Incremental Computation of Succinct Abstractions of Mixed Discrete-Continuous Systems

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Mixed Discrete-Continuous Systems

State space: Cross product of

- finite (e.g., Boolean functions, floating point)
- infinite, but countable (e.g., integers, dynamical data structures)
- uncountable (reals)

component state spaces.

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Formal details: open (see e.g., Bouissou et. al.: HybridFluctuat)

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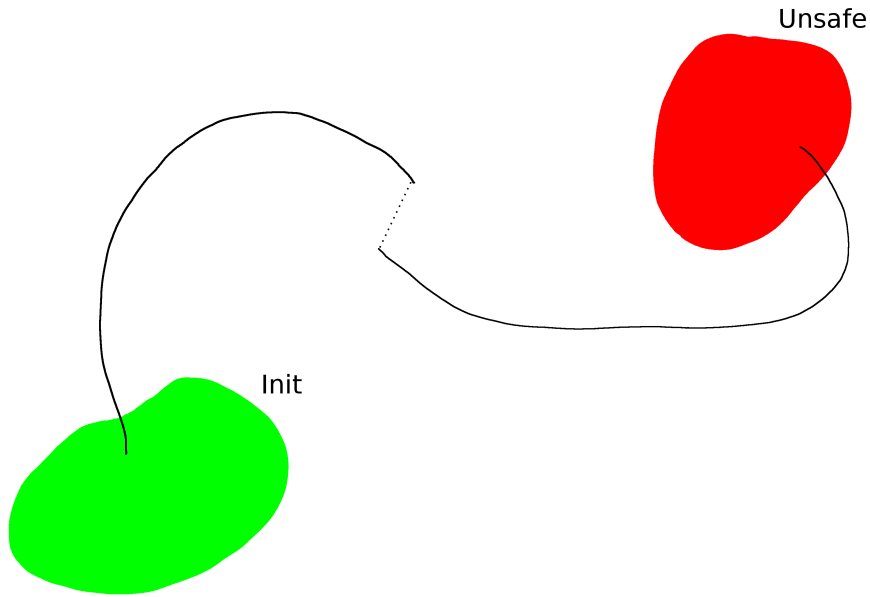

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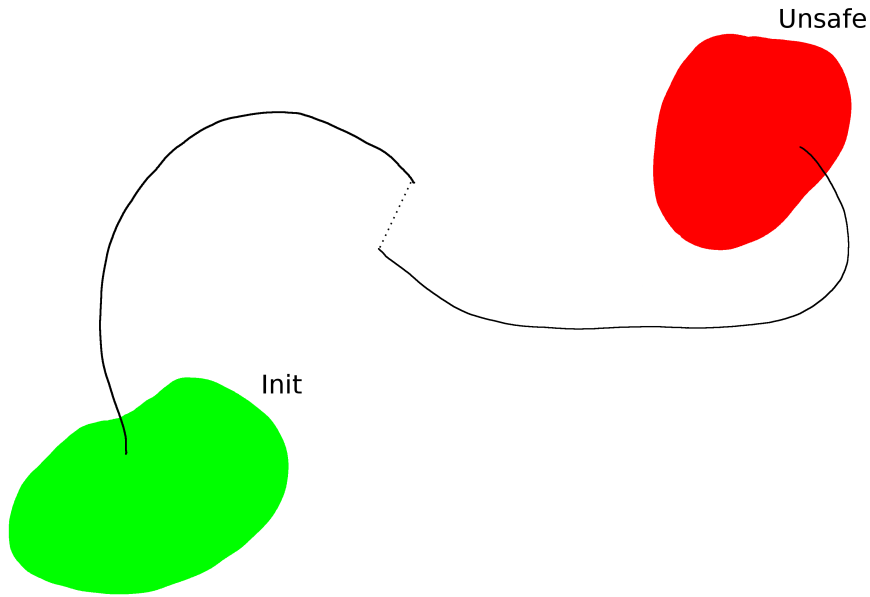

do:

- If system has an error trajectory
  (i.e., trajectory from initial to unsafe states), find it,
- otherwise (the system is safe), detect this.

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$\Rightarrow$:

incrementally improve abstraction
(abstraction refinement)

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$\Rightarrow$:

high-dimensional abstractions should be efficiently computable
(even if the refinement loop may not terminate)

Summary of Resulting Design Principles

- Use some form of abstraction refinement
- Compute as much information as possible
  without increasing the abstraction size
- Compute single abstractions in such a way that
  this scales with problem dimension

Abstractions

Based on decomposition of state space
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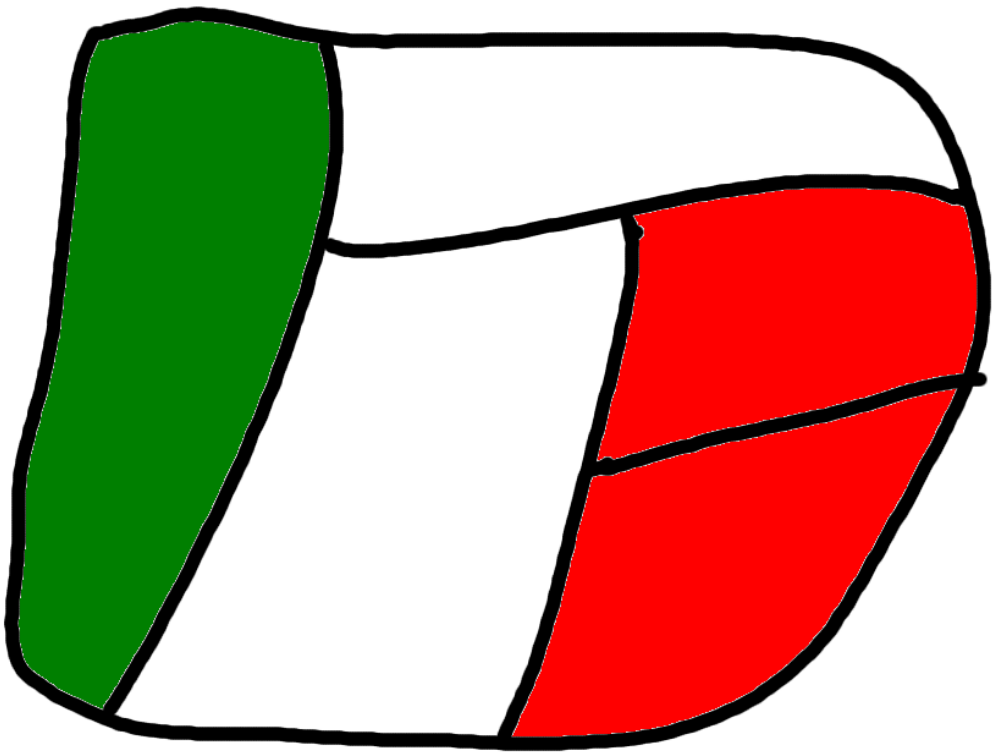

