Continuity Analysis of Floating Point Software

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Software Continuity: What is it? Why is it Important?

Continuity:

Small changes have small effects!

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Peter Wegner, 1979

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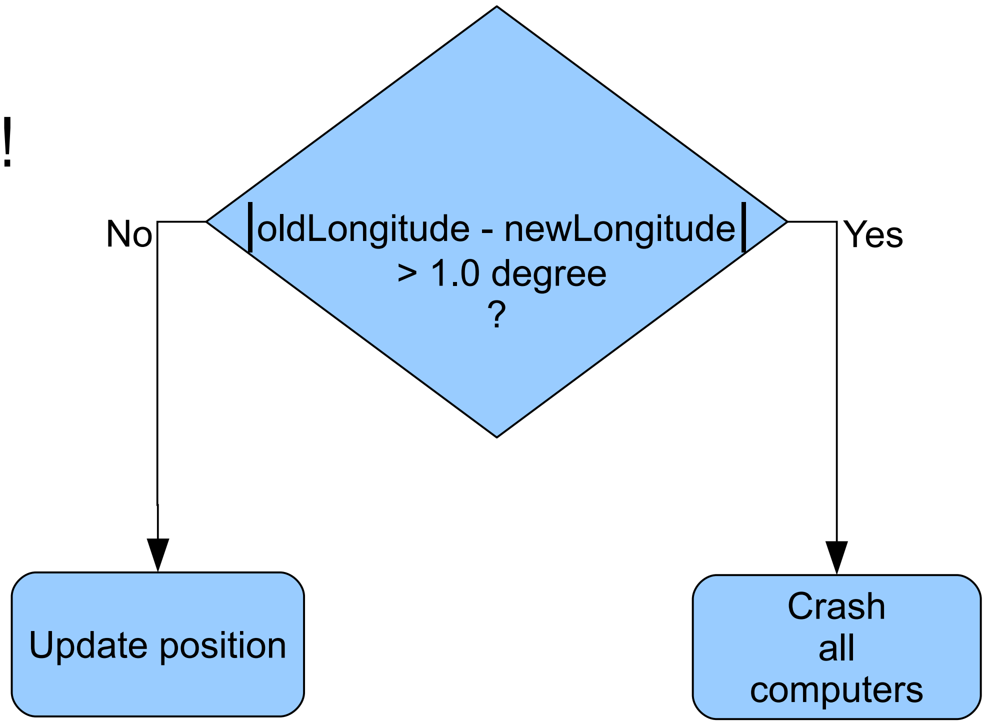
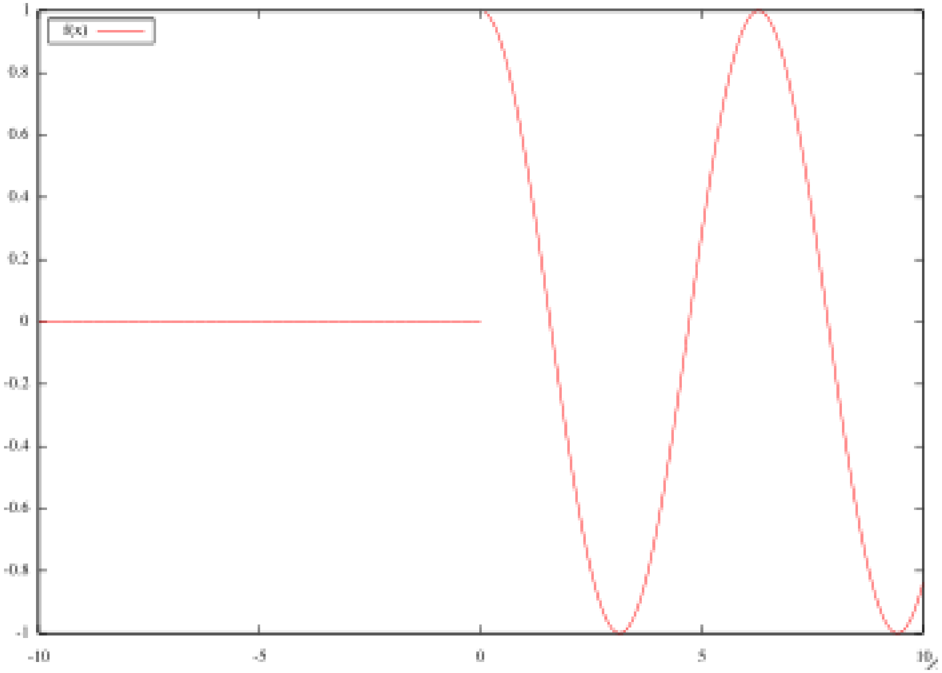


F22 Raptor
Crossing International Date Line
February 11, 2007

Problem Statement

```
double f(double x) {
  if (x < 0.0)
    return 0.0;
  else
    return cos(x);
}
```



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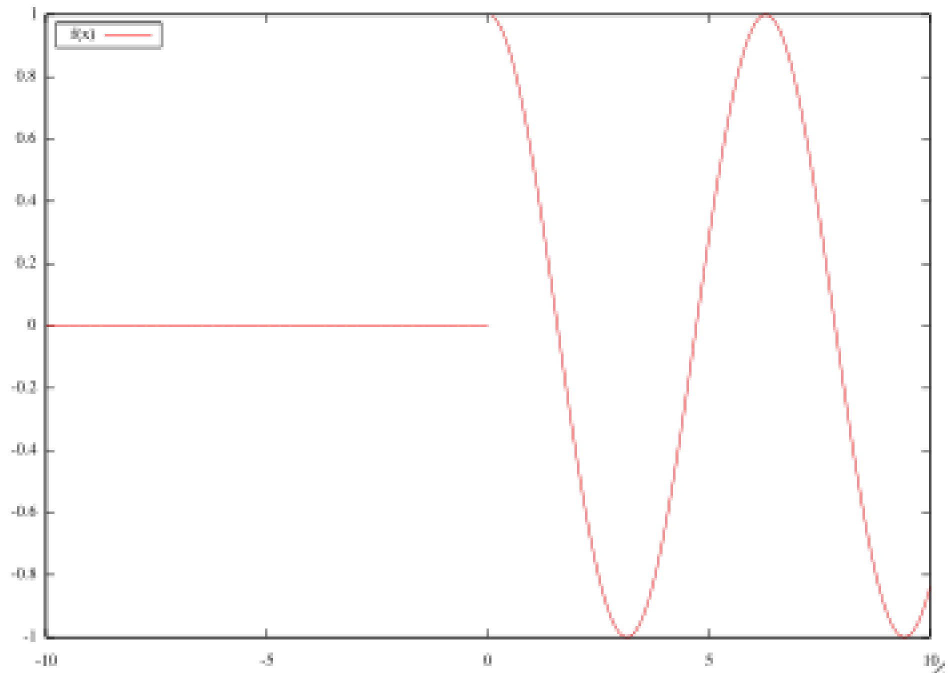


