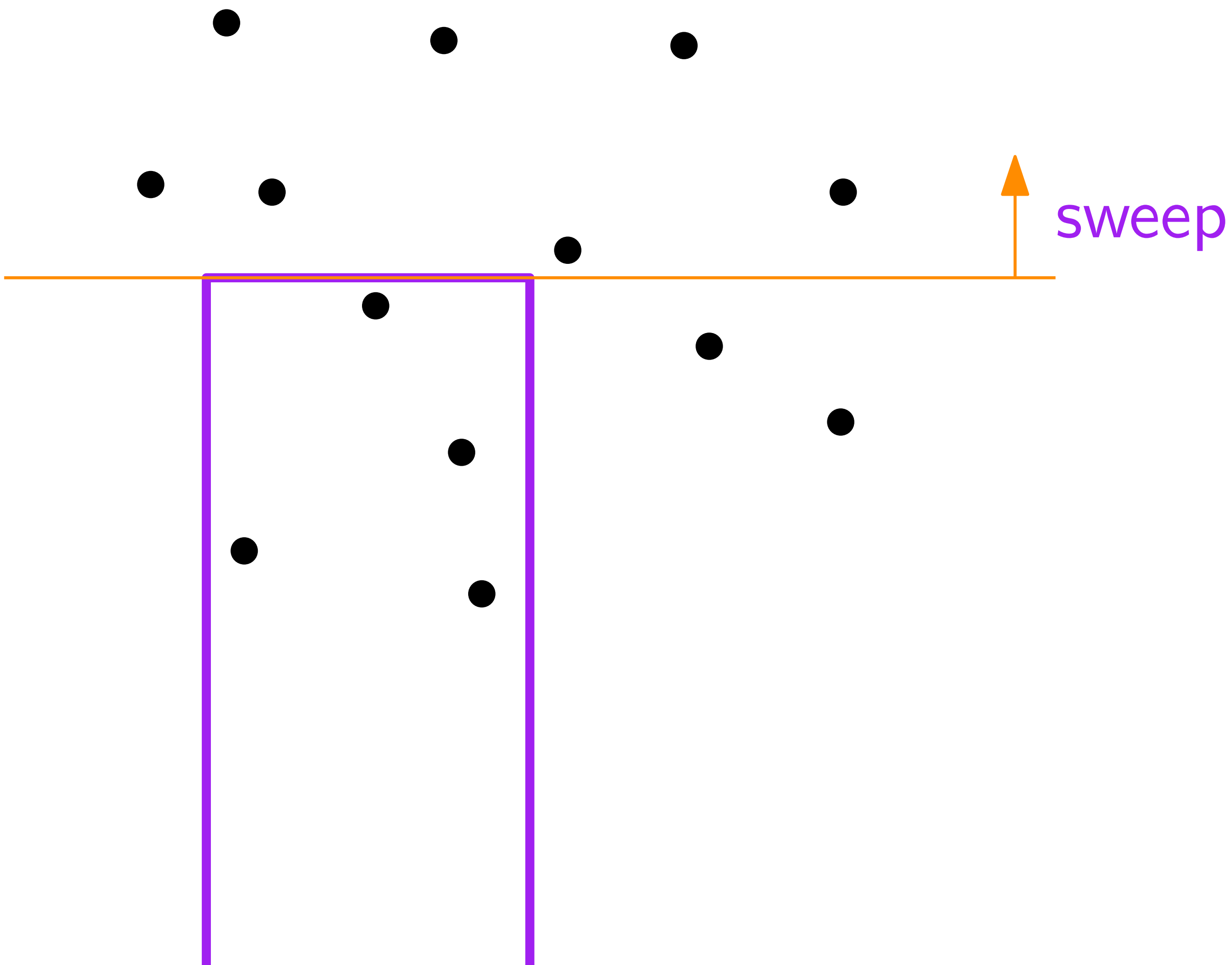

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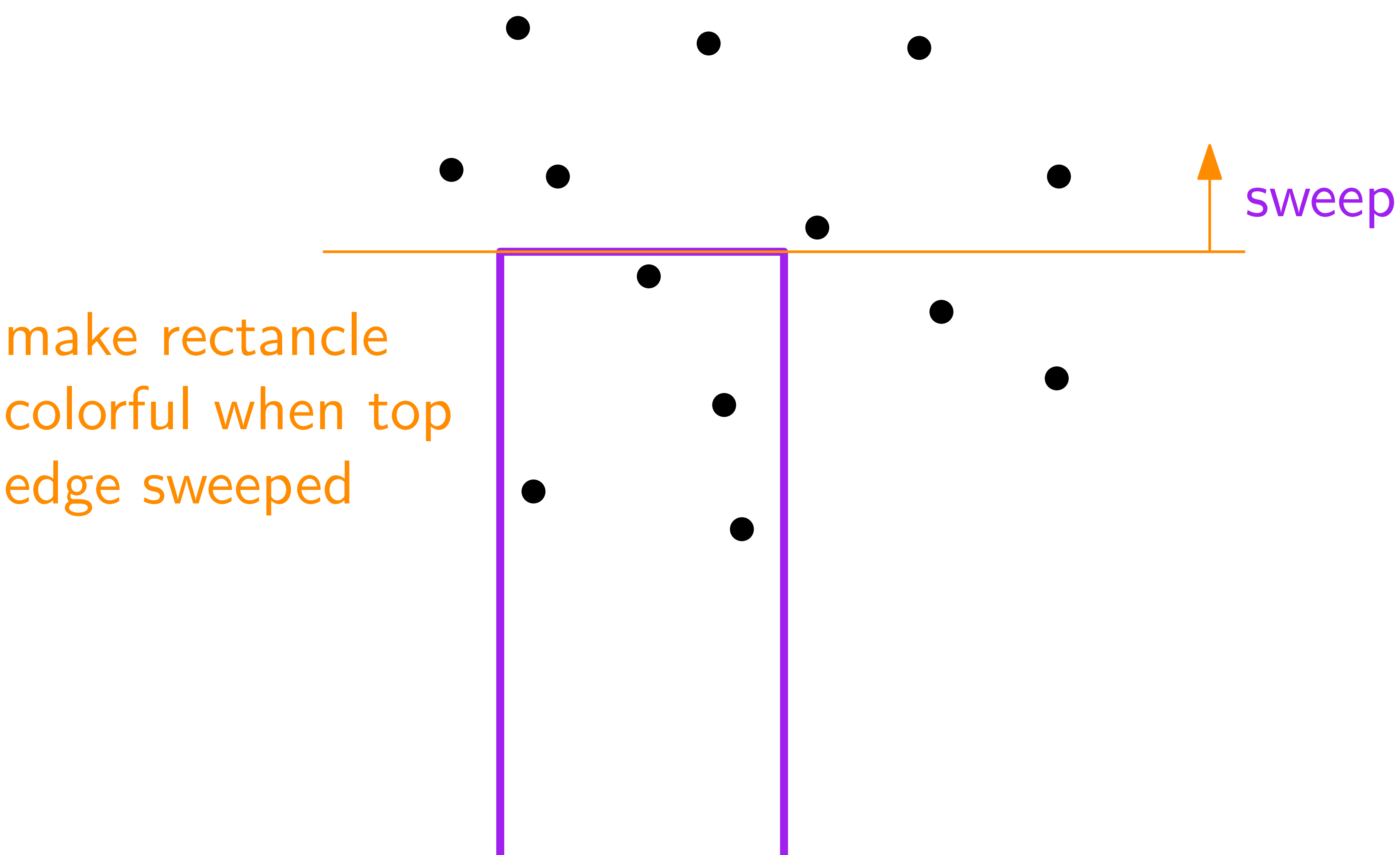

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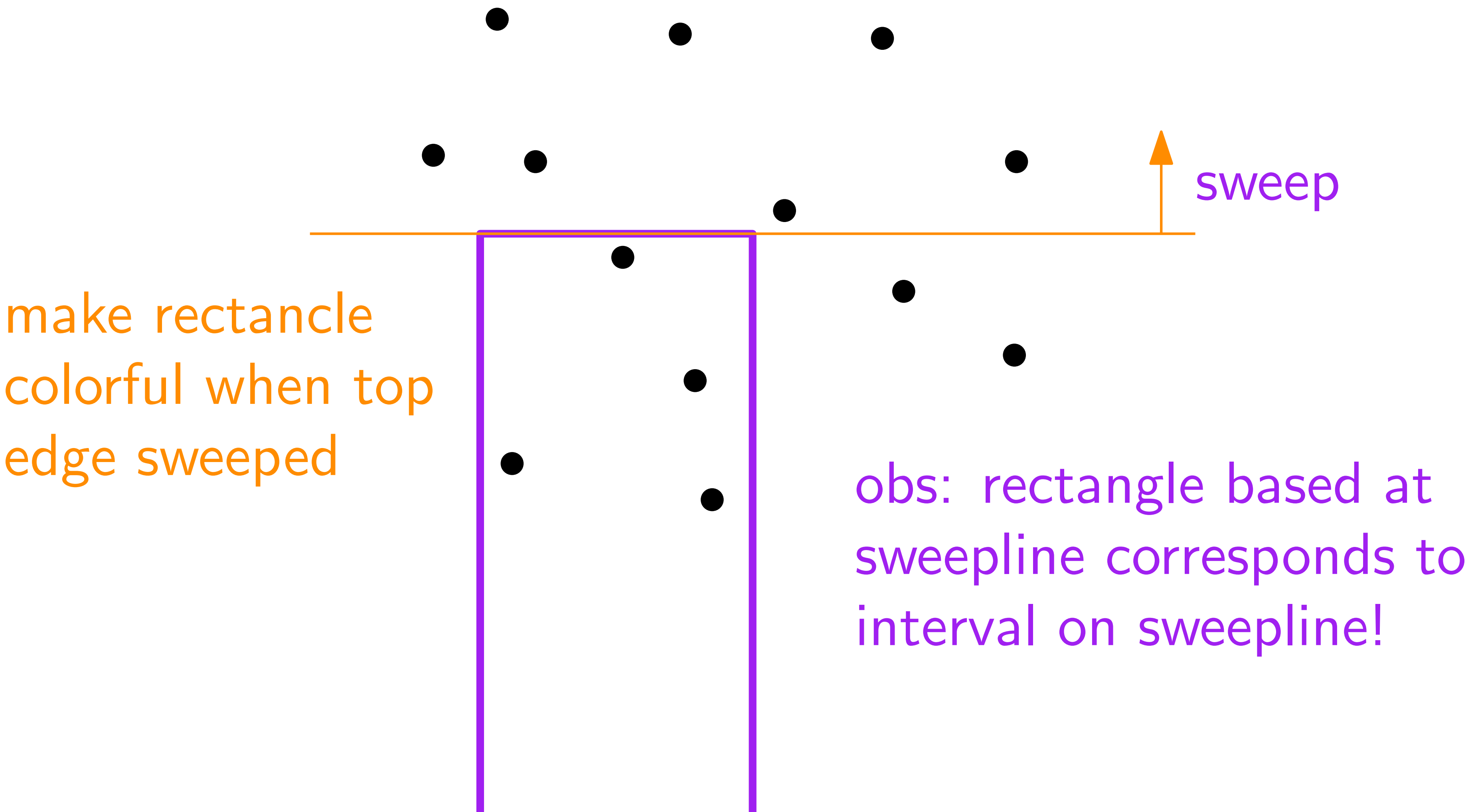