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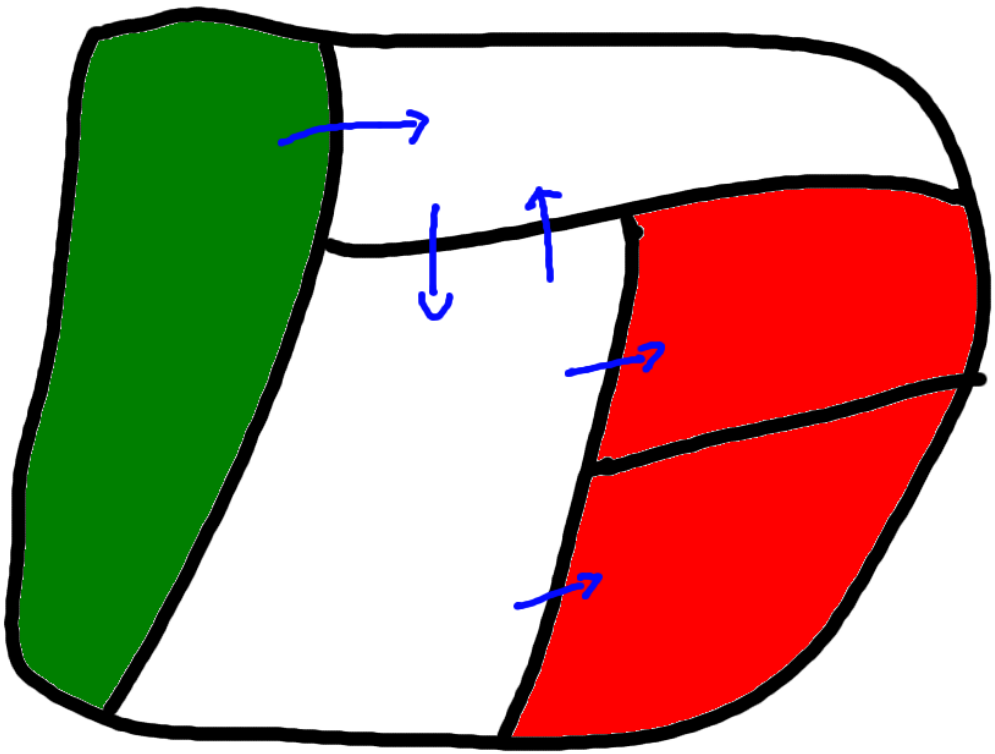

with marks and transitions.

