Rumour spreading

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LIX

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Plan

- Rumor spreading game
- 2 players
- 3 players
- n players

Table of Contents

Introduction - Content

In this talk :

- Rumour spreading in social networks

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$\Rightarrow$ Game on graphs

Introduction - Content

In this talk :

- Rumour spreading in social networks

$\Rightarrow$ Game on graphs

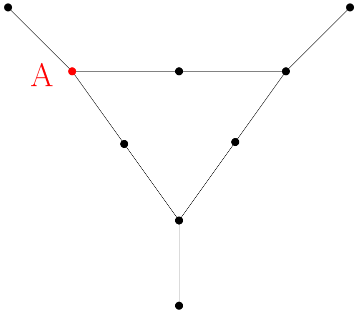
- Different cases: who can win?

Introduction - Setting

Rumor spreading:

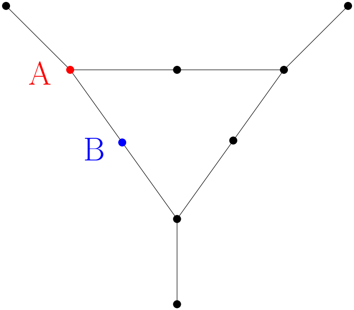
- Distributed algorithm
- Fast propagation of rumor in social network

Introduction - Setting



- Friendship graph
- Each player picks a vertex in the row

Introduction - Setting



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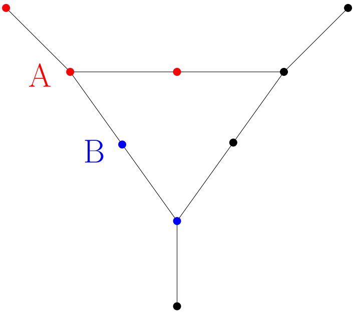
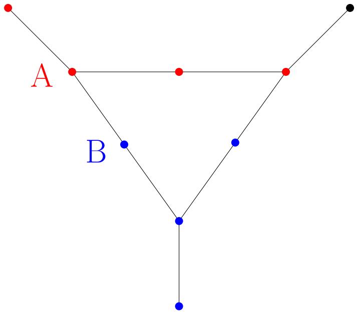


- Rumors are spreading

Introduction - Setting



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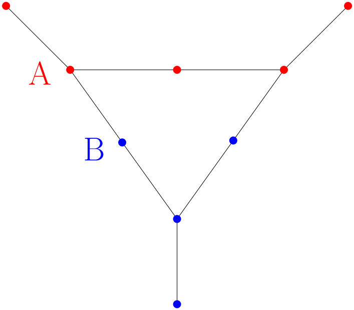
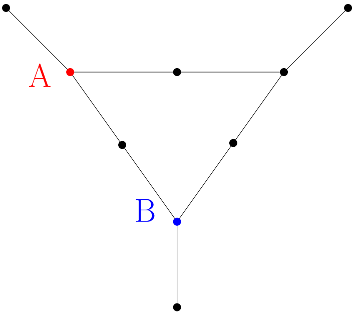


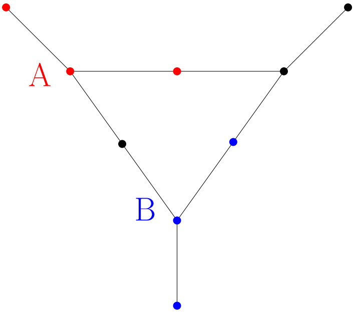
- A convinced 5 vertices
- B convinced 4 vertices