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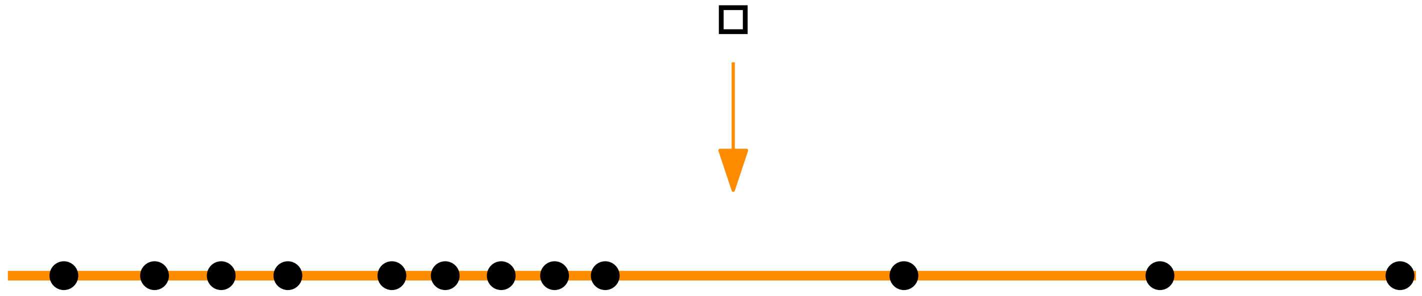
via semi-online k-coloring



invariants: for every color $i \in [k]$

- not more than $3k - 3$ consecutive points without i
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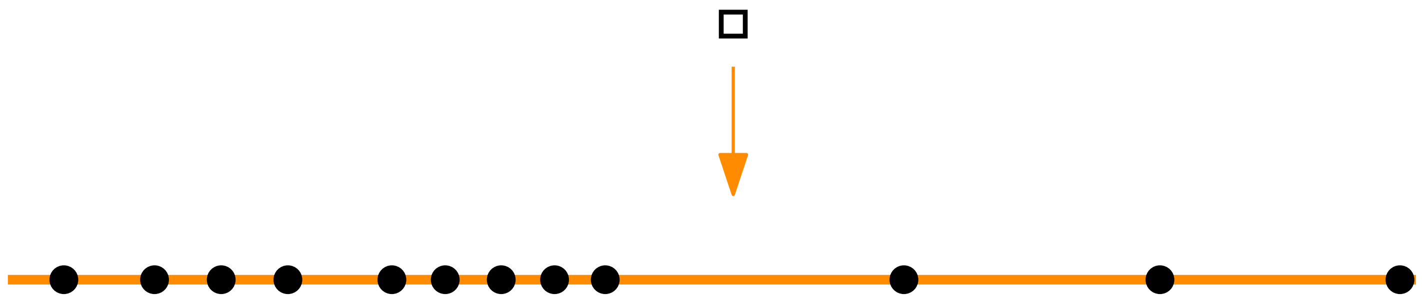

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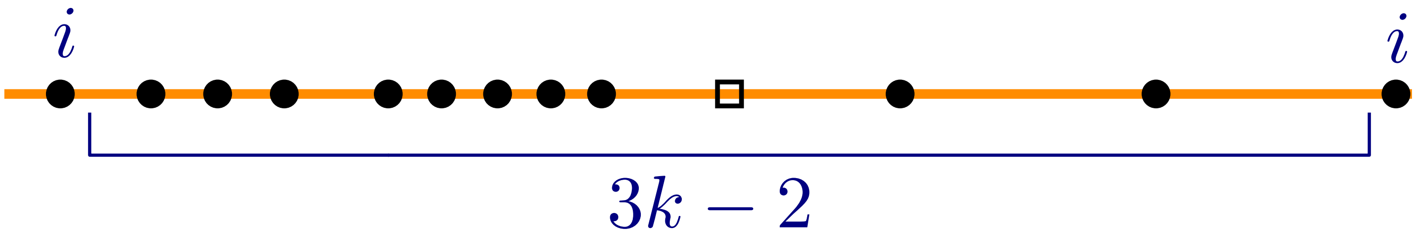

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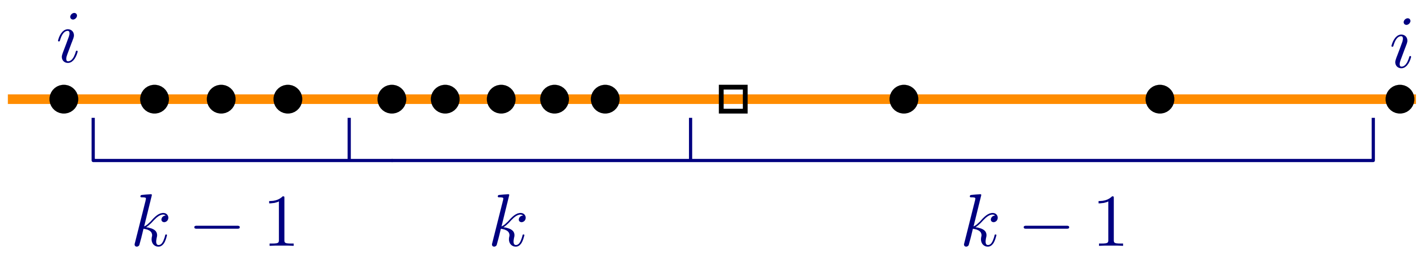

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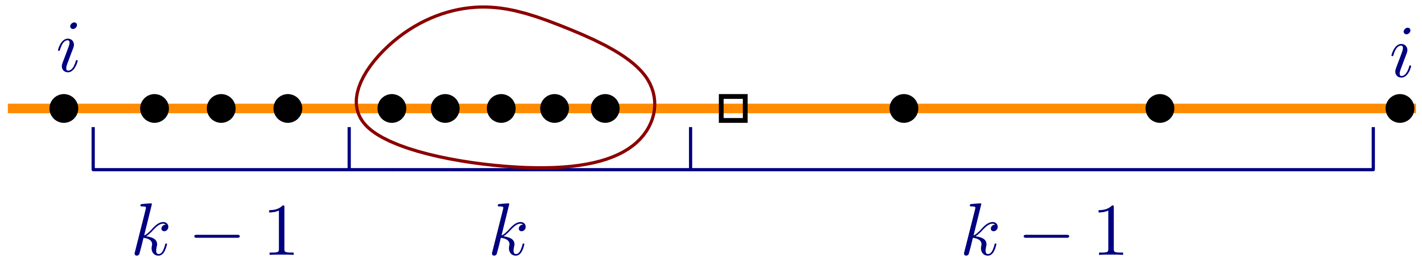

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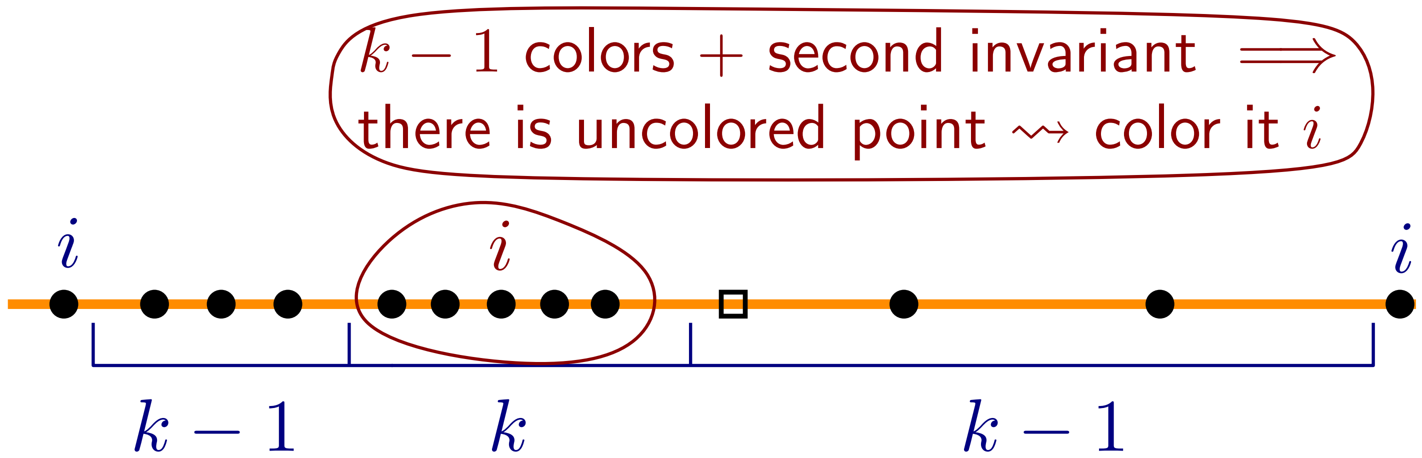