Abstraction Pruning

Reflect more information in abstraction,
without creating more abstract states

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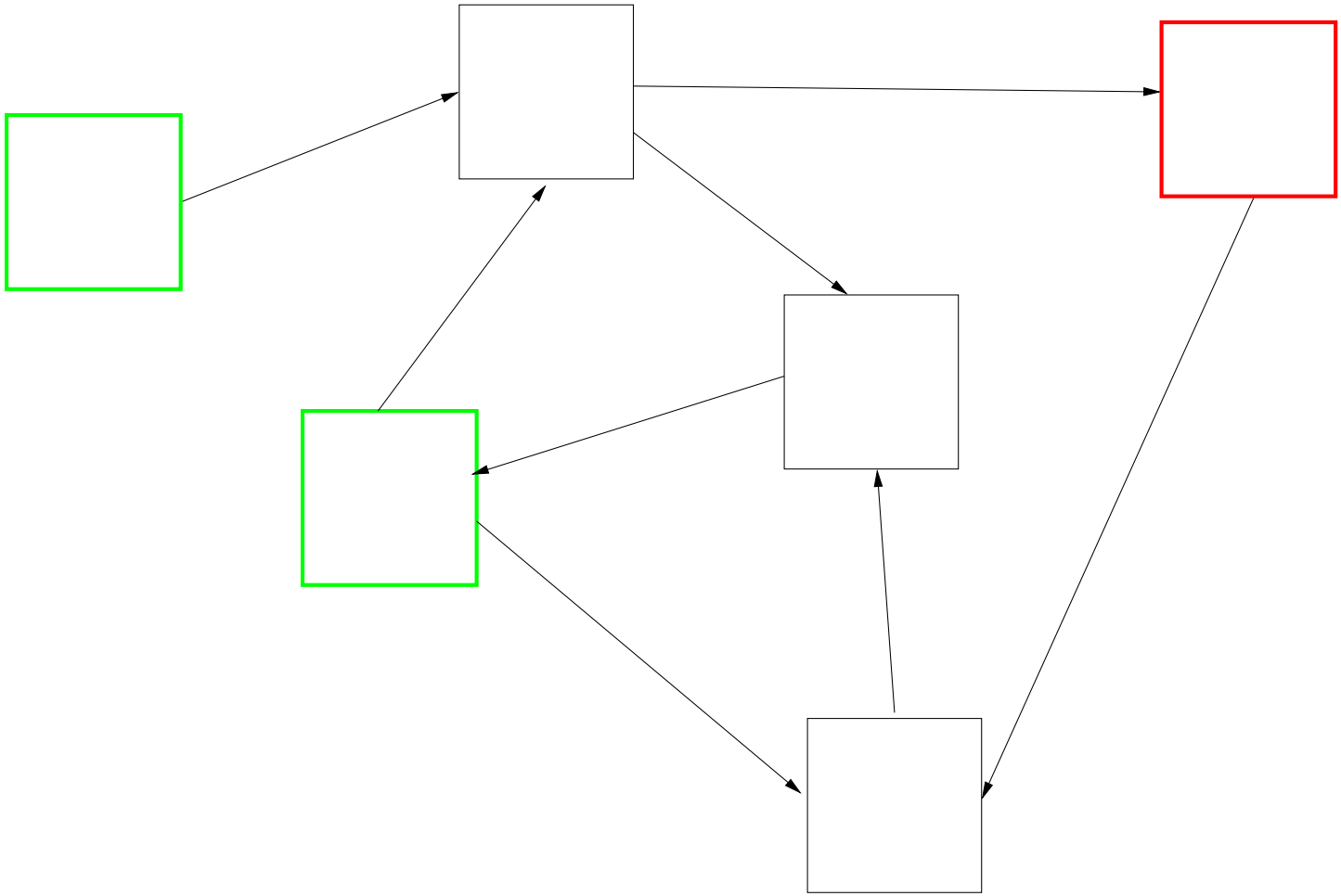
Observation:
  parts of state space not lying on an error trajectory not needed,
    remove such parts from regions

Algorithm for Abstraction Pruning



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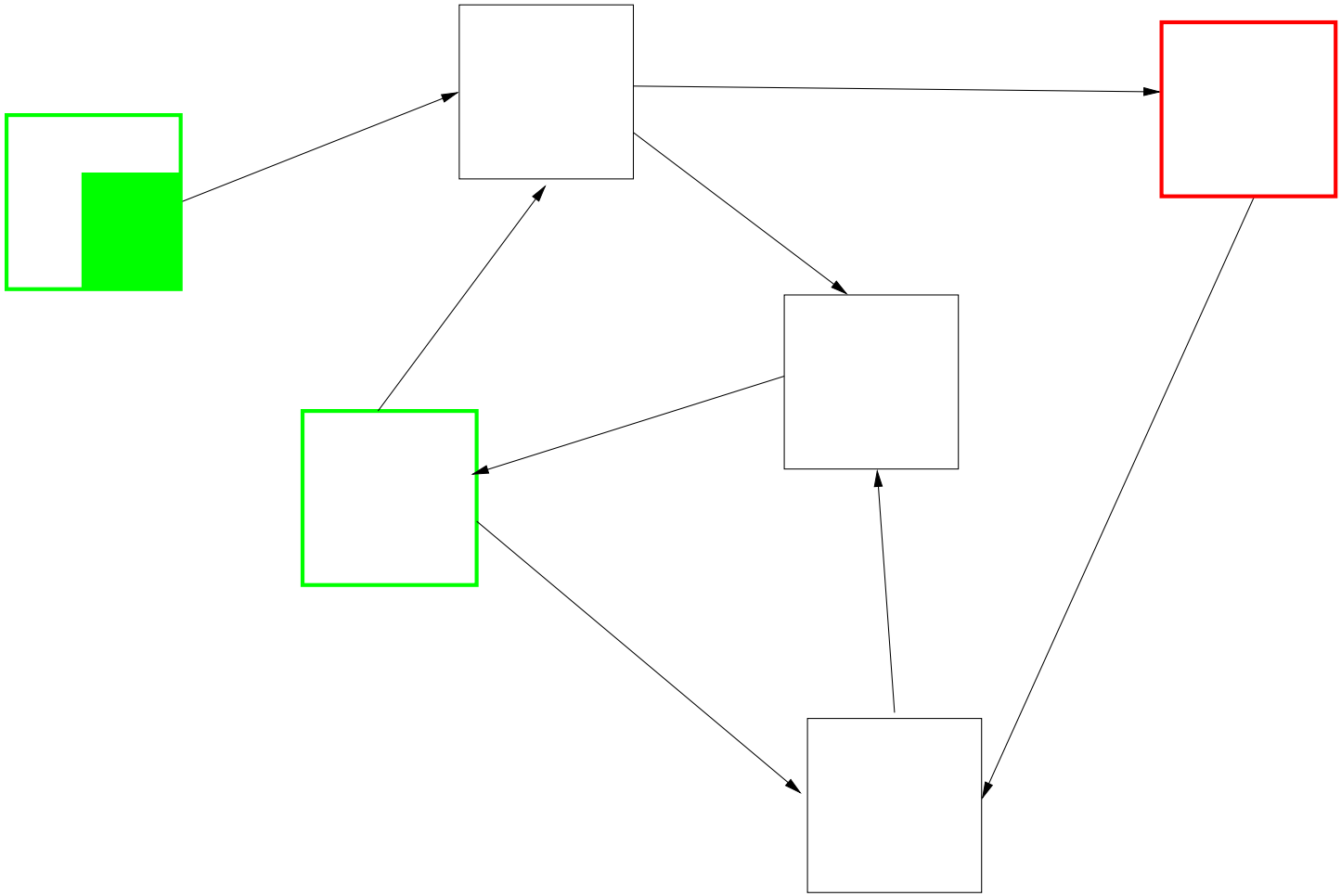

