Synchronous vs asynchronous stochastic π-calculi: a feedback from bioinformatics

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Abstract. In this article we compare the expressivness of the synchronous and asynchronous π-calculi in the stochastic settings. We first prove that the former is stricly more expressive than the latter. We then extend the result to the π-calculi with infinite rates. We also show that under a small restriction the asynchronous with infinite rates can encode the synchronous without infinite rates. Interestingly the separation results are proved using a bioinformatics problem.
Finally we propose and motivate a stochastic π-calculus with rates of different orders of magnitude: *the multi-scale* π-calculus to which we generalize our results.

1 Introduction.

The stochastic process algrebra [13, 5] have been successfully used over the last decades as languages of modeling and analysis in various fields among which one counts perfomance analysis [5] and formal system biology. Since the seminal works [15, 14] the stochastic has been widely used as a basis for the development of this last domain (see for instance [3, 6]). So studying its theorical properties has more than a theoritical importance.

In the classical and probabilistic settings much attention has been devoted to the expressivness of its numerous variants (see for instance [10, 9, 8]). An important stone in this edifice has been added by [10] in which the synchronous is proved to be strictly more expressive than the asynchronous one. In this article we adress the same problem in the stochastic settings.

It is well-accepted that the fragment of the with mixed choice (i.e. which can have both inputs and outputs among the same choice) is synchronous. The asynchronous usually refers to the fragment without choice and where the outputs have no continuation. However [9, 8] showed that output-prefix and separate choices (i.e. which are made either of outputs or of inputs) can be encoded in the asynchronous . Therefore in order to compare the synchronous and asynchronous stochastic we choose to compare the mixed and separate choices fragments.

In [12] it has been proved that in the probabilistic settings the mixed choice can be encoded by the separate choice. Could this work be extended to the stochastic settings? The idea of the encoding is the following: it divides the mixed choice into two separate choices and uses τ transitions to navigate from a choice to the other in order to keep both of them available. The τ-divergence induced by the τ transitions is discarded because it has probability 0. Such an idea could not work in the stochastic settings because one not only observes the probability to fire a transition but also the elapsing of time before the firing of the transition. Therefore the τ-transitions wouldn't be unobservable (this idea is studied more in detail at the end of part 2).

Structure of the article In part 2 we detail the specifities of stochastic semantics and discuss the properties that an encoding between stochastic languages should preserve. The part 3 presents the syntax and semantics of the stochastic . In part 4 we detail our results: part 4.1 introduces the bioinformatics problem used to prove the separation results, part 4.2 present the separation between separate and mixed stochastic choices, part 4.3 extends this results to the case of infinite rates and part 4.4 consider the case of infinite rate separate choices versus finite rate mixed choice. And finally in part 5 we propose a with rates of different order of magnitude: the multi-scale and extend our results to this settings.

2 On stochastic semantics and encodings.

The semantics of stochastic languages is usually given in terms of *Continuous Time Markov Chains* (in the following): this is a graph with a node for each term of the language and a transition from P to Q whenever P evolves into Q; each transition is labelled with a rate representing the parameter of an exponential distribution characterising the delay until the associated transition is enabled. More precisely, for rate r, the probability that the transition is enabled within t time-units is given by $1 - e^{r.t}$. In this article we consider rates strictly positive of infinite. A transition with an infinite rate takes no time ($1 - e^{\infty.t} = 0$) and is said to be *instantaneous*. Besides the infinite transitions have higher priority than the finite ones and are always executed first, this is know as the *maximal progress* hypothesis.

Stochastic semantics differ from classical semantics by the importance of the quantity of redexes yielding a transition from a term to another. Indeed the rate of the transition from a term P to a term Q is equal to the sum of the rate of the redexes yielding from P to Q. For instance, suppose that channel x has rate λ, then $x|x\lambda 0$ and $x + x|x 2 * \lambda 0$. In particular and in opposite to the classical settings P and $P + P$ are not bisimilar in general.

Therefore the derivation of the stochastic semantics of a language has to take care of occurences of redexes. It is computed in two steps. First the terms of the language are decorated with tags which permit to distinguish between the occurrences of redexes and an intermediary transition relation is derived where each transition corresponds to a unique redex. Then the stochastic semantics is computed by collecting into one the transitions having the same source and target terms and by summing their rates (assuming $\infty + \lambda = \infty$). This approach is illustrated in the section 3 with the semantics of the .