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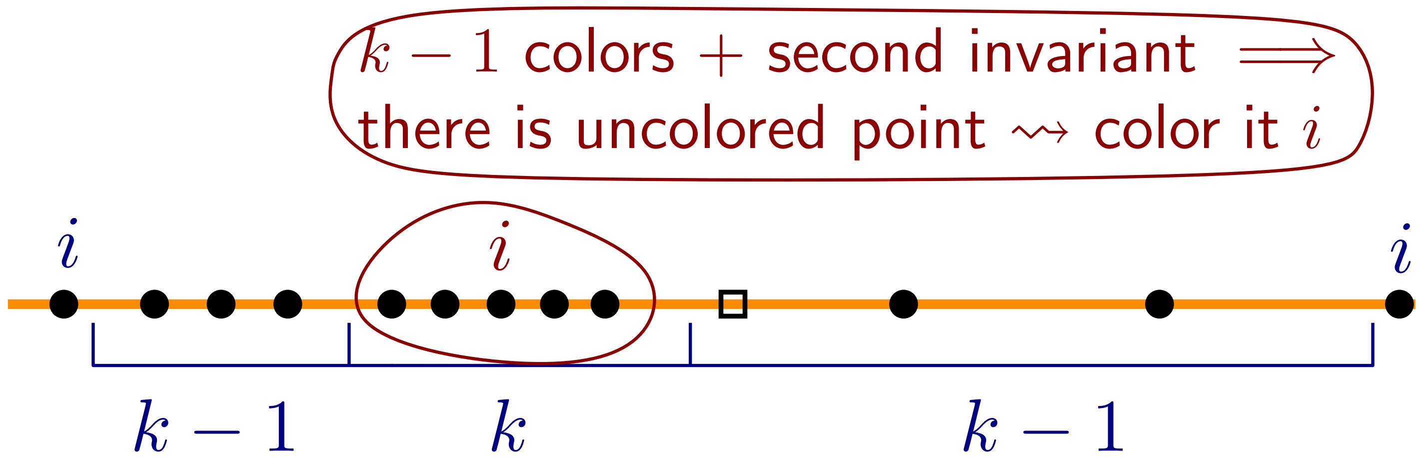

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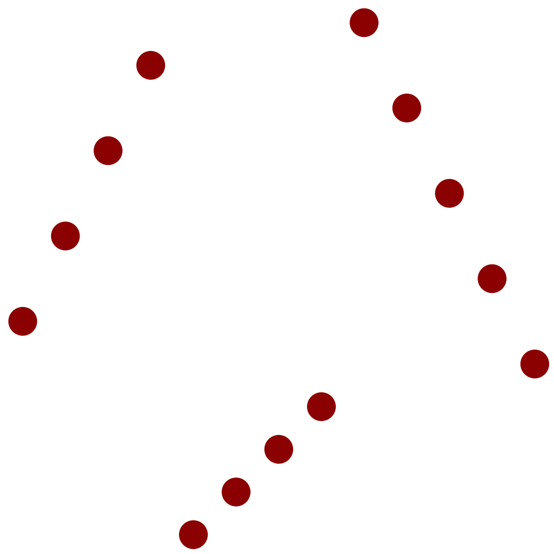

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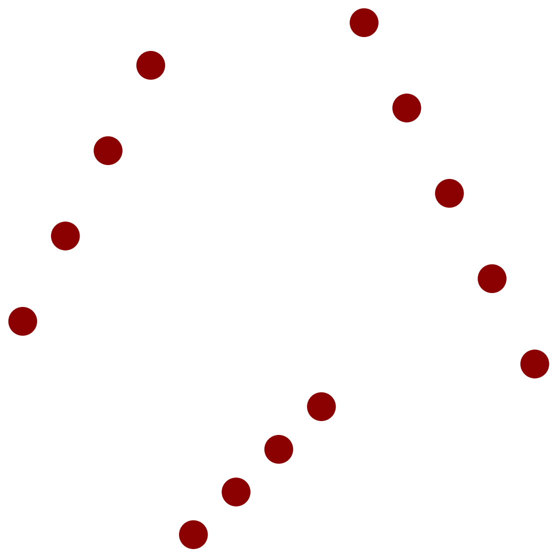
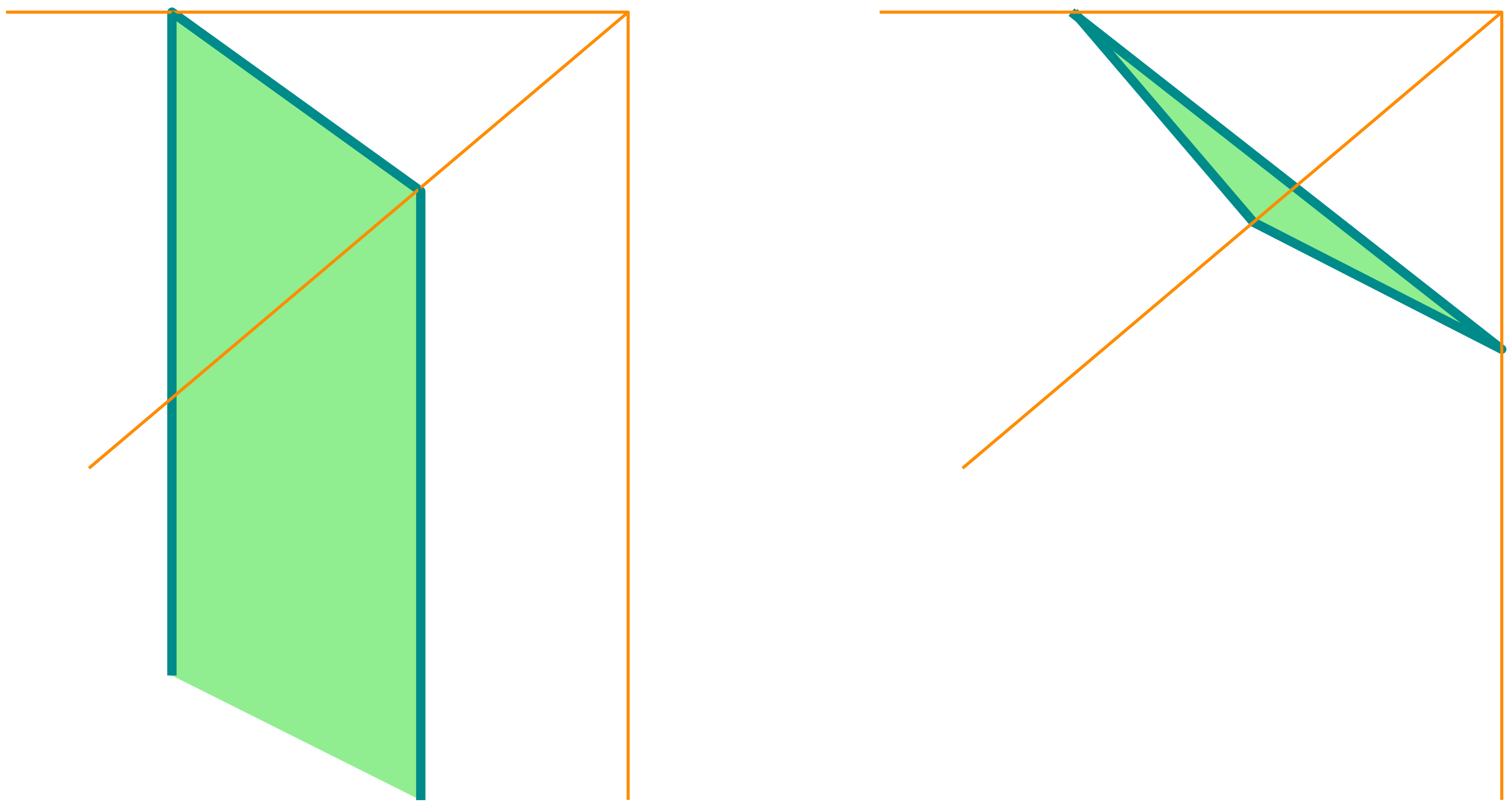


Question: $p(k) \sim 2k$?