For each region marked as initial:

over-approximate set of states reachable from an initial state

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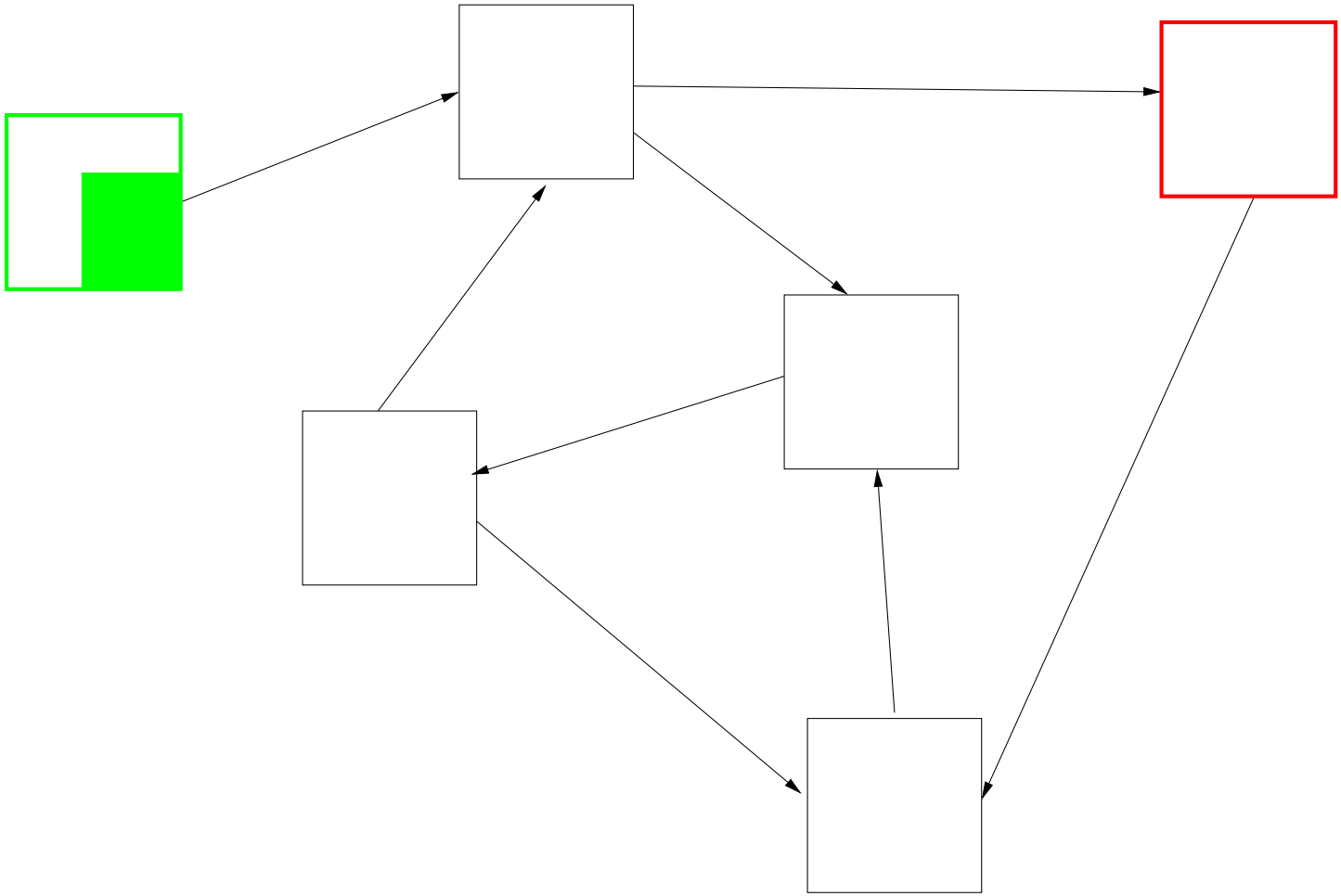

