# SPARSE: <br> Quadratic Time SA\&F of RNAs without Sequence-Based Heuristics 

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## Simultaneous Alignment and Folding [Sankoff]

Given: $\begin{aligned} \mathrm{A} & =\text { GCUGACGAGCACGCUCAUCGGUAAAUCUACCGAUCGUCAGCACU } \\ \& \quad B & =\text { AUUGCCGCUGACCGGCACGCCAUCGGAAUCCCGAUCGGGUCAGCGGCA }\end{aligned}$

Find:

sequence similarity + energy $A+$ energy $B \rightarrow$ opt
where alignment, structure $A, \&$ structure $B$ are COMpatible

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## Sankoff's Algorithm

## Dynamic Programming

## RNA Energy Minimization [Zuker]

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Sequence Alignment
$O\left(n^{6}\right)=$ "extreme computational cost"

## Sankoff-style Approaches <br> HEAVY <br> LIGHT

Dynalign
FoldAlign

- Sankoff implementations
- heavyweight energy model
- sequence-based heuristics


## PMcomp

- lightweight energy model
- base pair probabilities


## LocARNA

+ sparsifies structure space (ensemble-based)
- improves time and space


## RAF

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SPARSE

- strong sparsification w/o secuence-hased heuristics


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## Sankoff: sequence similarity + energies of $A$ and $B \quad \rightarrow$ opt

- energies composed of loop energies

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Base Pair Maximization [Nussinov] $\times$ Sequence Alignment

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Sankoff: same shape

## compatibility

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- lightweight (PMcomp pseudo-energy)
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complete (Sankoff's compatibility)
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We need "complete" for strong sparsification, please be patient.

## PARSE Algorithm



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- Sparsify structure ensemble

- improves time and space; each by $O\left(n^{2}\right)$


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## SPARSE: Novel Ensemble-based Sparsification*



- only base pairs with probabilities $>\theta_{1}$
- only bases with unpaired probabilities in loops $>\theta_{2}$ - only base pairs with probabilities in loops $>\theta_{3}$ requires complete prediction (Sankoff/PARSE)
(*) confer LocARNA's "old" sparsification:
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$a_{3}$ in loop $a_{2} \sqrt{ }$
but $a_{3}$ in loop $a_{1} X$
$a_{2} \boldsymbol{X} \Longrightarrow a_{3}-b_{2} \boldsymbol{X}$


## Thresholds in Recursions Cases



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## all base pairs $\theta_{1}$



## Modify Evaluation to Save Time



Quadratic Time


Q: How many matrices $M^{a b}$ compute $(i, k)$ ?


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A: each $(i, k)$ in only constant number of matrices

## Run-times and Speedup

| Tool | Sparsification |  |  | Mean Time | Speedup |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  | $\theta_{1}$ | $\theta_{2}$ | $\theta_{3}$ | per Instance | vs. LocARNA |
| LocARNA | $1 \mathrm{e}-3$ | - | - | 2.02 s | 1.0 |
| SPARSE | $1 \mathrm{e}-3$ | $1 \mathrm{e}-5$ | $1 \mathrm{e}-4$ | 0.92 s | 2.2 |
| RAF | $2 \mathrm{e}-3$ | - | - | 0.37 s | 5.5 |

Bralibase 2.1, pairwise alignments

## Alignment and Prediction Accuracy (Bralibase 2.1, 3-way alignments)



SPS: alignment quality


MCC: prediction quality

## Conclusions

SPARSE: very efficient RNA alignment without sequence-based heuristics

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## Thanks

...for your attention
... to my coauthors

- Christina Schmiedl
- Milad Miladi
- Mathias Möhl
- Rolf Backofen
... and the German Research Foundation $\boldsymbol{D} \boldsymbol{F}$

Appendix

## Computing "In Loop" Probabilities

from McCaskill matrices: $Q_{b}, Q_{m}$

similar: $\operatorname{Pr}[k$ unpaired in loop of $(\mathbf{i}, \mathbf{j})]$
[ExpARNA-P; Schmiedl et al., RECOMB 2012]

## SPARSE Improves Over LocARNA for Specific Families



(shown: IRES HCV, pairwise)