Weak transitions. In the classical settings, one-step transitions are often considered as a much precise level of observation. The so-called τ-moves, meaning the internal synchronization of a process, are often not considered as observable. The weak transitions permit to obtain a more realistic point of view. Let be the one-step transition relation, and * its reflexive and transitive closure, then the weak-transition relation is defined by $P\alpha Q \Leftrightarrow P\tau^*\alpha\tau^*Q$.

In the stochastic settings the important observable is the elapsing of time. Thus we choose to consider the finit rate transitions as observable. The instantaneous transitions are unobservable: since such transition takes no time, one cannot tell how many of them have occured. So we define the weak stochastic transition in the following way:

Definition 1 (Weak stochastic transitions). *The weak transition relation is defined by* $P\lambda Q \Leftrightarrow P\infty^*\lambda\infty^*Q$.

Stochastic encodings. What properties do we require for an encoding between stochastic languages ? It certainly depends on wether we want to prove a positive result or a separation result. Stronger results are obtained by asking weaker conditions in the latter and stronger conditions in the former. It doesn't seem to be a definitive answer, still one can identify a set of minimum conditions an encoding should satisfy. Here again we adapted existing work to the stochastic settings (see [11] for a review on classical encoding criterions).

- An encoding should be *uniform* i.e. homomorphic with respect to the parallel composition, namely $P|Q = \delta_{P,Q}(P|Q)$ where $\delta_{P,Q}$ is a declaration of new names. This ensures that two parallel processes are translated into two parallel processes, possibly with the introduction of new names: no coordinator process is introduced.
- In the classical settings the preserving of τ-divergence, that is infinite sequence of τ-moves, is not always required. However in the stochastic settings such divergence represents a serious problem in the stochastic behaviour: since the silent transitions are the instantaneous ones, and since such transitions have priority on the finite rate ones, it means that one will keep doing instantaneous transitions forever. Time is frozen. Therefore we ask that a process can do an infinite sequence of instantaneous transitions if and only if its encoding can do the same. If P can do an infinite sequence of instantaneous transitions we write $P\infty^\Omega$.
- An encoding should preserve a reasonable semantics. We reviewed the various criterion proposed in [11], we choosed the operational correspondance since it is one the weakest and we adapted it to the stochastic settings. More precisely we ask that any transition from P to Q is mimicked by the encodings of P and Q, and that any sequence of transitions starting from the encoding of P is the beginning of a sequence of transitions that matches a sequence of transtions of P.

These conditions are formalized in definition 2. They are the ones we use for our separation results. The properties satisfy by our positive encoding are detailed in the corresponding section (see 4.4).

Definition 2 (Admissible encoding). *An encoding . is* admissible *if and if only:*

1. $\forall P, Q.\, P|Q = \delta_{P,Q}(P|Q)$ *where* $\delta_{P,Q}$ *is a name declaration;*
2. $P\infty^{\Omega}$ *if and only if* $P\infty^{\Omega}$*;*
3. *if* $P\alpha Q$ *then* $P\alpha Q$*;*
4. *if* $P\lambda R$ *then* $\exists Q$ *such that* $R\infty Q \wedge P\lambda Q$*.*

Remarkably this notion is compositional:

Proposition 1. *Given two admissible encodings . and . their composition . is also an admissible encoding.*

3 The stochastic π-calculi: syntax and semantics.

In this section we present the syntax and semantics of the .