Where does the software's control structure cause `f(x)` to be discontinuous?

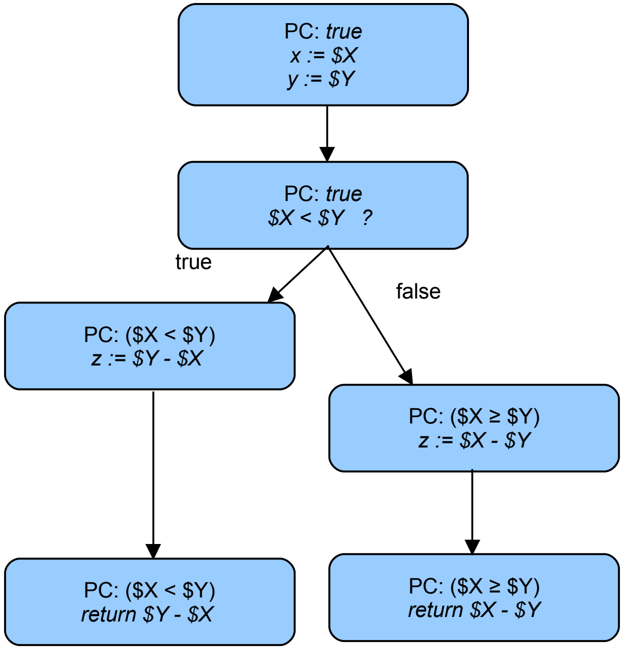
How to generate test cases near the discontinuities?

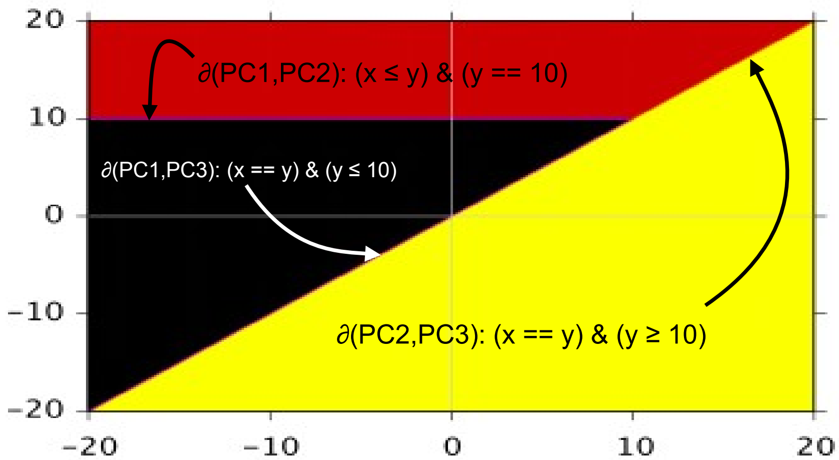
Approach

Symbolic Execution

+

Boundary Analysis





Concrete Execution for Testing

```
double g(double x, y) {

  double z;
  if (x < y)
    z = y-x;

  else
    z = x-y;

  return z;
}
```