Making negative octants colorful makes bottomless rectangles and homothetic triangles colorful



just put all points inside the green plane

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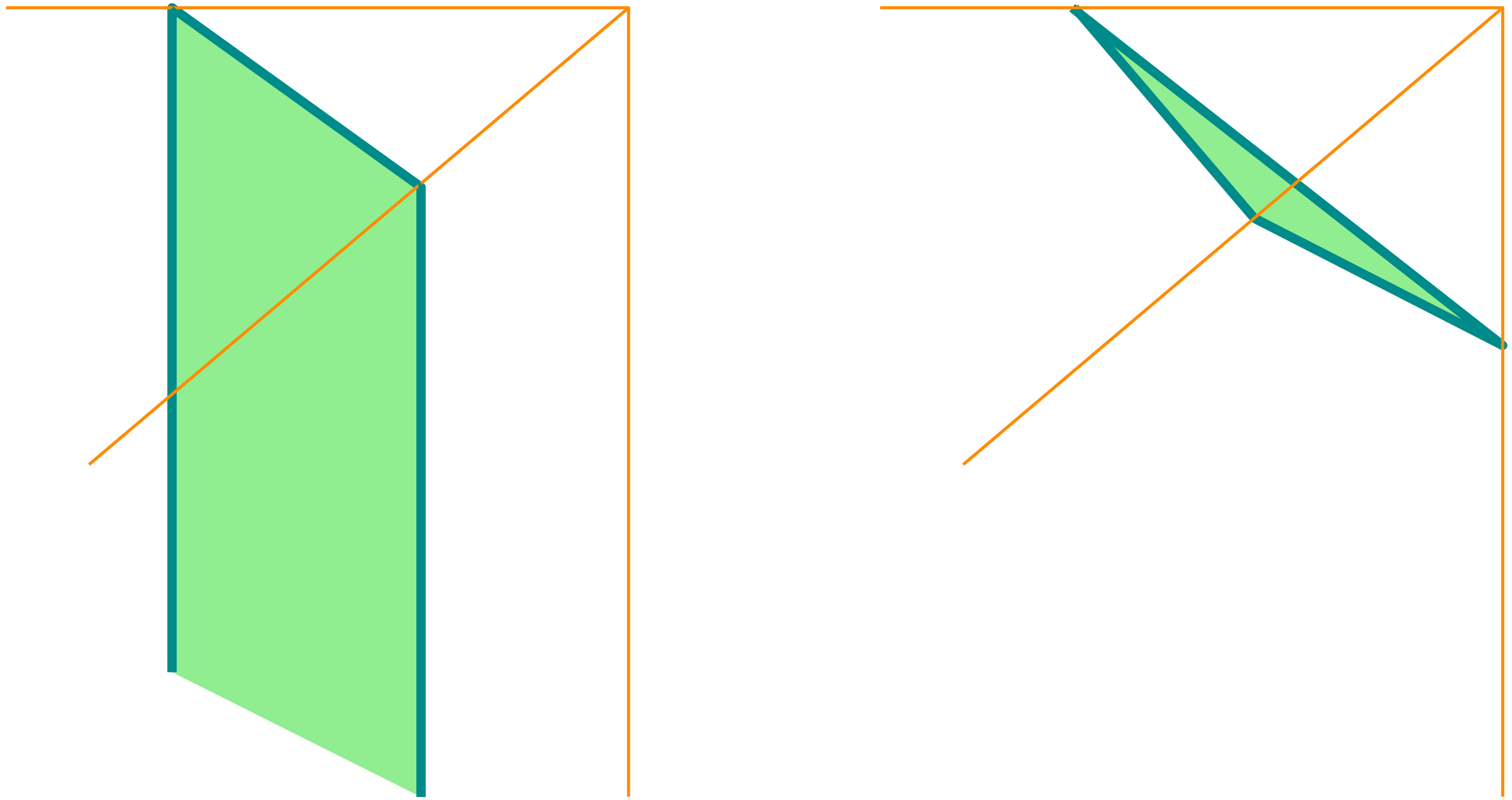

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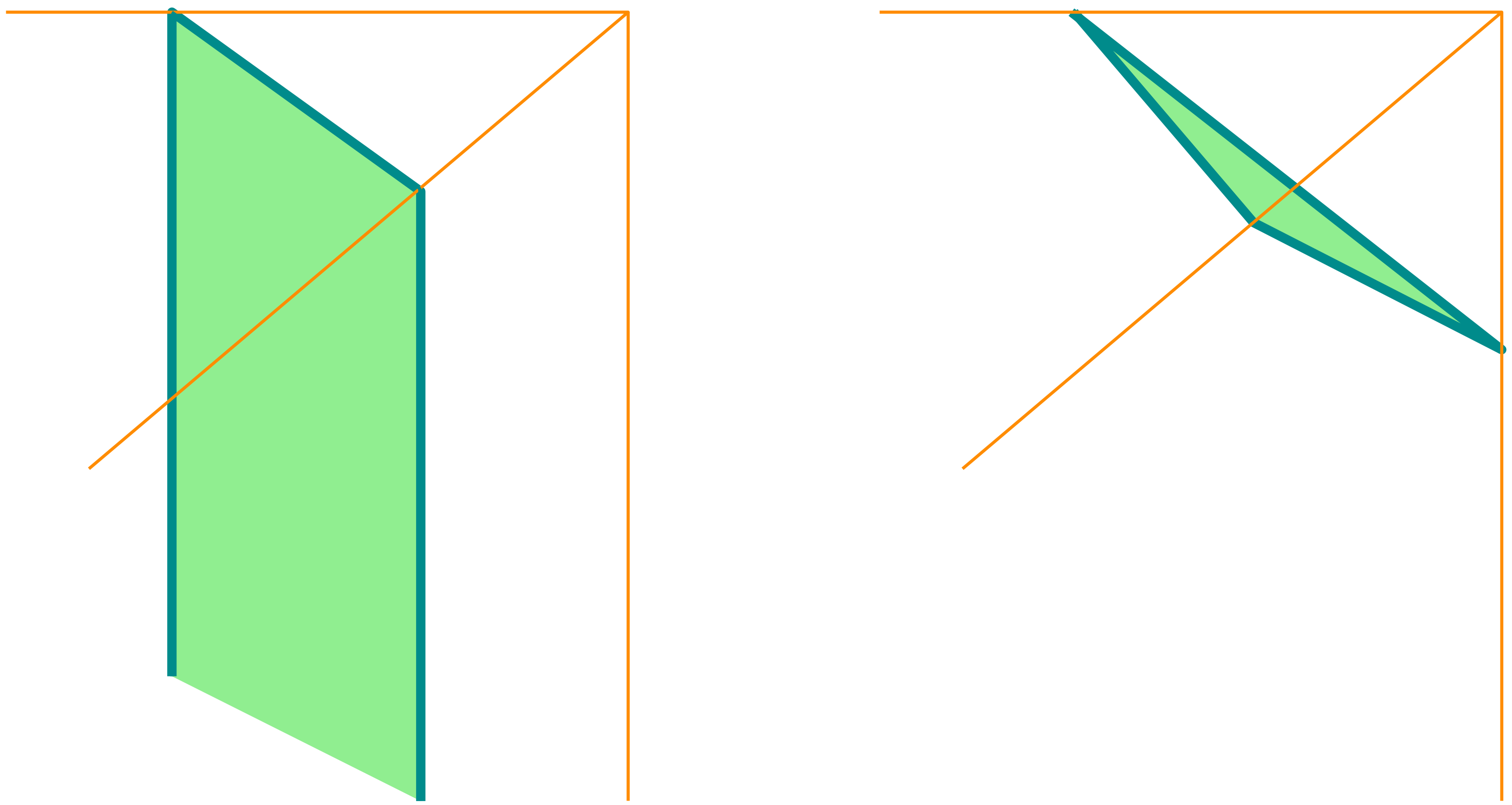
$\rightsquigarrow p(k)$ for octants is upper bound for both

Cover decomposition of octants is cover decomposition of bottomless rectangles and homothetic triangles



set of bottomless rectangles or triangles corresponds to set of octants, point in c of the first $\implies$ point in c of the latter

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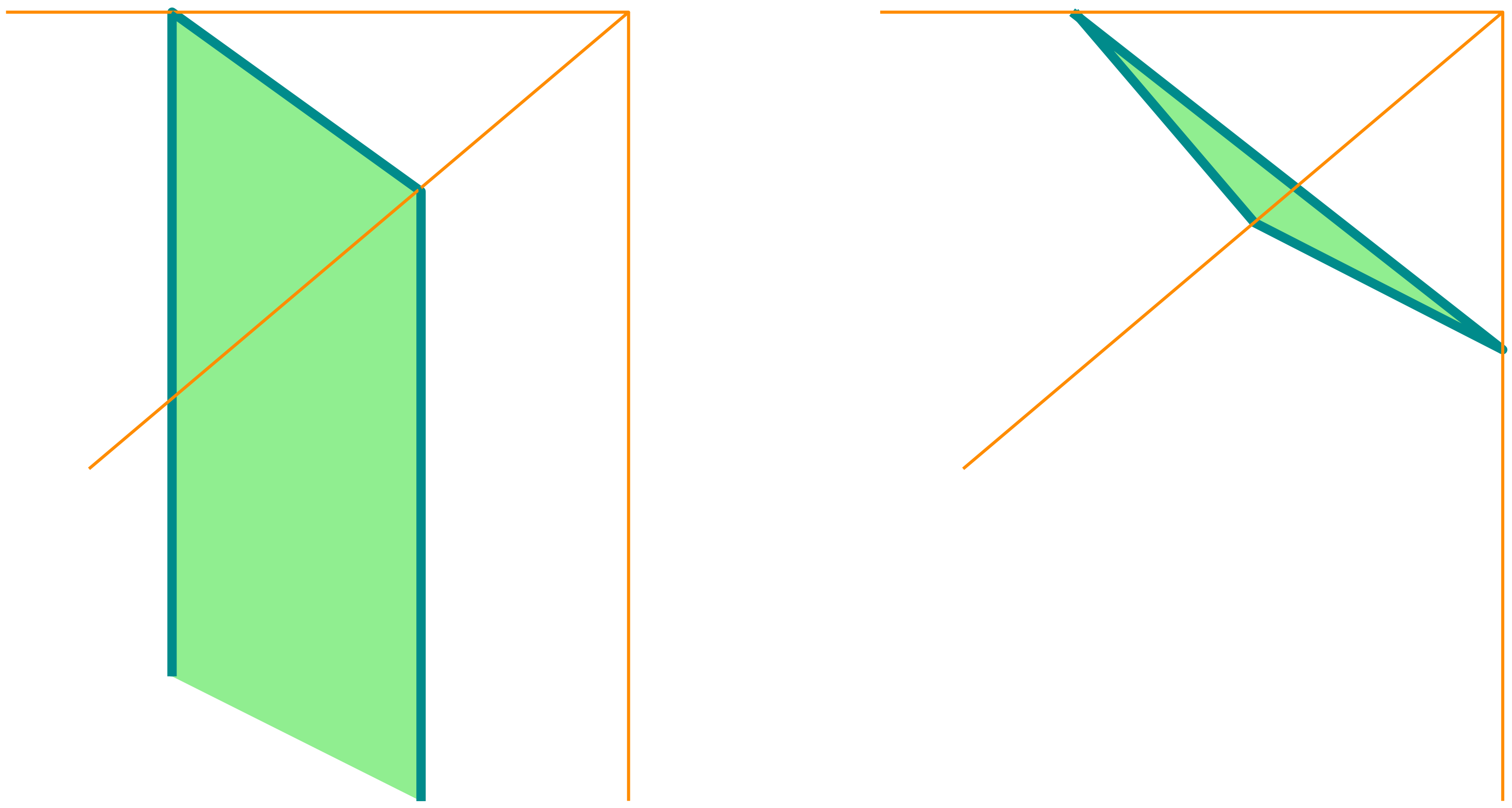