If empty set, remove initiality mark

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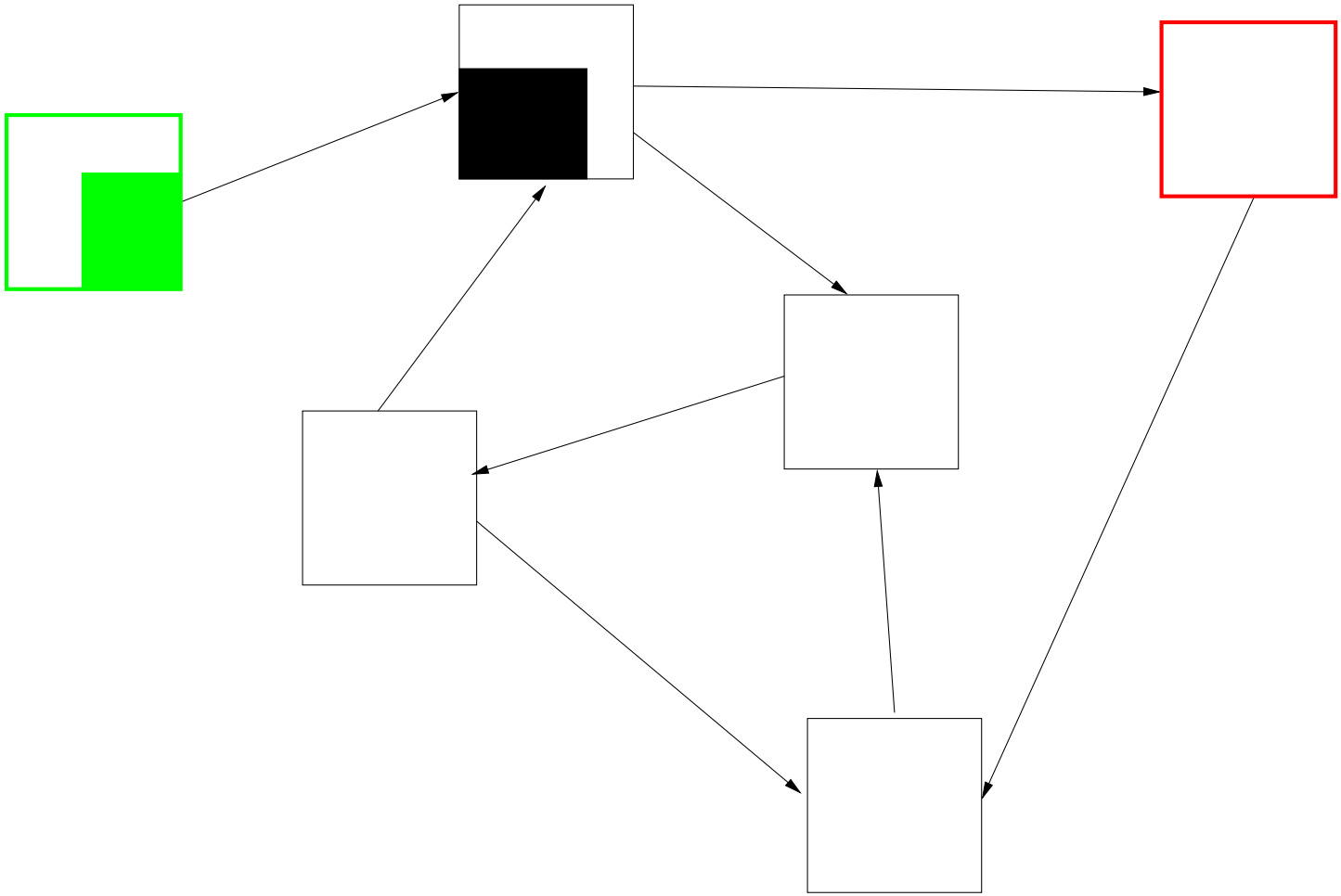

while some new states reachable through a transition,
  add them

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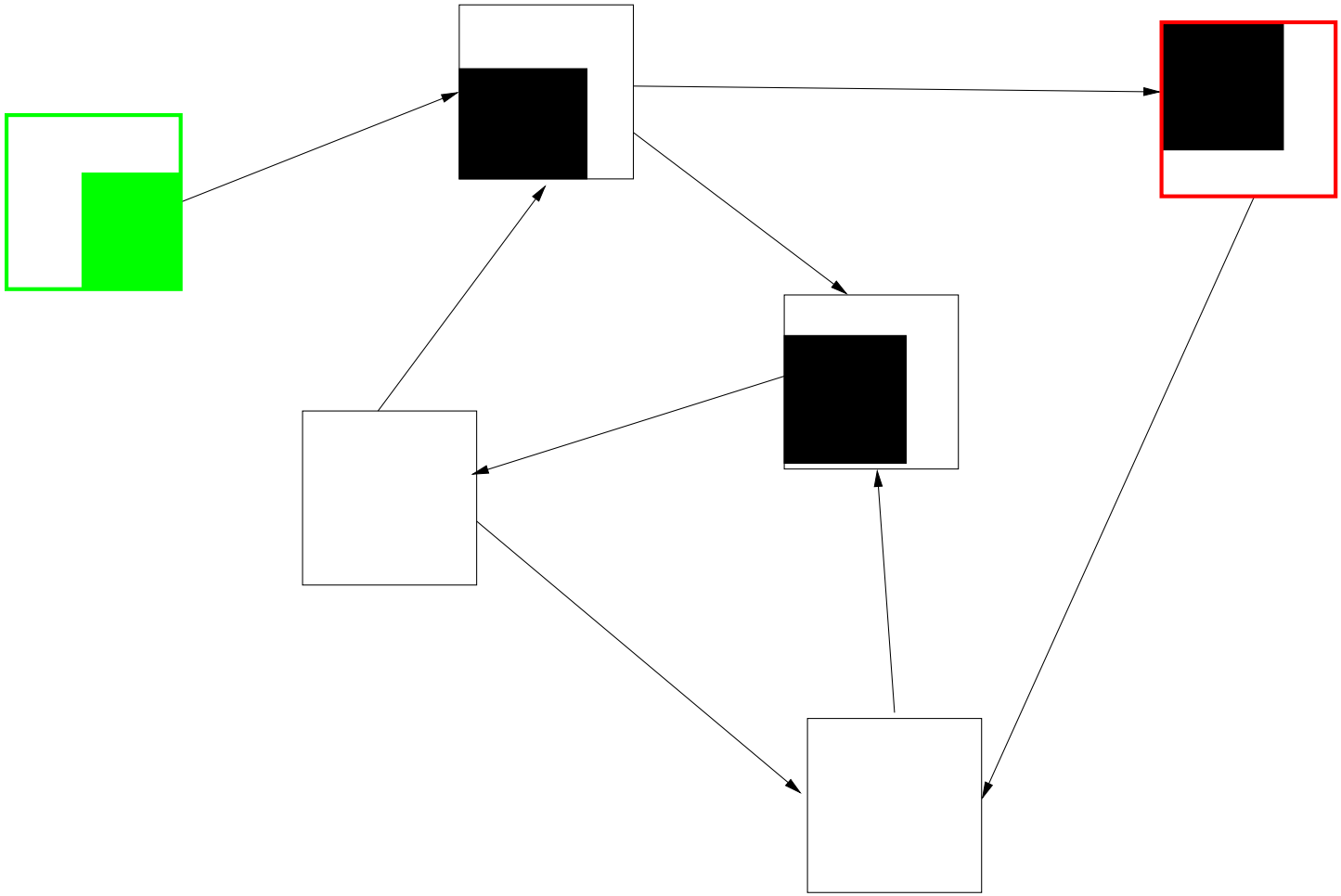