x := -5.3
y := 3.2

Concrete Execution for Testing

```
double g(double x, y) {

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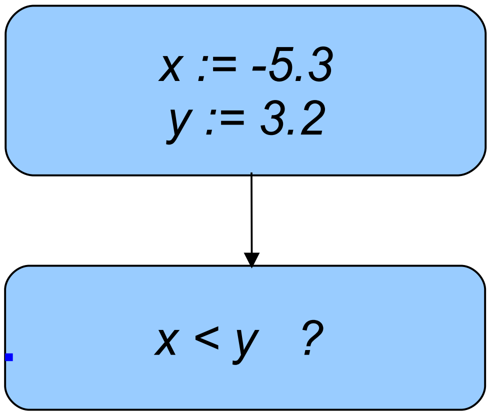

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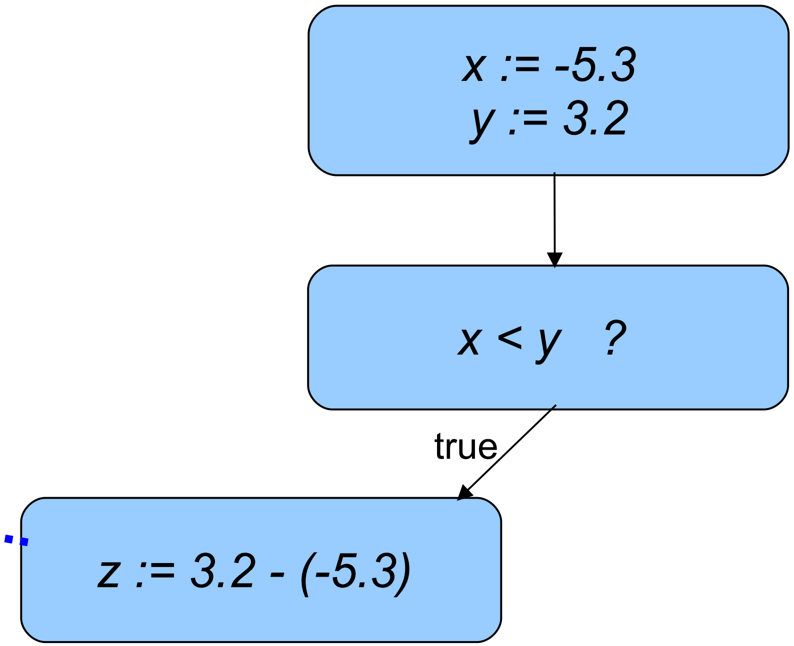

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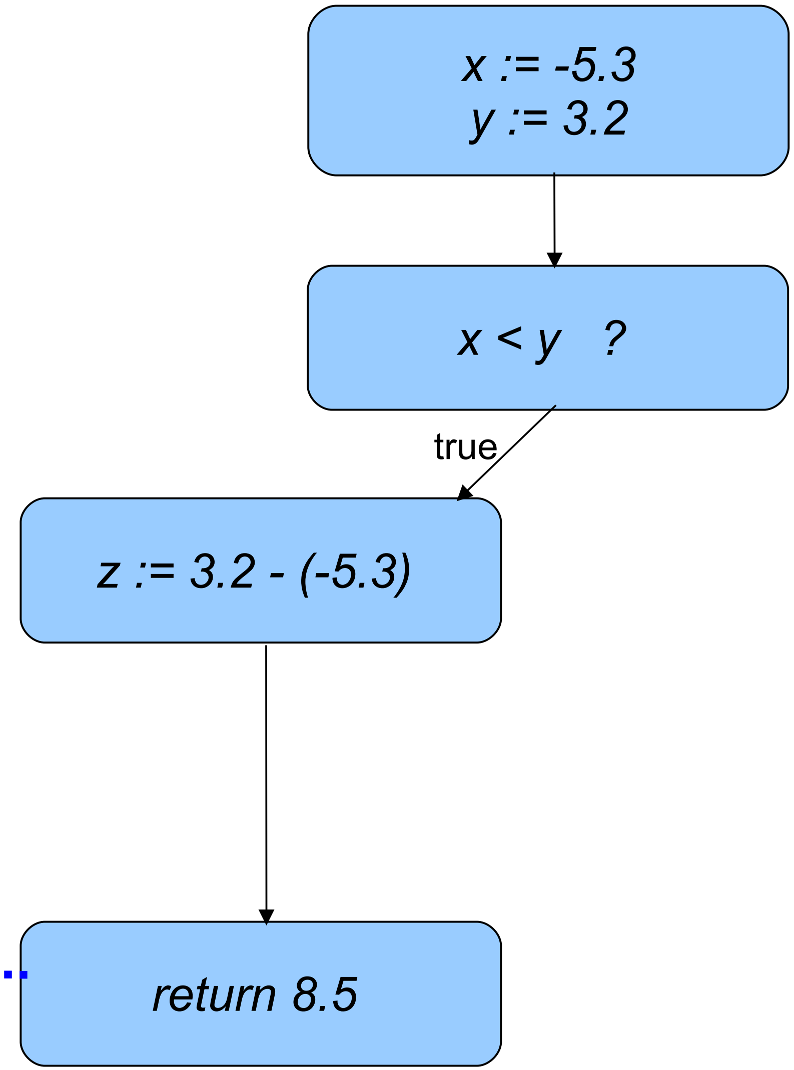

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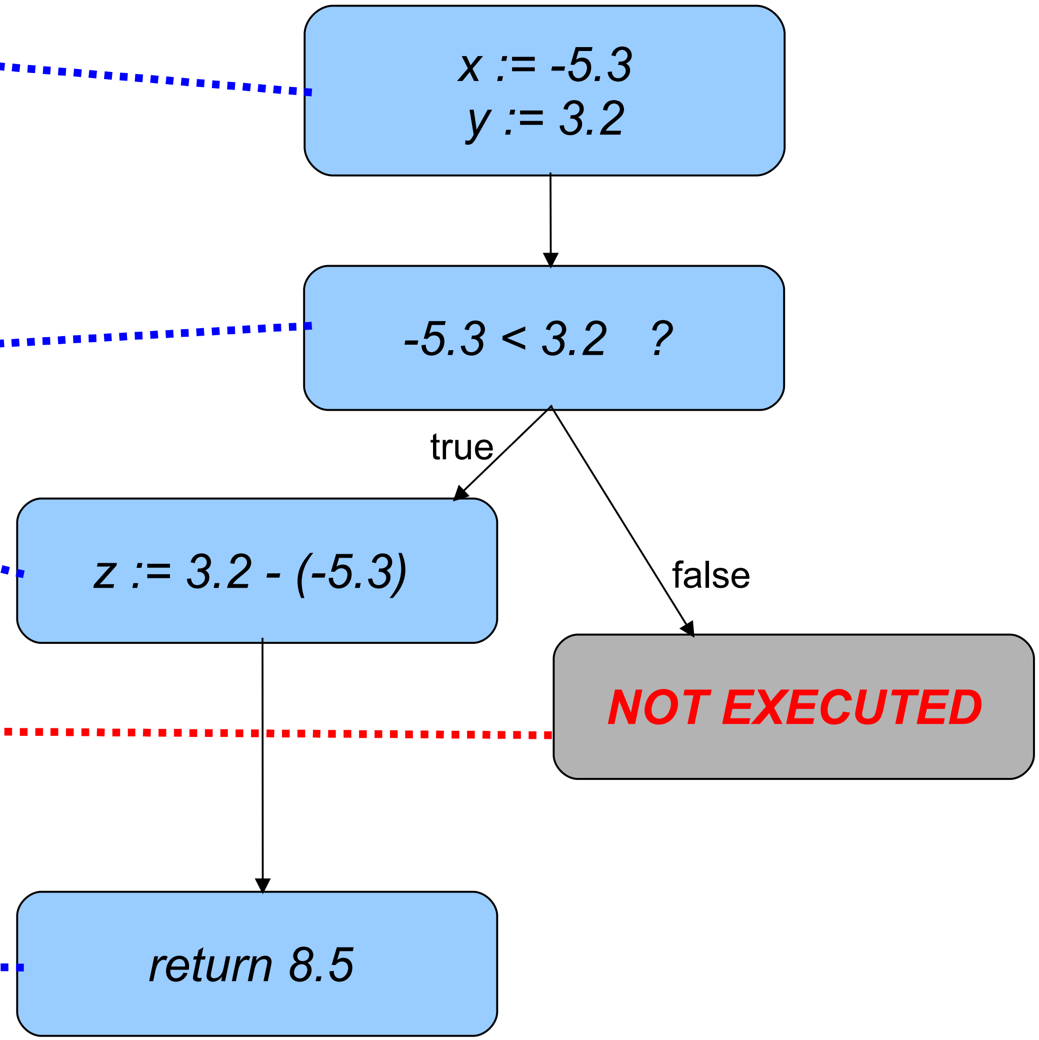

Symbolic Execution