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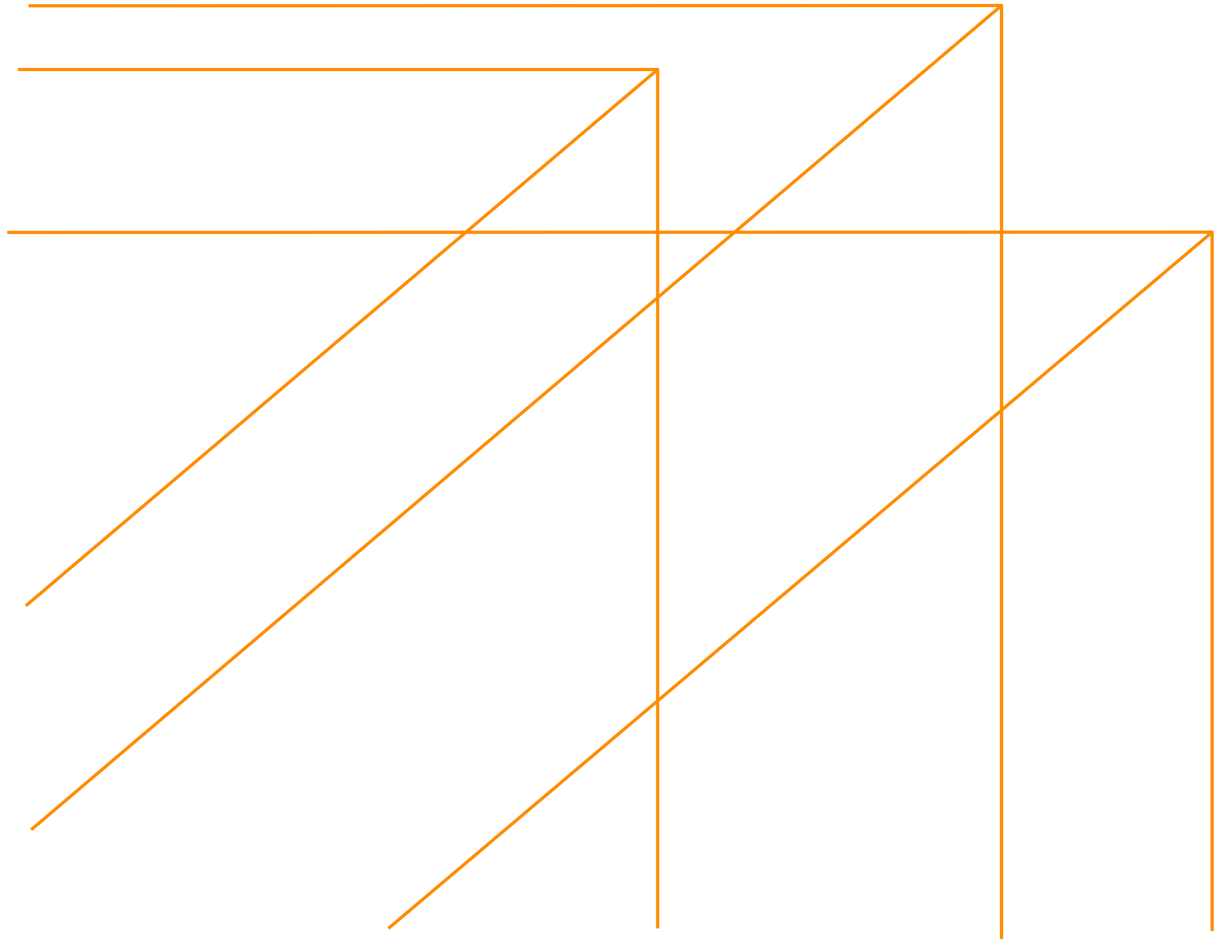
Negative octant covering decomposition is the same as making positive octants colorful



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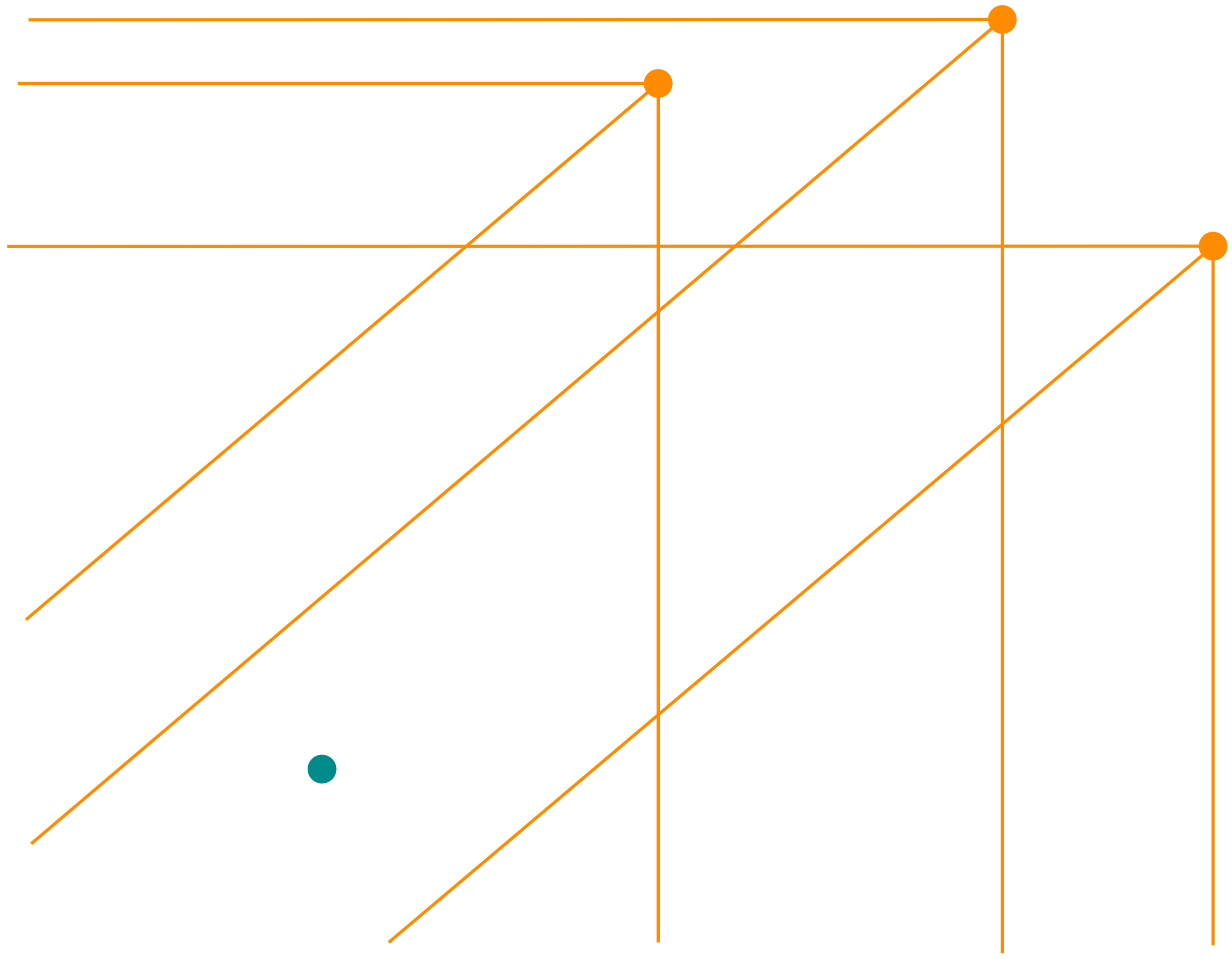