Definition 3 (Syntax). *The following grammar depicts the syntax of the stochastic* π*-calculus:*

$$P ::= \alpha.PP|Pi \in I\Sigma P_i(\nu x : \lambda)Pu = vPA(z)$$
$$\alpha ::= xyxy$$

This syntax is very close to one of the classical , the only addition being the labeling of channels with stochastic rates. Intuitively the meaning of the operators can be described as follows. Prefixes represent the atomic actions: xy is the output of the tuple of names y on the channel x and xy is the input of the tuple of names y from channel x. $P|Q$ is the parallel composition of P and Q: they can either proceed asynchronously or synchronyze on complementary output/input actions. $i \in I\Sigma P_i$ is a choice between the processes P_i: it proceeds as one of the P_i; it is noted 0 when the sum is empty and I is assumed to be finite. $(\nu x : \lambda)P$ is the declaration of the tuple of channels x labelled with rates λ in the process P. $u = vP$ is a match: if u and v are equal it proceeds as P and as 0 otherwise. $A(z)$ is the invocation of the agent A with the parameters z :in which case we ask for a unique equation $A(x) := P$ defining A.

The terms of the are considered up to the a structural congruence $\equiv$. Typically the parallel and choice operator are commutative and associative. For a formal definition see [2] for instance.

Definition 4 (Guarded, mixed, separate and proper choices). *A choice* $i \in I\Sigma P_i$ *is a* guarded choice *if each* P_i *is of the form* $\alpha_i.Q_i$*. Moreover it is a* mixed choice *if it exist* i *and* j *such that* α_i *is an input and* α_j *is an output, and it is a* separate choice *if no such* i *and* j *exist. Finally a mixed choice is* proper *it doesn't contain an output and an input on the same channel.*

Notations. In the following and respectively refer to the stochastic π-calculus where all choices are mixed or separate respectively, and where the rate of all channels are finite. The corresponding languages with infinite rates are noted and respectively.

Stochastic semantics. The reduction rules of the classical are collected in the following definition.

Definition 5 (Reduction rules of the).

1. *$xy.P + i \in I\Sigma P_i \,|\, xz.Q + j \in J\Sigma Q_j$ rate(x)$P|Qy/z$,*
2. *if $P\lambda Q$ then both $P|R\lambda Q|R$ and $(\nu x : \lambda)P\lambda(\nu x : \lambda)Q$,*
3. *If $P \equiv P' \wedge P'\lambda Q' \wedge Q' \equiv Q$ then $P\lambda Q$*

As depicted in the previous section, in order to compute the stochastic transitions of a given term P we proceed in two steps. First we tag each action of P with a unique identifier; this new process is noted $\hat{P}$. We derive the tagged-transitions of $\hat{P}$ using the rules of the classical , but we also tag each τ-move with the pair of the identifiers of the actions to whom it corresponds. Since each identifier is unique this permits indeed to distinguish every redex. Then each group of tagged-transitions going to a structurally congruent term are collected in one transition, whose rate is the sum of these tagged-transitions. It is worth remarking that even if the stochastic transitions are obtained by summing rates of tagged-transitions, they correspond to only one synchronization.

The version of the stochastic presented here defers from the original one [13] where rates are attached to actions rather than to channel names. We choosed to study this version of the stochastic because as far as we know it is nowadays much more used, especially in the formal system biology field.

4 Gaps and bridges between synchronous and asynchronous stochastic π-calculi.

In this section we collect our expressivness results. We prove that without infinite rate mixed choice is strictly more expressive than separate choice (part 4.2), we extend this result to the case of infinite rates (part 4.3) and we show that separate choice with infinite rates can encode finite rate mixed choice assuming that (part 4.4). Interestingly the separaton results are proved using a problem originated from bioinformatics, namely:

Does an admissible encoding exist from the to a given language?

Remark 1. In [7] it has been proved that a subset of the with mixed choice can encode the . This encodings works both in the case of infinite rates or without. Actually it satisfies the very strong following property: $S\lambda T$ if and only if $S\lambda T$. This corresponds to the stochastic bisimilarity where the relation is also a bijection.

We begin by shortly recalling the essential feature of the . For more details and examples see [4].

4.1 The .

The is a reaction-based language designed for the modeling of nano-machines. Its terms are called solutions and are a list of molecules in parallel. A molecule is a term $Au\sigma$ where A is the species to whom belongs the molecule, u is a tuple of fields and σ is an interface: a tuple of sites. The fields record the internal state of the molecule: each field is associated with a value, typically an integer. A site can either be linked to a site of another molecule or be free. For instance in the molecule $As^1 + r^2 1 + 2^x$ the value of the fields s and r are 1 and 2 respectively, the site 1 is free and the site 2 is bound to a site of another molecule, on a link which name is x. The retains a formal graphical representation: the above molecule for instance is rendered in Figure 1(a).