```
double g(double x, y) {

  double z;
  if (x < y)
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  else
    z = x-y;

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}
```

PC (Path Condition): *true*
x := $X
y := $Y

Symbolic Execution

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y := $Y

PC: *true*
$X < $Y   ?

Symbolic Execution

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   else
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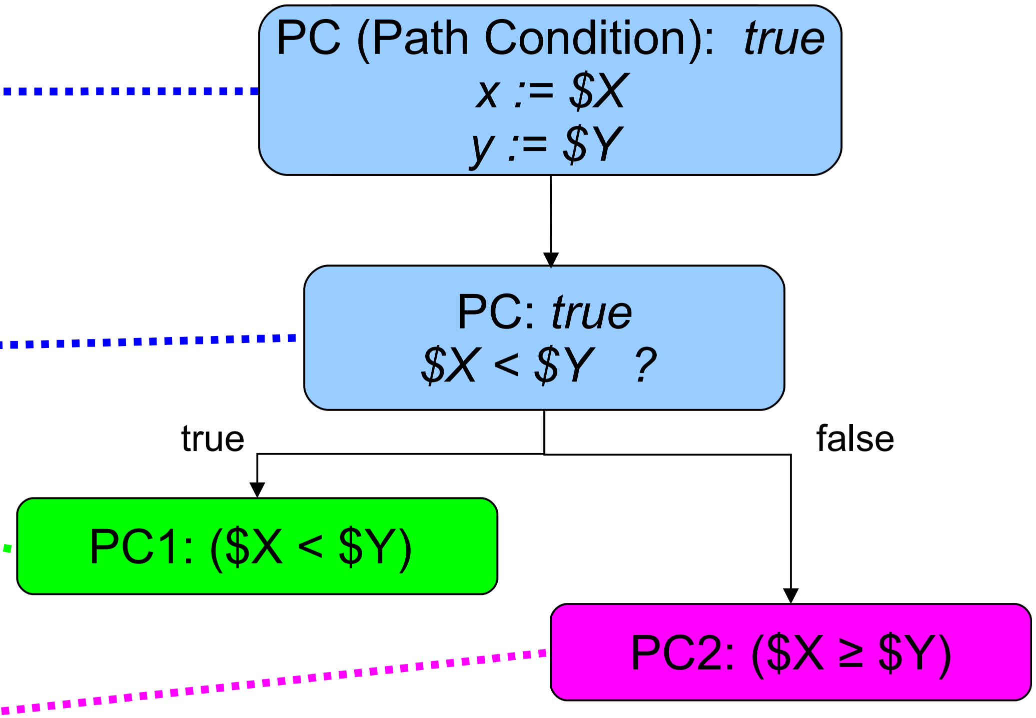

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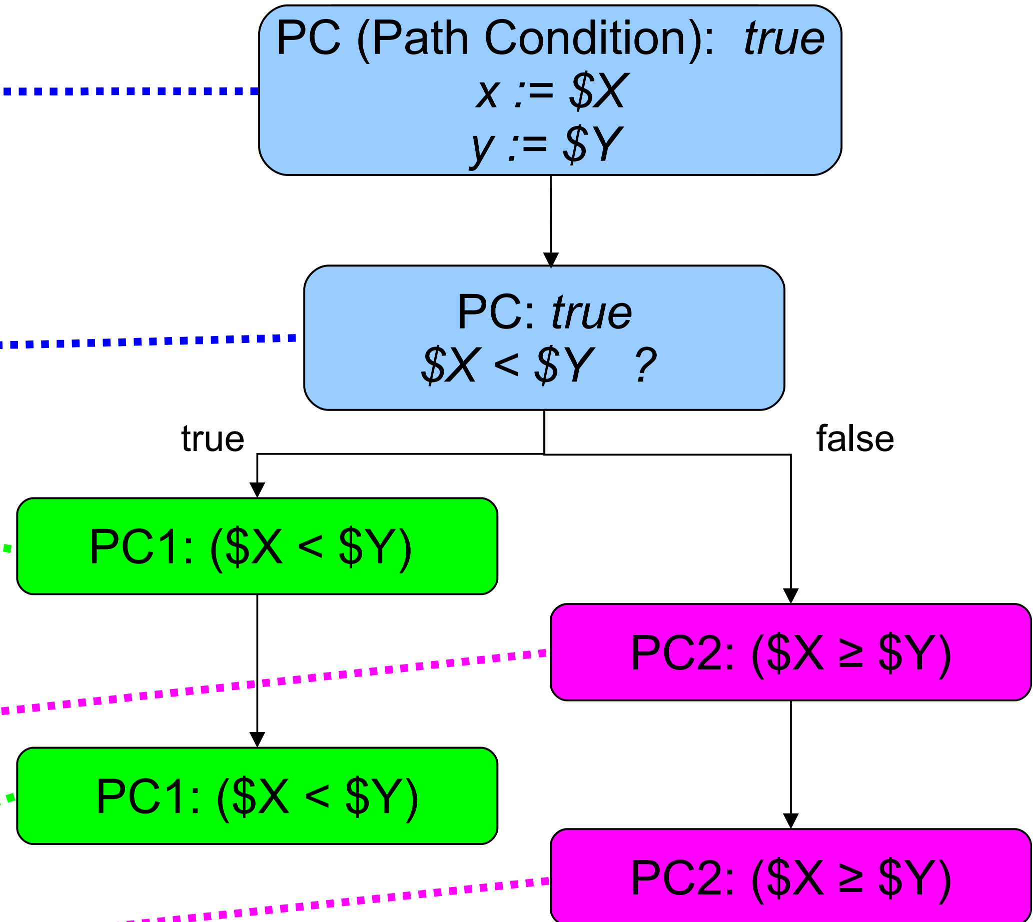

Symbolic Execution

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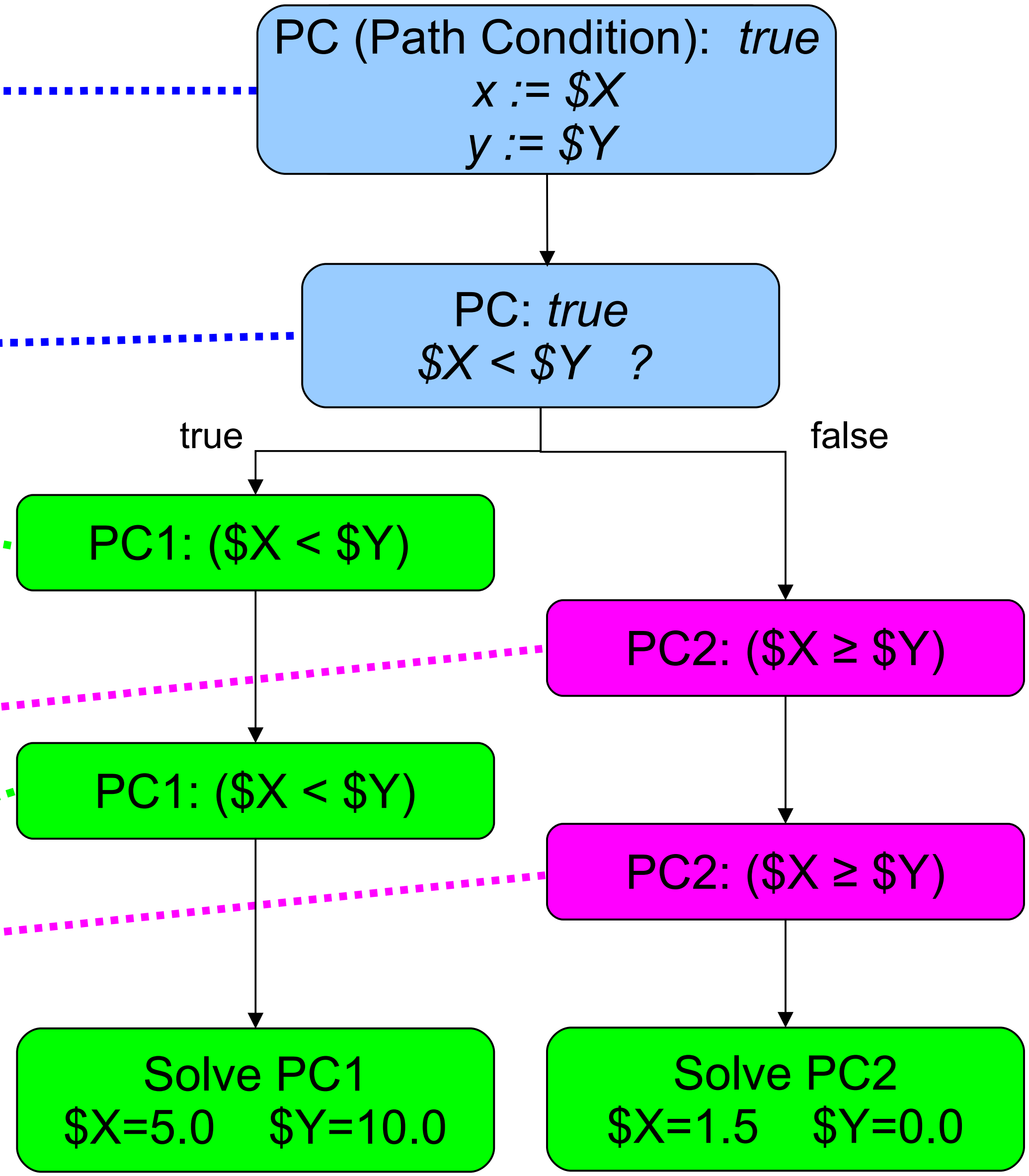


Java Pathfinder: JPF

- General purpose Java verification tool
  - Software model checking (concurrency bugs: deadlock, race conditions)
  - UML statechart verification
  - Floating point overflow/underflow/catastrophic cancelation
  - Symbolic execution (test case generation)
  - Many more ...
- Developed at NASA/Ames