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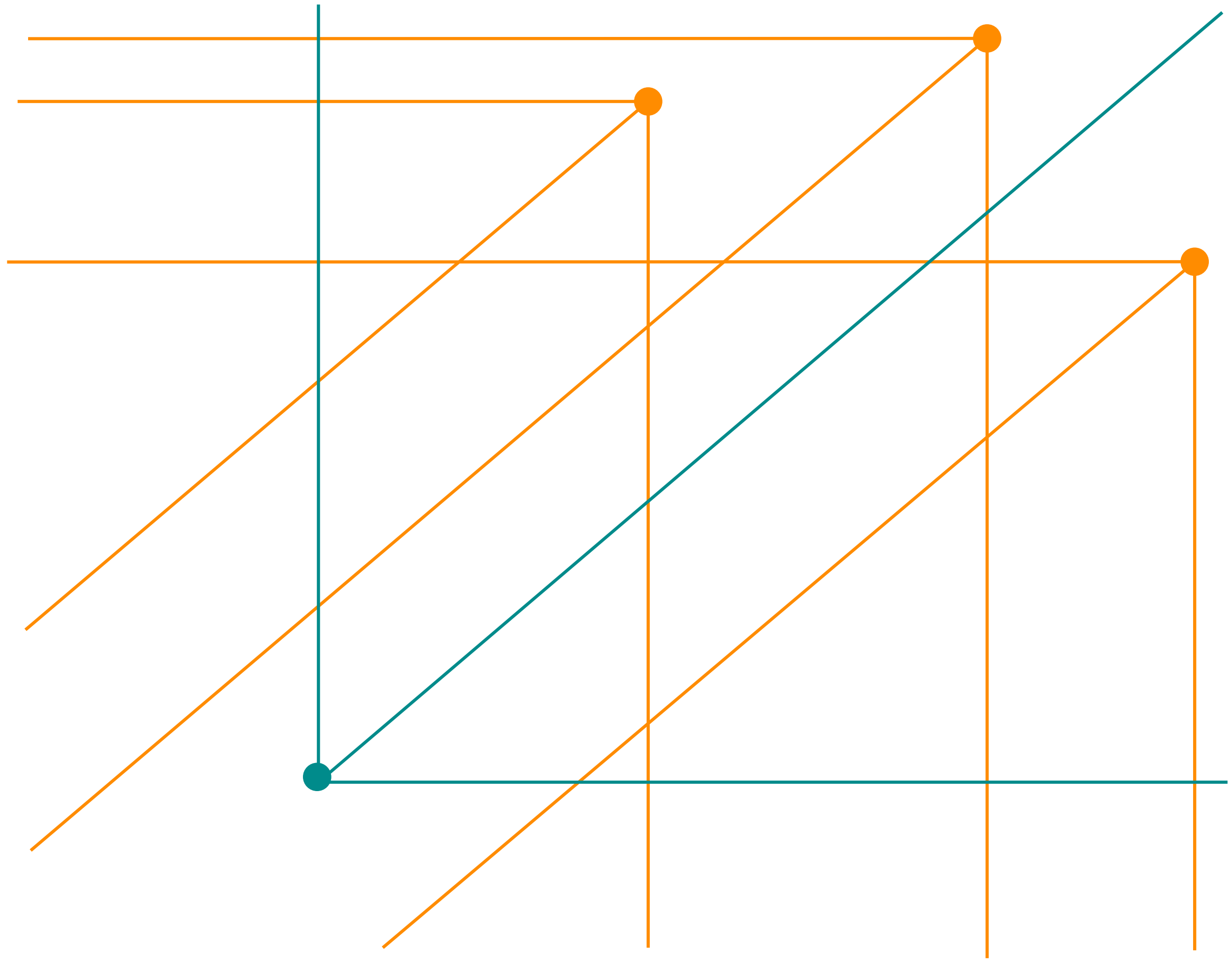

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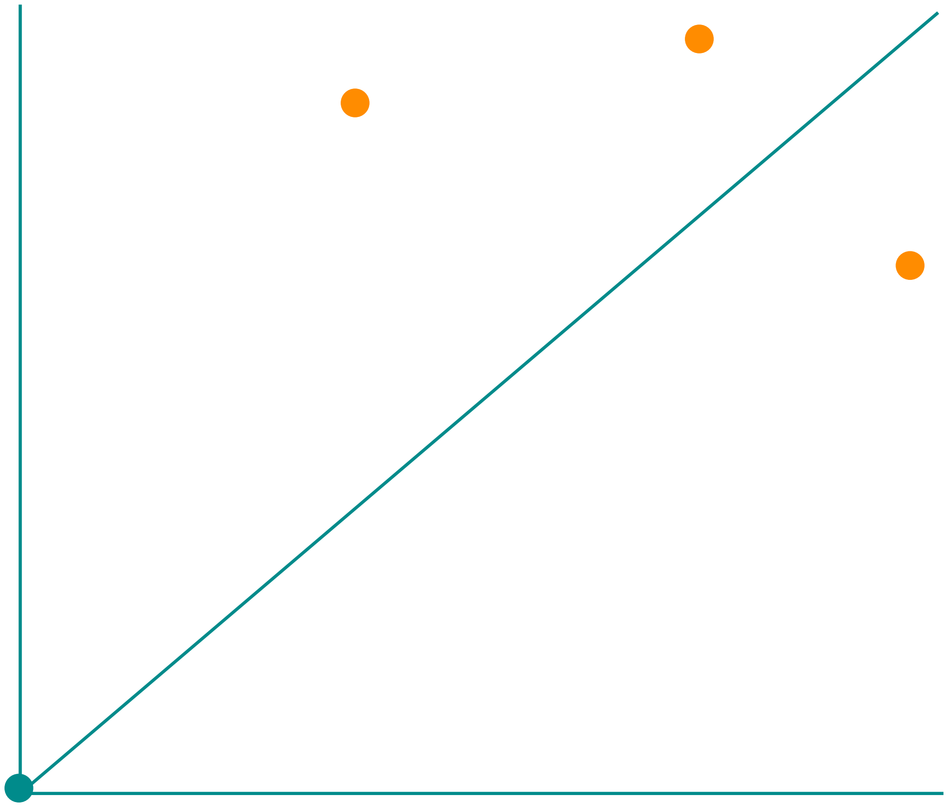

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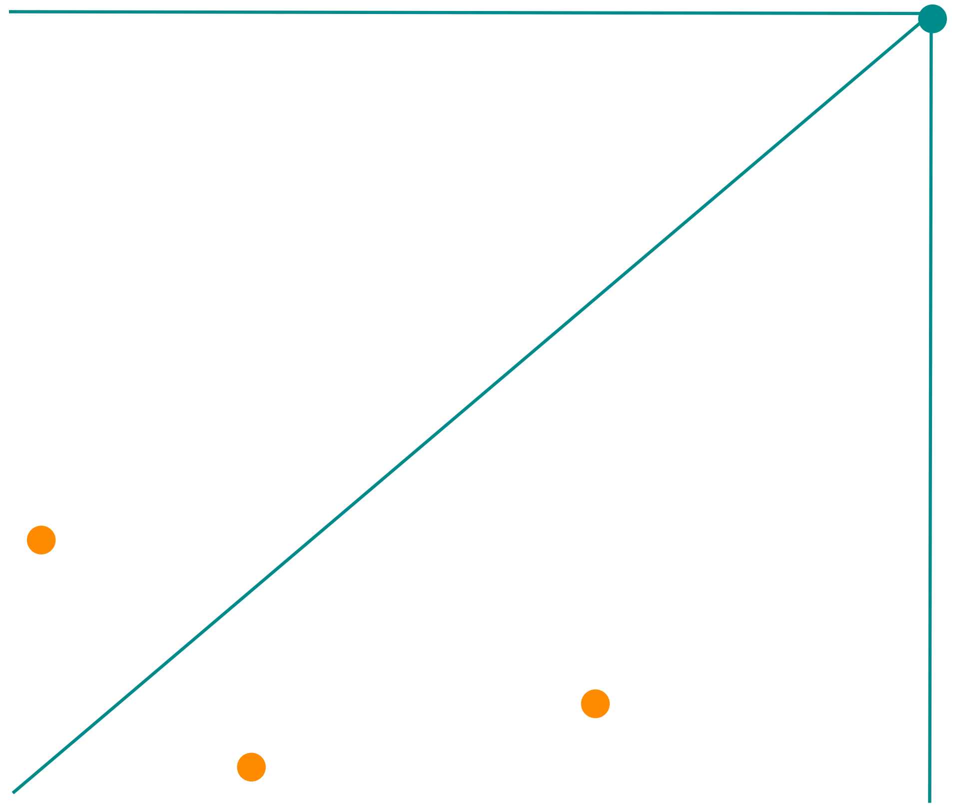

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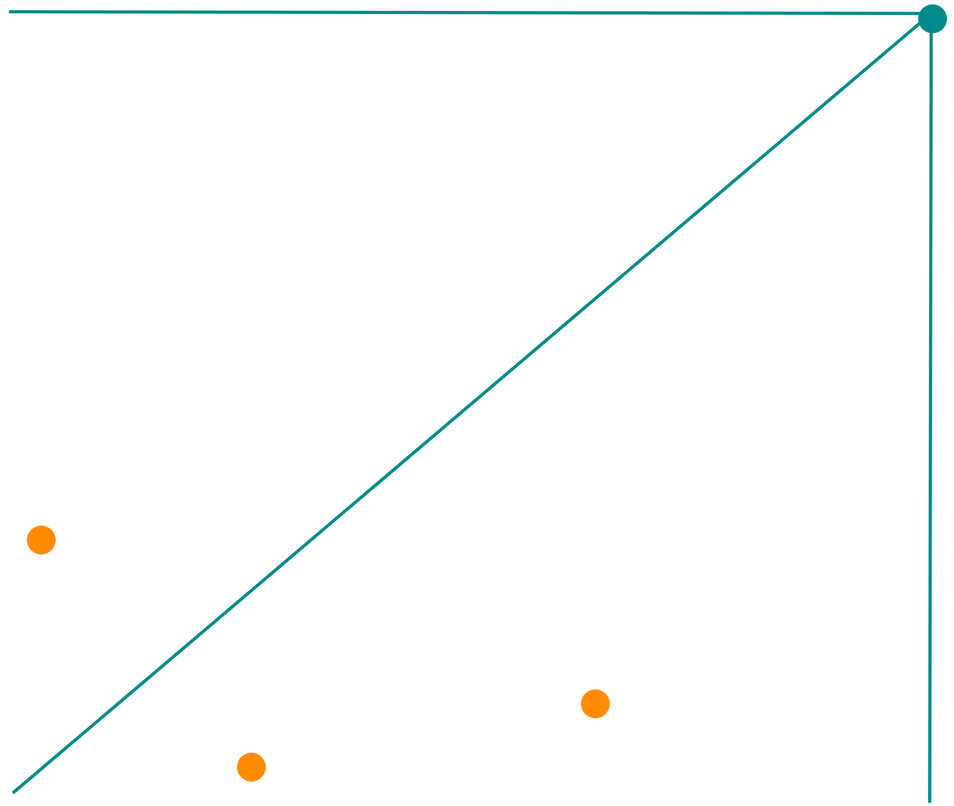

$\rightsquigarrow c(k) = p(k)$ for negative octants

Making Octants Colorful

Thm[Keszegh, Palvögly '12]: $p(2) \leq 12 =: a$ and $p(k) \leq 12^{2^k}$.