Introduction - Setting



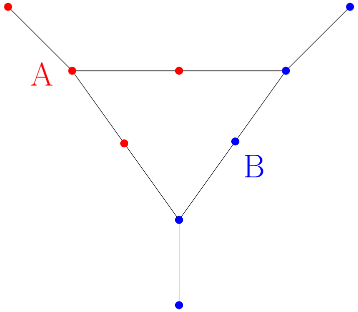
- Other case

Introduction - Setting



- Rumors are spreading
- A and B convinced 3 vertices

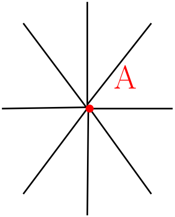
Introduction - Setting



- Last case :
- A convinced 4 vertices
- B convinced 5 vertices

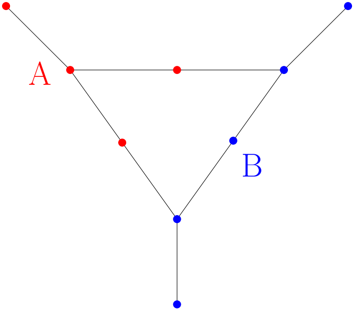
Table of Contents

2 players - First can win



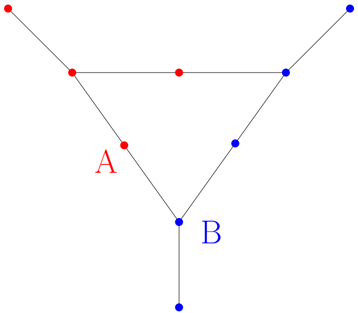
- A wins

2 players - Last can win



- B wins

2 players - Last can win



- B wins

Table of Contents

3 players - Middle cannot win

Assume by contradiction that B has a strategy for graph G

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• If A chooses k and C chooses 1 if not chosen by B then A wins !

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Assume by contradiction that B has a strategy for graph G

• If A chooses 1 then B chooses k and wins for any choice of C

• If A chooses k and C chooses 1 if not chosen by B then A wins !

• Can be extended: a player who is not last nor first cannot win

3 players - Last can win

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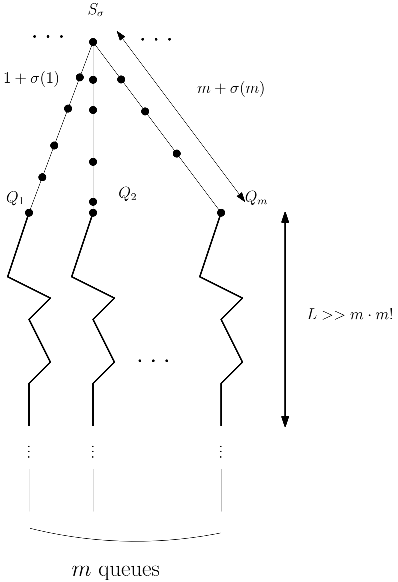
Build graph G:

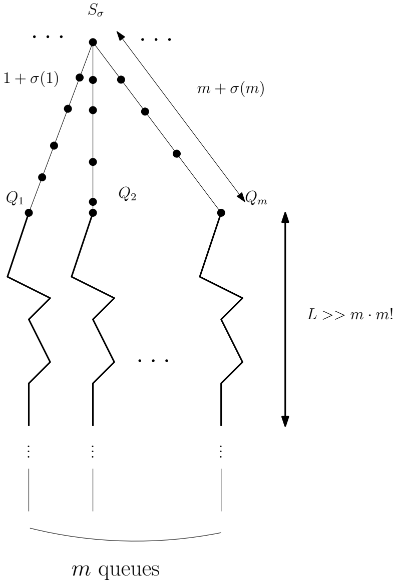
- $m >> 1$ queues of length $L >>> $ anything else
- $m!$ vertices S_σ for σ permutation of $\{1, \ldots, m\}$
- path from S_σ to the head of jth queue ; length $m + \sigma(j)$

3 players - Last can win



- players should choose some S_σ

3 players - Last can win



- After A and B have played, C can stay below them

Table of Contents

n players - Last can win

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- See the game without the last player

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- Use the same graph G as above, with $m >> n$
- See the game without the last player
- Best player conquers $v \geq \frac{m}{n-1}$ queues
- Last player can steal $v - 1$ queues
- It can be stolen evenly: other players keep at most $v\left(1 - \frac{1}{n-1}\right)$ queues

Table of Contents

Conclusion - Summary

- First player can always win, and get a ratio close to 1

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- First player can always win, and get a ratio close to 1
- Last player can always win, and get a ratio close to $\frac{1}{n-1}$
- Other players cannot

Conclusion - Open questions

- Can we reduce the size of graphs involved?

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$\rightarrow$ $\frac{1}{n-1}$ can be improved to $\frac{2}{n}$