  - Started in 1999, work continues to this day
  - Recognized by numerous awards from both NASA and the verification community
  - Open source since 2005

Symbolic Pathfinder: SPF

Symbolic Pathfinder:

Extends JPF for symbolic execution

Supports booleans, integers, floating point, complex data structures, strings

Supports many constraint solvers:

- Coral

  High performance non-linear floating point constraint solver

- Choco

  Solver for linear/non-linear, integer/floating point constraints, mixed constraints

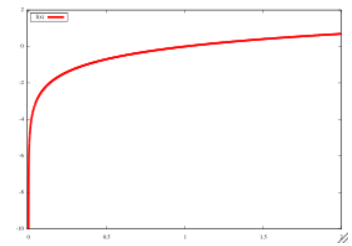
- CVC3

  Solver for integer/real linear constraints

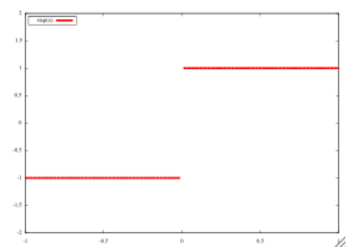
Discontinuities

Discontinuities come in many forms, including:

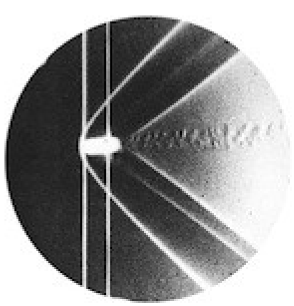
- Non-convergence: $y = -\sum_{1}^{\infty} \frac{(1-x)^n}{n}$



- Blackbox: `y = step(x)`



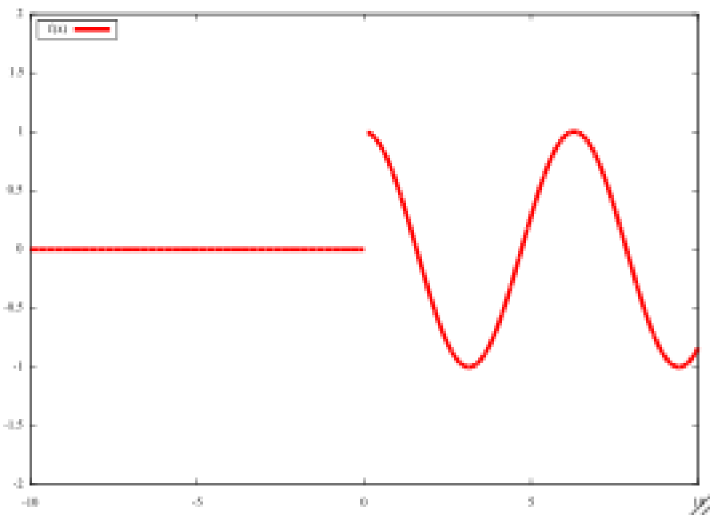
- Physical discontinuities in the model e.g. shockwaves:



Discontinuities

We analyze discontinuities of another type:
Those that arise from the structure of the software