Thm[CMKU '13]: $p(k) \leq k^6$.

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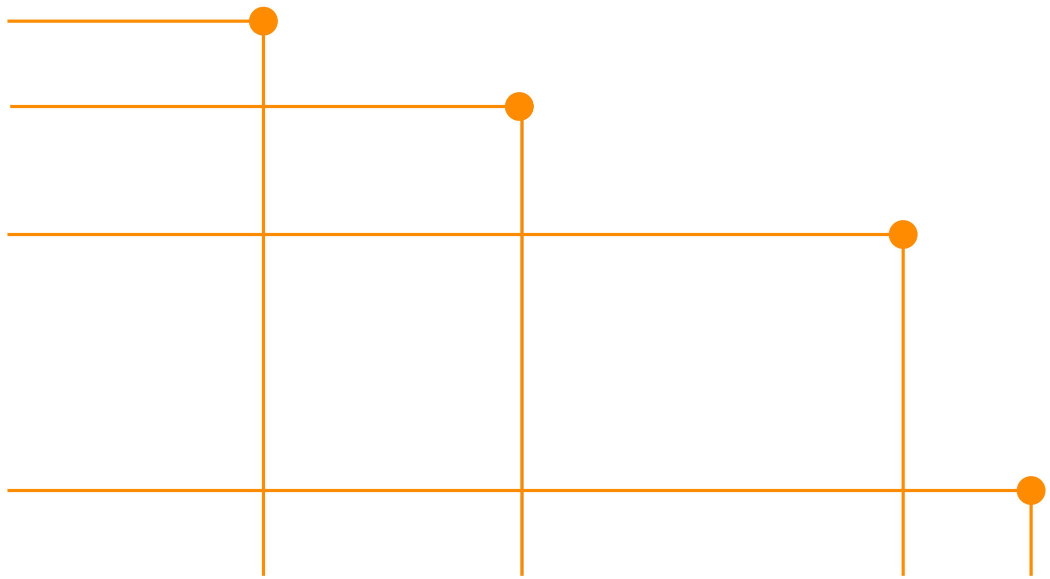
no point in P above other point in P wrt componentwise order



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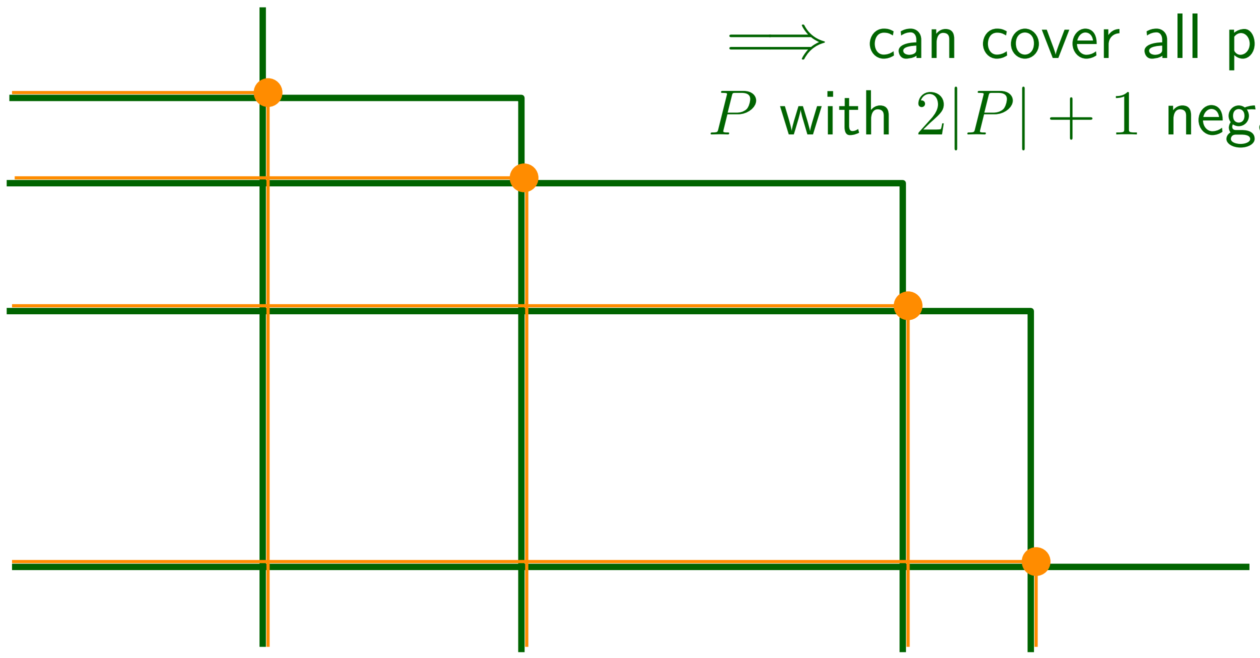
$\implies$ can cover all points not above P with $2|P|+1$ negative octants



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2-color each color-class i of ϕ using Keszegh and Palvögly $\rightsquigarrow i', i''$

Show: octants containing $(2a-1)f(k)$ points are colorful.

if $|Q| \geq (2a-1)f(k)$ but no color i', then Q contains at most $a-1$ points of color i''.

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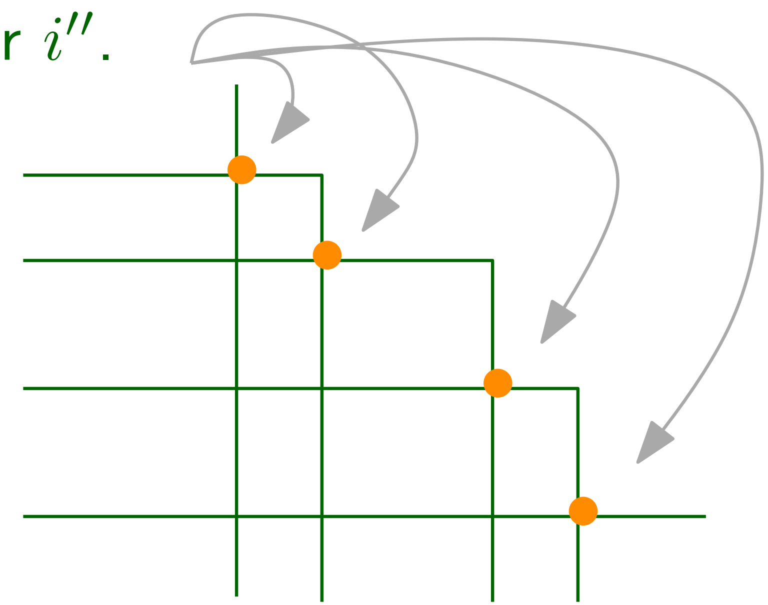
Show: octants containing $(2a-1)f(k)$ points are colorful.

if $|Q| \geq (2a-1)f(k)$ but no color i', then Q contains at most $a-1$ points of color i''.



Q without i'' covered by $2a-1$ octants

Making Octants Colorful

Thm[Keszegh, Palvögly '12]: $p(2) \leq 12 =: a$ and $p(k) \leq 12^{2^k}$.

Thm[CMKU '13]: $p(k) \leq k^6$.

Lemma: $P \subset \mathbb{R}^3$ *independent* $\implies$ k-color such that every octant containing $ak^{\log_2(2a-1)}$ points is colorful.

