

Comparing \Box and $!$ via polarities

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In [3], Martini and Masini exhibit a translation of the modal logic $S4$ into full propositional linear logic LL using the exponential $!$ (and $?$) in correspondance with the modality \Box (and \Diamond resp.). In combination with a forgetful interpretation of LL into $S4$, this allows them to show that any cut-free proof of an $S4$ sequent can be faithfully transformed into a cut-free proof of its LL translation, and therefore to conclude that "from a proof-theoretical point of view exponentials behave exactly like $S4$ modalities". The goal of this work is to extend the comparison of the exponentials and the modalities behaviour by studying the way they can be *polarised*.

Following from the study of their focusing behaviour [1], one can differentiate two classes of linear logic formulas, the *positive* and the *negative* ones, given by the grammar:

$$\begin{aligned} P &::= x \mid A \otimes A \mid 1 \mid A \oplus A \mid 0 \mid \exists x.A \mid !A \\ N &::= x^\perp \mid A \& A \mid \top \mid A \multimap A \mid \perp \mid \forall x.A \mid ?A \end{aligned}$$

where x is an atom, x^\perp denotes its dual, and A ranges over the usual grammar for linear logic formulas.

On the other hand, in the recent studies of the focusing behaviour of classical modal logic formulas [2, 4], the polarities of the modalities have been distinguished as positive for \Diamond and negative for \Box . In this way, they can be seen as inherited from the standard translation of modal logic formulas into classical first-order logic, where \Box is interpreted as a universal quantification (therefore negative) and \Diamond as an existential quantification (therefore positive), but not anymore as in correspondance with the linear logic exponentials.

In this talk, we investigate whether this connection can be retrieved by importing the focusing procedure for LL to the ordinary sequent calculus for $S4$ replacing exponentials by modalities, that is, a positive \Box and a negative \Diamond . Then we report on the issues that appear when trying, conversely, to transform the focused proof system for classical modal logic $S4$ proposed in [2] into a proof system for propositional linear logic based *a priori* on the use of a positive $?$ and a negative $!$.

References

- [1] Jean-Marc Andreoli. Logic programming with focusing proofs in linear logic. *Journal of Logic Programming*, 2(3):297–347, 1992.
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