```
if (x < 0)
  return 0.0
else
  return cos(x);
```



Terminology

Path Condition: Boolean expression in symbolic inputs specifying a single execution path through the software.

Written as `PC, PC1, PC2, …`

All inputs satisfying a given path condition force the same execution path.

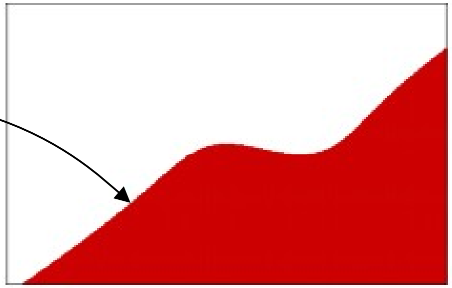
Example: `PC1: (x>0.0)∧(x+y<z)`

Terminology

***Boundary** between path conditions PC1 and PC2:*

The set of input values at which the software changes from the execution path for PC1 to the path for PC2.

Written as ∂(PC1,PC2)



Example:

if PC1:(x > y) and PC2:(x $\leq$ y)

then ∂(PC1,PC2): (x = y)

Terminology

Path Function: The function computed by all inputs satisfying a single path condition.

Example:

```
if (x < 0)
 return 0.0
else
 return cos(x);
```

For `PC1: (x < 0)`

`PF1: 0.0`

For `PC2: (x ≥ 0)`

`PF2: cos(x)`

Terminology

Discontinuity Condition: Boolean expression in symbolic inputs specifying a region where the function computed by the software is discontinuous.

Written as `DC`, `DC1`, ...

We are interested in discontinuity conditions associated with the software's control structures.

Calculating Boundaries of Path Conditions

First:

Calculate the *closure* of a relation.

For the floating point relations

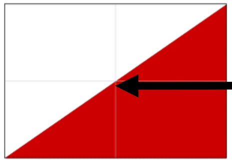
$\{<, \leq, =, \neq, \geq, >\}$

define the *closure* of the relation as in the table:

Relation	Closure
F < G	F ≤ G
F ≤ G	F ≤ G
F = G	F = G
F ≠ G	*true*
F ≥ G	F ≥ G
F > G	F ≥ G

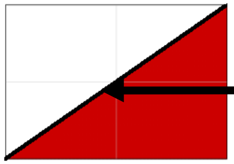
Example:

The region defined by (x < y) is:



Border missing

Its closure is (x ≤ y):



Border included

Calculating Boundaries of Path Conditions

Second:

Define the *closure of a path condition:*

Given a path condition `PC`

$$PC = R1 \wedge R2 \wedge \ldots$$

where $R1$, $R2$, ... are primitive relations.

Its closure is written `PC` or `cl(PC)`
and is given by

$$\overline{PC} = \overline{R1} \wedge \overline{R2} \wedge \ldots$$

Calculating Boundaries of Path Conditions

Finally:

Define the *boundary between two path conditions* `PC1` and `PC2` as:

$$\partial\left(PC1\,,PC2\right)=\overline{PC1}\wedge\overline{PC2}$$

Example: Boundaries