[width=0.15]figureqapl2.pdf (a) [width=0.7]figureqapl1.pdf (b)

Fig. 1. Molecules and reactions in

The dynamics of a system is governed by reactions describing how two reactants may evolve. For instance, the reaction

$$As^0 1, Bt^1 1 \lambda As^1 1^x, Bt^0 1^x$$

illustrated in Figure 1(b), specifies that every molecule A whose internal state s equals 0 and whose site 1 is free may react with every B whose internal state t equals 1 and whose site 1 is free. The result is a complex where A and B are connected by a bond, called x, and the two internal states have updated values. The label λ of the reaction is its stochastic rate. It is worth to notice that this reaction applies to the molecule A in Figure 1(a), as well as to every other A with a different value of r and/or with an unbound site 2.

In molecules may react by means of three types of reactions: creations, destructions, and exchanges. A creation adds a new link between two molecules, a destruction removes a link between two molecules, and an exchange either doesn't affect interfaces or flip the extremity of a link between two molecules. Moreover all reactions can modify the fields of the molecules involved. They all can be presented in the format $Au\sigma, Bv\phi\lambda Au'\sigma', Bv'\phi'$ and we suppose that the reactant always share at most one bond.

The structural congruence of the is composed of the reflexivity and associativity of the parallel composition together with the injective renamings of bonds.

The stochastic semantics of the can be derived in a similar way to the one of the [1]. Given a set of reactions R, the reduction rules of the are given by the following definition:

[1] The stochastic semantics of the originally presented in [4] is different but can be seen as a particular case of the one proposed here.

Definition 6 (Reduction rules for the).

- *if* $Au\sigma, Bv\phi\lambda Au'\sigma', Bv'\phi'$ *is a reaction of* R *then for any* w_1*,* w_2*,* τ_1 *and* τ_2 *and for any edge name renaming* $\imath$ *one has the following transition:* $Au + w_1\sigma \circ \imath + \tau_1, Bv + w_2\phi \circ \imath + \tau_2\lambda Au' + w_1\sigma' \circ \imath + \tau_1, Bv' + w_2\phi' \circ \imath + \tau_2$*;*
- *If* $S\lambda T$ *then for any* R *one has* $R, S\lambda R, T$*;*
- *If* $S \equiv S' \wedge S'\lambda T' \wedge T' \equiv T$ *then* $S\lambda T$*.*

As for the , the stochastic transitions are obtained by first labeling each molecule of a solution with a unique identifier, then deriving tagged-transitions using the above rules and finally agregating groups of transitions leading to structurally congruent solutions.

4.2 is stricly more expressive than

Before presenting our first result we need to prove the following key-lemma:

Lemma 1. $\forall P \in$ *, if* P *can be written* $P = (\nu x : r)(Q|Q)$ *and if* $\exists P' \in$ *such that* $P\lambda P'$ *then* $\exists P'' \in$ *such that* $P'\lambda' P''$*.*

Proof. Such a transition $P\lambda P'$ corresponds to a set of tagged-transitions which are either internal to Q or are synchronizations between the two Qs.

If there is one transition of the first type, then it means that P' can be written $(\nu y : t)(Q'|Q)$, with $Q\lambda Q'$. Since P' contains another Q it can do another transition.

Otherwise there is one transition of the second type, and it means that Q contains an input and an output on a same channel, say x. Since we have only separate choices here, this implies that Q can be written $(\nu y : t)(R \,|\, xz.Q_1 + \Sigma \,|\, xw.Q_2 + \Sigma')$ where Σ and Σ' are the remaining part of the choices. Thus P' can be written $(\nu z : s)(R \,|\, Q_1 \,|\, xw.Q_2 + \Sigma' \,|\, R \,|\, xz.Q_1 + \Sigma \,|\, Q_2)$, from which a synchronization on the channel x is again possible.

We can now state and prove our first result:

Theorem 1. *There is no admissible encoding from to .*

Proof. First as recalled in remark 1, the without infinite rates can be encoded by in an admissible way. Let us suppose that it exists an admissible encoding from to , we can compose it with the one from to , to obtain an encoding from to . By proposition 1 it is again an admissible encoding. We now prove that no such encoding exists, which yields the theorem.