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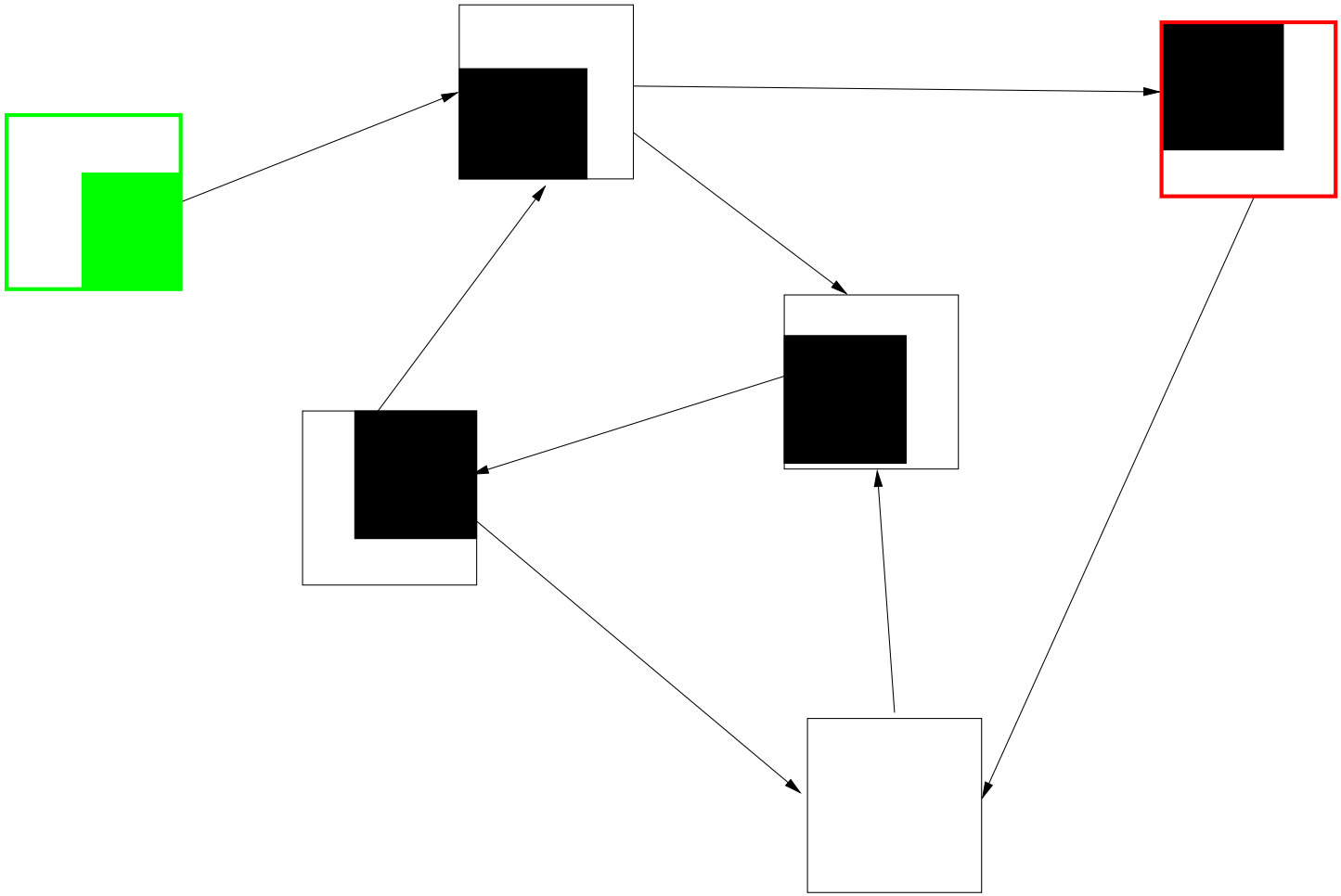