```
double step2(double x, double y)
   if (x < y)
     return 0.0;
   else
     return 1.0;
```

Path Conditions: $PC1 : (x < y)$ $PC2 : (x \geq y)$

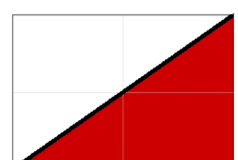
Boundary:

$$\partial(PC1, PC2) \equiv \overline{PC1} \wedge \overline{PC2}$$
$$\equiv (x \leqslant y) \wedge (x \geqslant y)$$
$$\equiv (x = y)$$

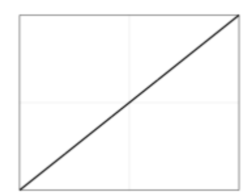
PC1:

$\wedge$

PC2:



$\equiv$ (x=y)



Discontinuity Conditions From Control Structures

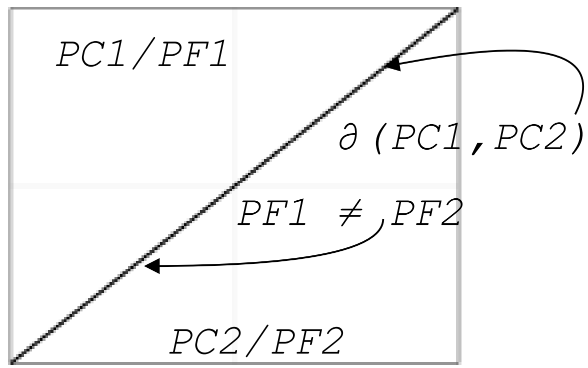
How can the output be discontinuous when control switches from one execution path to another?

Answer: when the values computed along Path 1 `(PC1)` do not match the values computed along Path 2 `(PC2)` at the crossover

Discontinuity Conditions From Control Structures

Crossover happens at the boundary between Path 1 and Path 2

The values computed are the path functions `PF1` and `PF2`



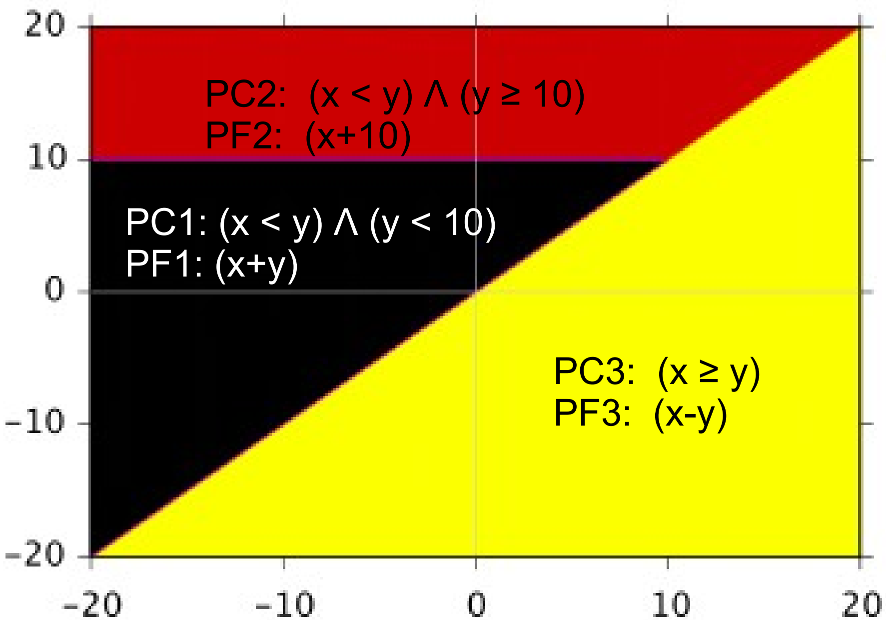
So when control switches from Path 1 to Path 2, the software will be discontinuous at all points satisfying:

$$\partial(PC1, PC2) \wedge (PF1 \neq PF2)$$

Example: Discontinuity Conditions

```
double sa(double x, double y)
    if (x < y)
     if (y < 10)
      return x+y;
     else
      return x+10;
    else
      return x-y;
```

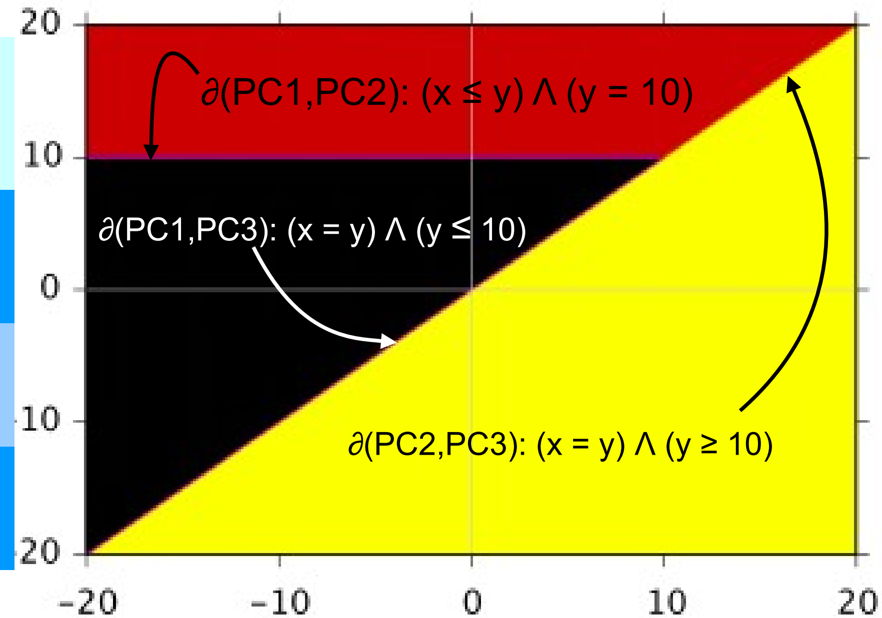
Path Condition	Path Function
PC1: $(x<y) \land (y<10)$	$x + y$
PC2: $(x<y) \land (y\geq 10)$	$x + 10$
PC3: $(x\geq y)$	$x - y$



Example: Discontinuity Conditions