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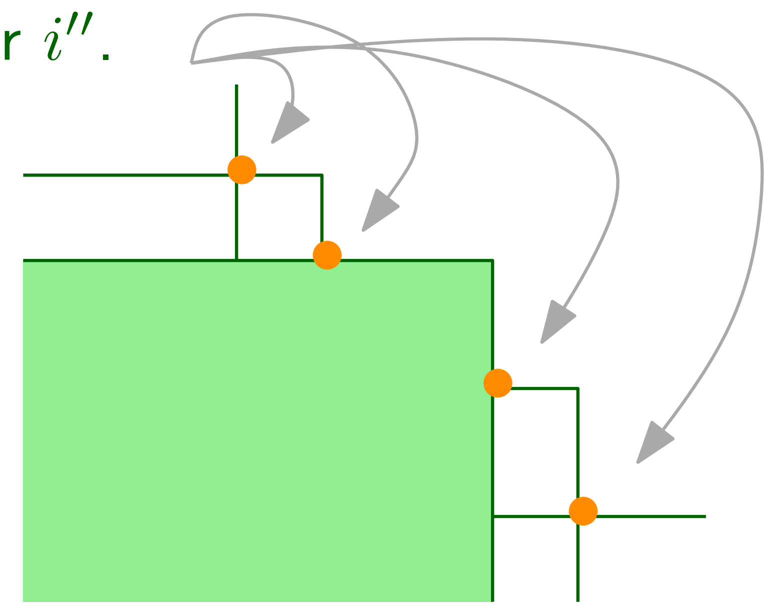

Q without i'' covered by $2a-1$ octants

one contains at least $\lceil \frac{(2a-1)f(k)-(a-1)}{2a-1} \rceil = f(k)$ many points

should have contained color i

Making Octants Colorful

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solve recurrence:

$f(2) = a$

$f(2k) = (2a-1)f(k)$

$\rightsquigarrow f(k) \leq ak^{\log_2(2a-1)} \leq 12k^{4.6}$

Making Octants Colorful

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this gives $p(k) \leq 12k^{4.6}$ for homothetic triangles and bottomless rectangles...

Making Octants Colorful

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Making Octants Colorful

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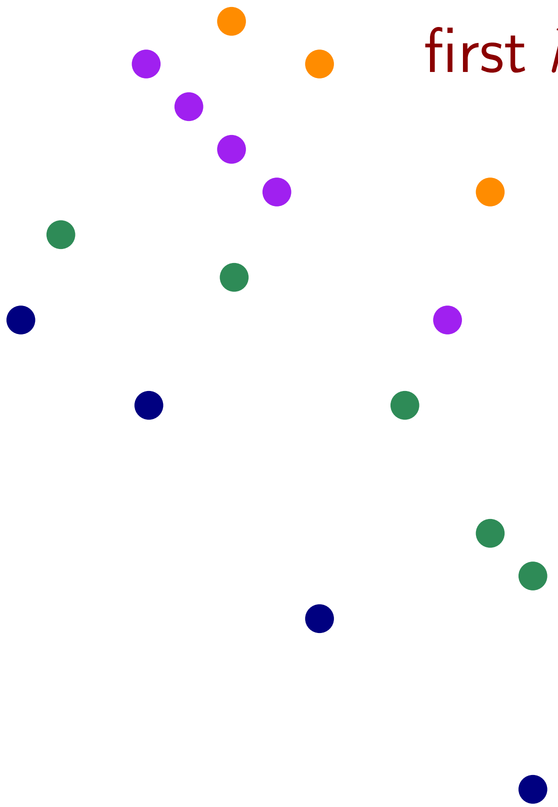

first k independent layers, each precolored with Lemma

Making Octants Colorful

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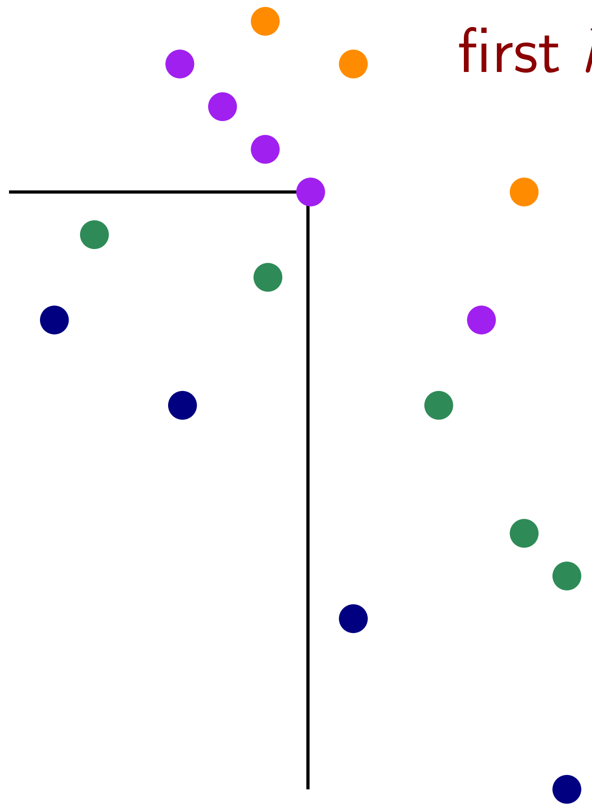

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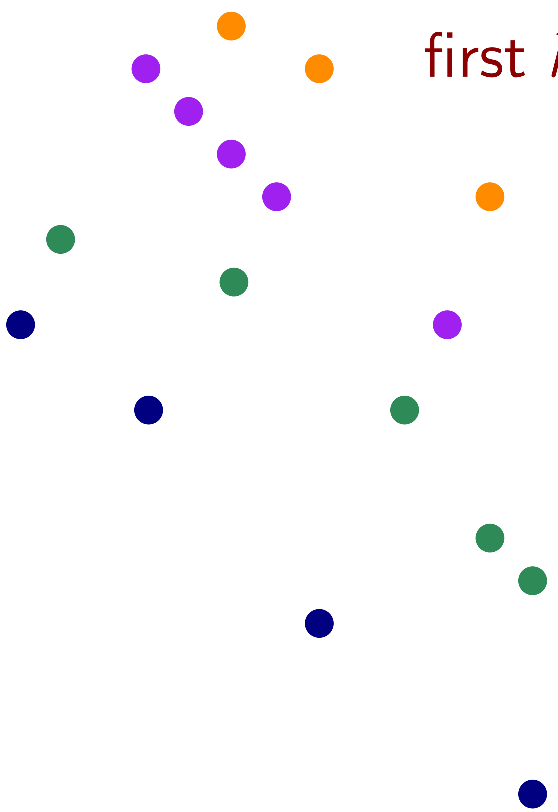

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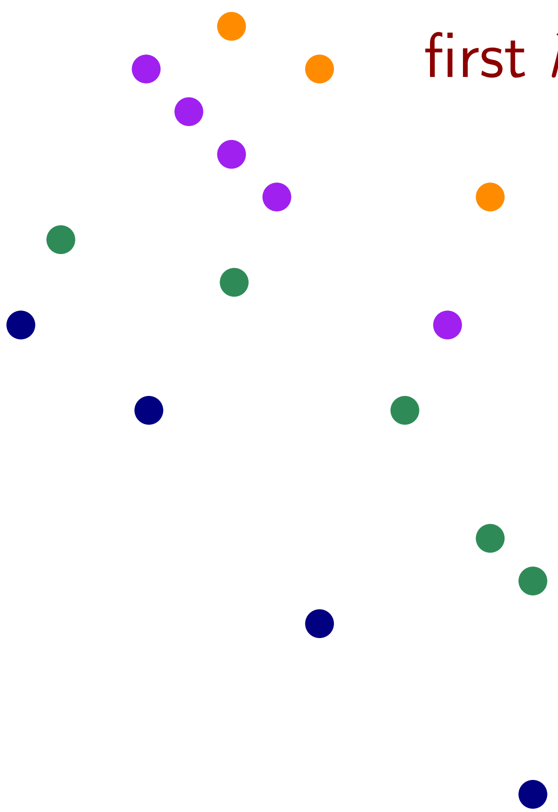

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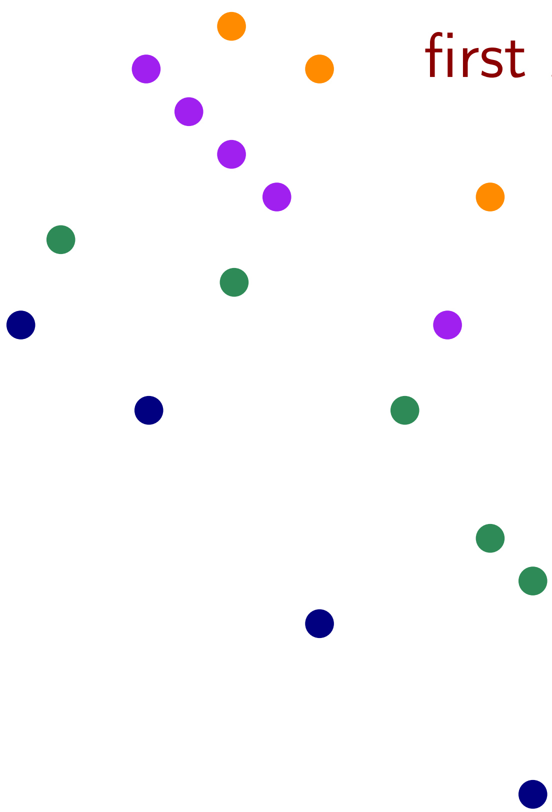

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□

What is done and what is not:

geometric ranges	$\mathbf{c(k)}$	$\mathbf{p(k)}$
all halfplanes in $\mathbb{R}^2$	$k=2 \implies =3$ otw $\le 4k-3$	$2k-1$
all halfplanes in $\mathbb{R}^3$	?	∞
translates of disk	$k=2 \implies \le 33$	otw ?
translates of convex polygon	$O(k)$	
translates of negative octant in $\mathbb{R}^3$	$k=2 \implies \le 12$	otw $\le k^6$
homothetic copies of bottomless rectangles	$k=2 \implies =3$	$1.6k \le \cdot \le 3k-2$
homothetic copies of triangle	$\le k^6$	$\le k^5$
homothetic copies of polygon	?	?
homothetic copies of disk	?	∞

Smorodinsky, Yuditsky '10 Fulek '10 Pach, Tardos, Tóth '05 Keszegh '12

Mani-Levitska, Pach '86 Gibson, Varadarajan '11 Keszegh, Palvögly '12

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Question: What is $c(k)$ or $p(k)$ for squares?