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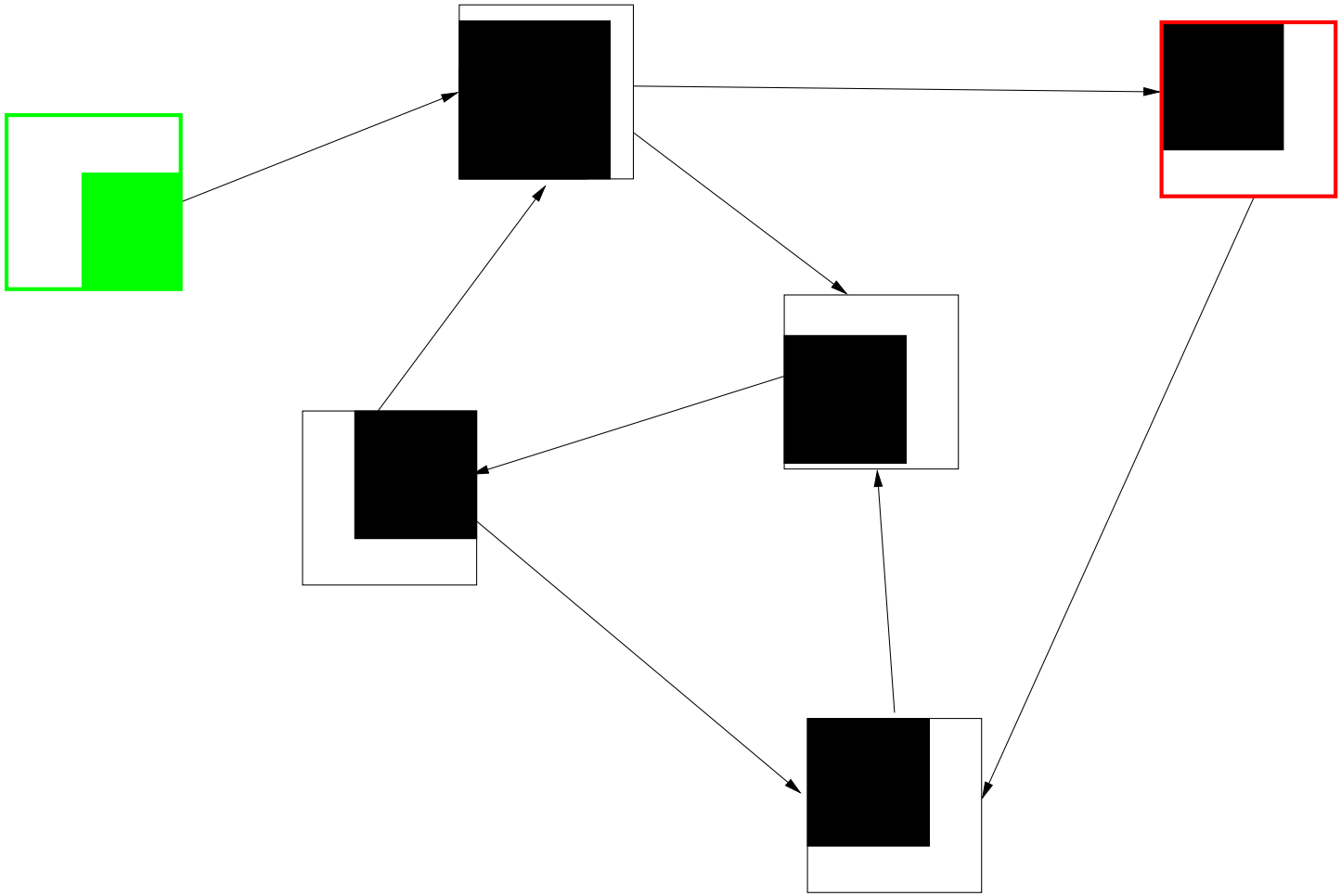

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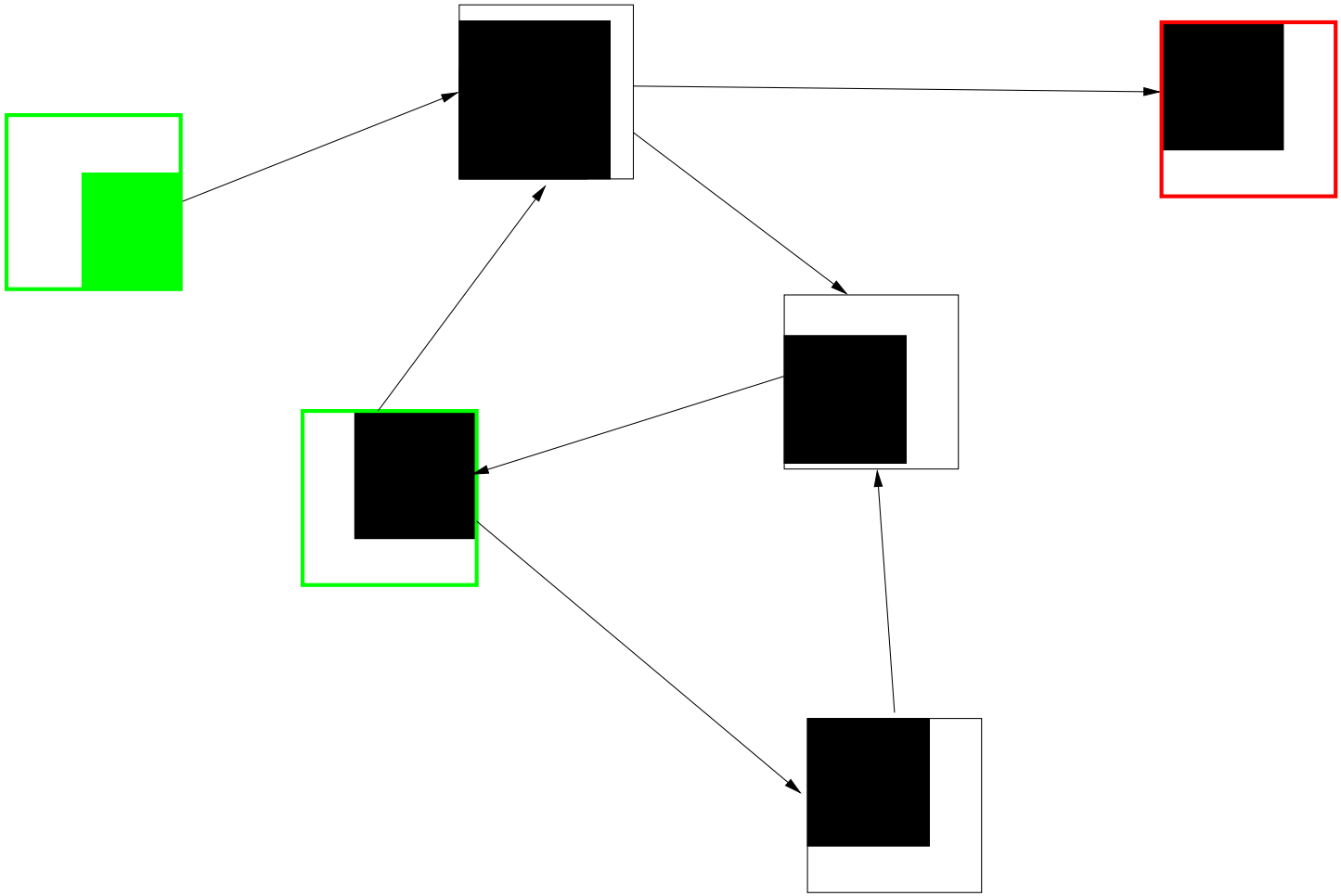