```
double sa(double x, double y)
    if (x < y)
     if (y < 10)
      return x+y;
     else
      return x+10;
    else
      return x-y;
```

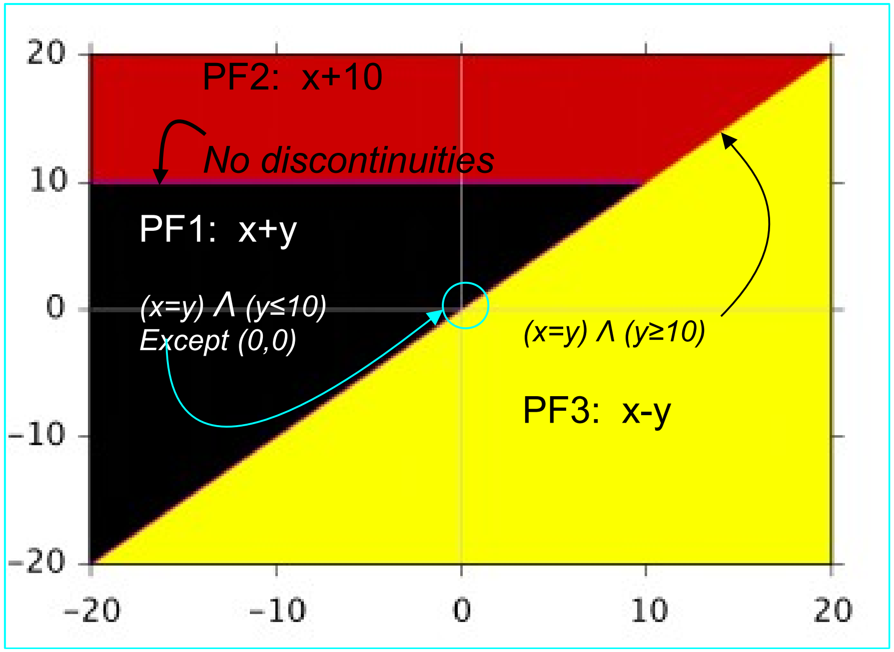
PCa PCb	∂(PCa,PCb) = cl(PCa) Λ cl(PCb)
PC1: (x<y) Λ (y<10) PC2: (x<y) Λ (y≥10)	(x≤y) Λ (y=10)
PC1: (z<y) Λ (y<10) PC3: (x≥y)	(x=y) Λ (y≤10)
PC2: (x<y) Λ (y≥10) PC3: (x≥y)	(x=y) Λ (y≥10)



Example: Discontinuity Conditions

```
double sa(double x, double y)
    if (x < y)
     if (y < 10)
      return x+y;
     else
      return x+10;
    else
      return x-y;
```

PCa PCb	∂(PCa,PCb) ∧ (PFa ≠ PFb)
PC1: $(x < y) \land (y < 10)$ PC2: $(x < y) \land (y \geq 10)$	$(x \leq y) \land (y = 10) \land (x + y \neq x + 10)$ $\equiv (x \leq y) \land (y = 10) \land (y \neq 10)$ $\equiv$ false
PC1: $(x < y) \land (y < 10)$ PC3: $(x \geq y)$	$(x = y) \land (y \leq 10) \land (x + y \neq x - y)$ $\equiv (x = y) \land (y \leq 10) \land (y \neq 0)$
PC2: $(x < y) \land (y \geq 10)$ PC3: $(x \geq y)$	$(x = y) \land (y \geq 10) \land (x + 10 \neq x - y)$ $\equiv (x = y) \land (y \geq 10)$



Status

Implemented on top of Symbolic Path Finder:

- Open source JPF extension: jpf-continuity
- Computes boundaries between path conditions
- Computes constraints along borders:
  - Discontinuity
  - Continuity
- Integrated with Coral constraint solver
  - Can solve highly non-linear floating point constraints
- Computes symbolic derivatives of path functions

Status

Demonstrated automatic generation of test cases at border discontinuities

- Applied to selected code from TSAFE
  (Tactical Separation Assisted Flight Environment)
- Automatically generates test cases demonstrating discontinuities along path condition borders

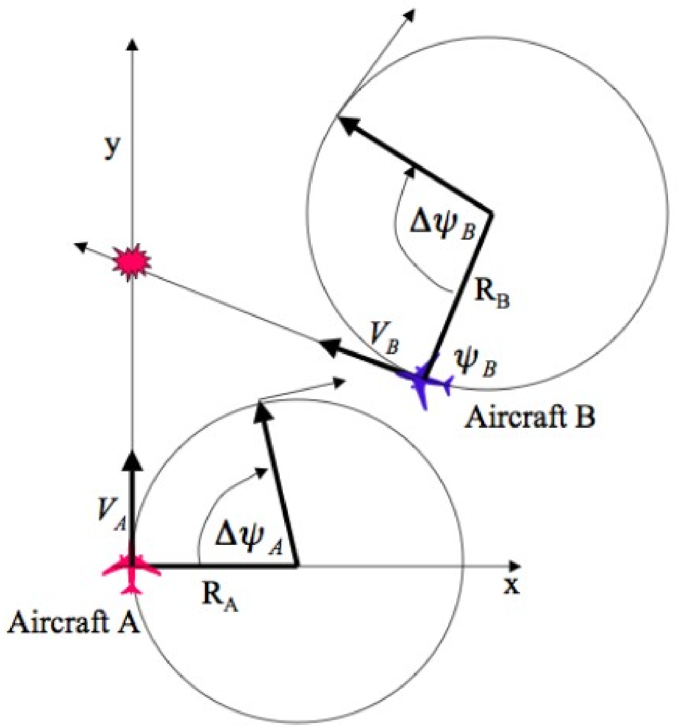


Diagram from: H. Erzberger, 2009

Beyond Discontinuity Analysis

Robustness:

- Where along the border are the path functions "extremely discontinuous"?
  Discontinuous: `PF1≠PF2`
  Non-Robust: `|PF1-PF2|>ε`

Global border behavior:

- How bad do the discontinuities get?
- How badly to the continuous points behave?

Beyond Discontinuity Analysis

- Compositional Analysis
  - If $f(x1,\ldots,xn) = h(g(x1,\ldots,xn))$ and we know the discontinuities of $h()$ and $g()$, what are the discontinuities of $f()$?
- Guided Test Generation
  - How to verify regions for safe execution of code?
- Polynomial Approximation
  - Useful when finding min, max, zeros:
    - Approximate with low-degree polynomials
    - Need to know discontinuities

Availability

JPF (Java Path Finder), SPF (Symbolic Path Finder), and jpf-continuity are all open source software

They are available for download at

- JPF: http://babelfish.arc.nasa.gov/trac/jpf/
- SPF: http://babelfish.arc.nasa.gov/trac/jpf/wiki/projects/jpf-symbc
- jpf-continuity: https://jpfcontinuity@bitbucket.org/jpfcontinuity/jpf-continuity

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Questions?