Suppose then that an admissible encoding . from to exists. Since in this case there is no instantaneous transitions, the items 3 and 4 of the admissibility definition can be rephrased into $P\lambda Q \Leftrightarrow P\lambda Q$. Consider the system where there are two species A and A' with no field and no site, and only one reaction: $A, A\lambda A', A'$. And consider the encoding of the solution $S = A, A$ and $S' = A', A'$. Since $S\lambda S'$, $S\lambda S'$. Since the encoding is uniform, S can be written $(\nu x : t)(Q|Q)$ where $Q = A$. So by the lemma 1 this implies that it exists a $S'\lambda' T$. But the solution S' cannot match this transitions since no reaction can be applied in it.

4.3 is striclty more expressive than

We carry on with the extension of the previous result to the π-calculi with infinite rates. Similarly to the previous part we start with a key lemma. It defers from the lemma 1 by the fact that it concerns instantaneous transitions and because it asks that P'' is also a symmetric process.

Lemma 2. *$\forall P \in$, if P can be written $P = (\nu x : r)(Q|Q)$ and if $\exists P' \in$ such that $P \infty P'$ then $\exists P'' \in$ such that $P' \infty P''$ and such that P'' can be written $P'' = (\nu y : s)(Q'|Q')$.*

Proof. Such a transition $P \infty P'$ corresponds either to an instantaneous transition internal to Q or to an instantenous synchronization between the two Qs.

In the first case, P' can be written $(\nu z : t)(Q'|Q)$, with $Q \infty Q'$. So $P' \infty (\nu z' : t')(Q'|Q')$.

In the second case, Q contains an input and an output on a same channel, say x, whose rate is infinite. Since we have only separate choices here, this implies that Q can be written $(\nu y : t)(R \,|\, xz.Q_1 + \Sigma \,|\, xw.Q_2 + \Sigma')$ where Σ and Σ' are the remaining part of the choices. Thus P' can be written $(\nu z : s)(R \,|\, Q_1 \,|\, xw.Q_2 + \Sigma' \,|\, R \,|\, xz.Q_1 + \Sigma \,|\, Q_2)$. And so $P' \infty (\nu z' : t')(R \,|\, Q_1 \,|\, Q_2 \,|\, R \,|\, Q_1 \,|\, Q_2)$. It is now sufficient to take $Q' = R \,|\, Q_1 \,|\, Q_2$.

Theorem 2. *There is no admissible encoding from to .*

Proof. First as recalled in remark 1, the with infinite rates can be encoded by in an admissible way. Let us suppose that it exists an admissible encoding from to , we can compose it with the one from to , to obtain an encoding from to . By proposition 1 it is again an admissible encoding. We now prove that no such encoding exists, which yields the theorem.

Suppose then that an admissible encoding . from to exists. Consider the system where there are three species A, A' and A'' with no field and no site, and two reactions: $A, A \infty A', A''$ and $A, A' \lambda A'', A'$. And consider the encoding of the solution $S = A, A$ and $S' = A', A''$. Since the encoding is uniform, S can be written $(\nu x : t)(Q|Q)$ where $Q = A$ and since $S \infty S'$, it exists R such that $S \infty R$. By the lemma 2 this implies that it exists a Q' such that $R \infty (\nu x' : t')(Q'|Q')$. If this new process can again do an instantaneous transition the lemma can be reapplied. In this way we build two sequences of processes $(R_i)_{i \in I}$ and $(Q_i)_{i \in I}$ such that $Q_0 = Q$, $R_0 = R$ and $\forall i \in I$ it exists a declaration of name δ_i such that $\delta_i(Q_i|Q_i) \infty R_i \infty \delta_{i+1}(Q_{i+1}|Q_{i+1})$. Because the solution S doesn't diverge and since admissible encodings preserve divergence the set I is finite. Let i_0 be its maximum. We have that $S \infty \delta_{i_0}(Q_{i_0}|Q_{i_0})$, so by the last item of the admissibility definition it exists a solution T such that $\delta_{i_0}(Q_{i_0}|Q_{i_0}) \infty T$ and $S \infty T$. By maximality of i_0, and because there is only one instantaneous transition from S, we obtain that $\delta_{i_0}(Q_{i_0}|Q_{i_0}) = A', A''$.