remove unconfirmed transitions

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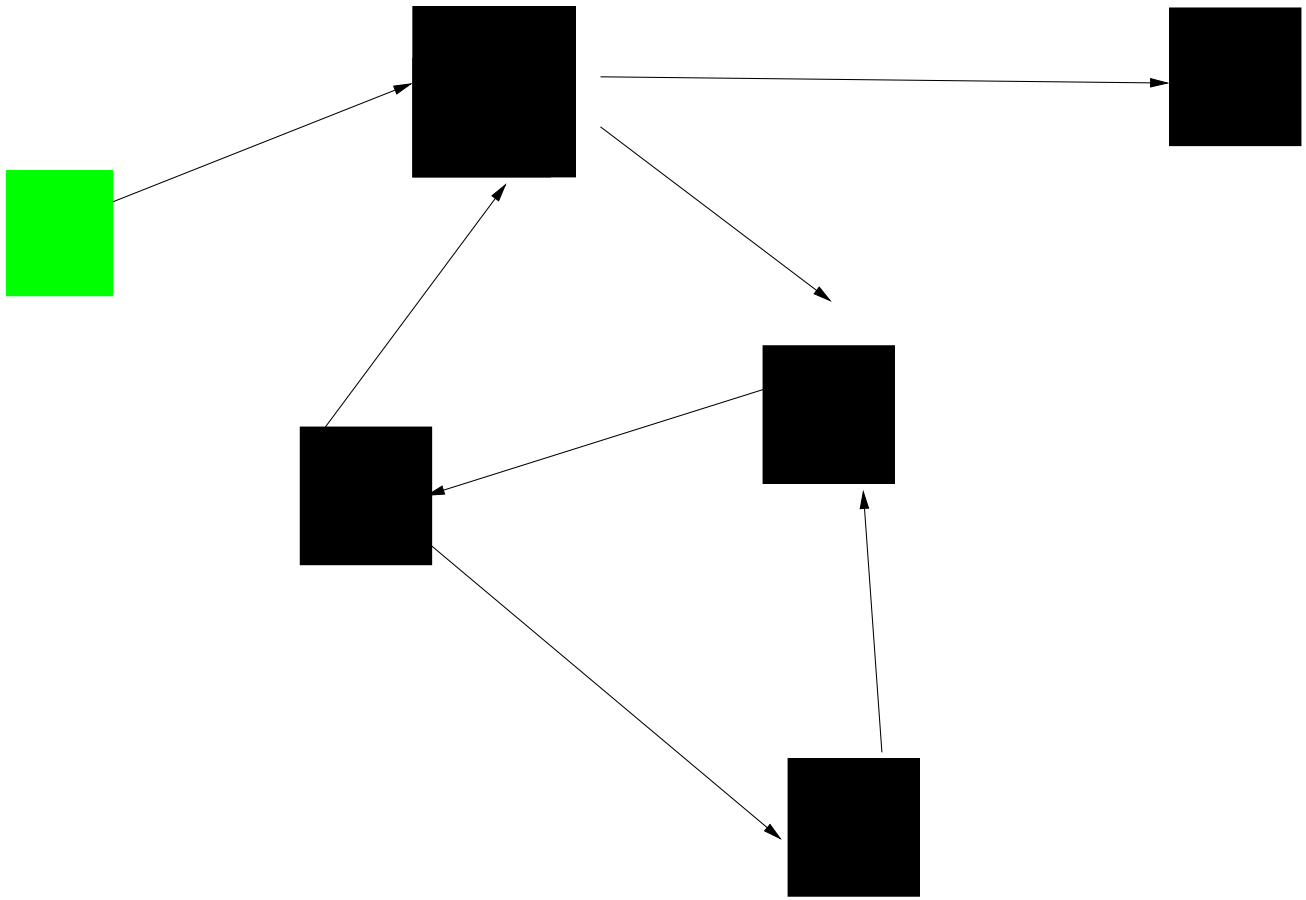


Replace regions by new ones

Discussion

Termination?

Discussion

Termination?

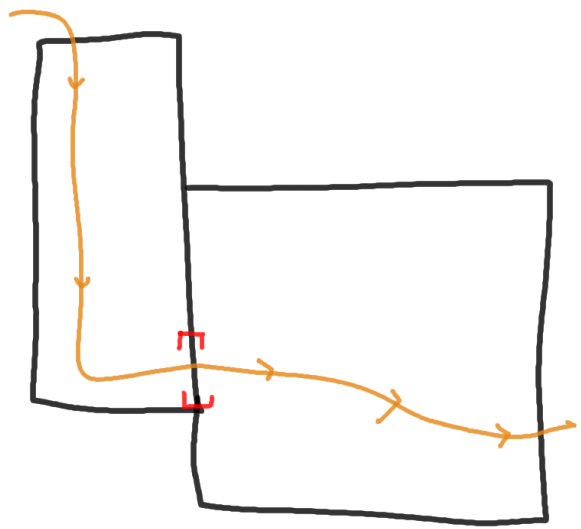
Backward in time

Discussion

Termination?

Backward in time

Exit regions (exploit continuous time nature):



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See our tool HSolver (`http://hsolver.sourceforge.net`)

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Implementation based on boxes available (open source) from

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