Extending the scope of labelled sequent calculi: the case of classical counterfactuals

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Labelled sequent calculi provide a versatile formalism for the proof-theoretical investigation of large families of non-classical logics, through the internalization of Kripke semantics. In recent work (Negri 2013) we have shown that the method covers frame conditions beyond geometric implication and the whole of Sahlqvist fragment of modal logics. The semantics of important intensional connectives such as counterfactual conditionals, however, is based on a more general, neighbourhood-style, semantics. A conditional implication $A > B$ is said to be true at a world x if either A is never possible, or if there is a neighbourhood of x of worlds similar to x where the antecedent is satisfied and the (classical) implication always holds. The strong assumption of existence of a minimal satisfying neighbourhood that witnesses the *ceteris paribus* similarity condition, criticized both on philosophical and mathematical grounds since the work of Lewis, permits to eliminate the quantifier alternation in the semantic explanation of counterfactuals and to obtain a truth condition structurally similar to that of the standard modal operator. Both labelled tableaux and sequent systems have been formulated on the basis of this assumption (Olivetti et al., 2007, Priest 2008).

It will be shown how complete sequent calculi for classical counterfactuals can be obtained without this simplifying assumption. In particular, the systems obtained enjoy invertibility of the rules, height-preserving admissibility of contraction, and syntactic cut elimination (Negri and Sbardolini 2013).

References

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