The room is missing to detail formally the last step of proof, we only sketch it. By tracking the residues of A along the sequences of transition one can prove that Q_{i_0} is equals to both A' and A''. In particular in the lemma 2 one can prove from $\delta_{i_0}(Q_{i_0}|Q_{i_0}) = A', A''$ that each Q' is the residue of a Q. We now have a contradiction because A' can perform a reaction which A'' cannot mimick, and thus their encodings must be different.

4.4 versus

We now consider the question of wether the addition of infinite rates to the separate choice allows to encode the mixed choice. Unfortunately we are not able to provide a full answer. However if we assume mixed choices to be proper (that is without input and output on the same channel) the encoding is possible. The following definition depicts it:

Definition 7. *Let . be the encoding from the proper mixed choice to defined on the proper mixed choice by:*

$$\begin{aligned} i \in I \Sigma x_i y_i.P_i + j \in J \Sigma x_j y_j.Q_j \;\; & (\nu z : \infty, z' : \infty) \\ & i \in I \Sigma x_i y_i.(P_i \mid z') + z \\ & | j \in J \Sigma x_j y_j.(Q_j \mid z) + z' \end{aligned} \tag{2}$$

and defined homomorphically on the other operators.

Intuitively the mixed choice is divided into two separate choices put in parallel. In order to ensure that after a synchronization, on the output $x_i y_i$ for instance, the input choice is disabled we develop a preemption mechanism. Once the output has been consumed the continuation P_i is put at top level as expected, but together with an output on the channel z. This output can synchronize with the corresponding input in the input choice, and since it has infinite rate, this synchronization has priority on any other transition.

This encoding doesn't work if the mixed choice has an input and an output on the same channel because it permits these actions to synchronyze. The following theorem states the correctness of this encoding.

Theorem 3. *For all terms P and Q of the proper mixed choice :*

$$P \lambda Q \Leftrightarrow P \lambda \infty \infty Q.$$

5 Generalization of the results to the case of the multi-scale π-calculus

In this section we introduce the *multi-scale* where the rates can be of different order of magnitude. Two main reasons motivate this proposition. Firstly biochemical systems often exhibit stochastic behaviours ranging over a very large scale of rates. It is common to assume some of them to be instantaneous or on the contrary to neglect some of them. The multi-scale is a first attempt to cope with such phenomena. Moreover the results of the previous section and in particular the result of part 4.4 suggest to introduce another degree of infinite rate.

Our proposition is to replace the strictly positive real number possibly infinite representing a rate by a pair (i, r) where i is a relative integer and r is a strictly positive real number. The order of magnitude of the rate is modeled by i, the higher i the higher the order of magnitude. A transition whose rate is (i, r) has priority on every transition of rate (j, s) such that $j < i$. Such priority are not unhearded of (see EMPA language for instance [1]) but as far as we know it is the first time it is used in terms of order of magnitude.

The stochastic semantics can easily be adapted to the settings of the multi-scale . The transitions are derived exactly in the same way, one just has to define the sum $(i, r) + (j, s)$ to be $(i, r + s)$ if $i = j$, (i, r) if $i > j$ and (j, s) if $i < j$. The so-called *race-condition* which determines the next transition to be executed according to the rates is made only between the transitions of higher order of magnitude.

We note n and n the mixed choice and separate choice fragments of the in which the degree of the rates is bounded by n. The extension of the results of the previous section to the multi-scale are presented in the following theorem:

Theorem 4. *For any relative integer i:*

1. *there is no admissible encoding from n to n;*
2. *there is no admissible encoding from* $\Omega to \Omega; there is an encoding . from the proper n to the n such that:$

3. $P \lambda Q$ if and only if $P \lambda Q$

Proof. 1. The result is proved similarly to the theorem 1.
2. Since the term of the multi-scale are finite so is the set of the order of magnitude appearing in each term. So the result is corollary of the previous item.
3. The encoding is obtained similarly to the one of the part 4.4.

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