

$$\begin{array}{c}
\overline{\langle \text{skip} \mid \sigma \rangle \longrightarrow \sigma} \text{ (SKIP)} \\
\\
\overline{\langle x := a \mid \sigma \rangle \longrightarrow \sigma[x \mapsto \llbracket a \rrbracket_{\sigma}^{\mathbf{Aexp}}]} \text{ (SET)} \\
\\
\frac{\langle c_1 \mid \sigma \rangle \longrightarrow \sigma' \quad \langle c_2 \mid \sigma' \rangle \longrightarrow \sigma''}{\langle c_1; c_2 \mid \sigma \rangle \longrightarrow \sigma''} \text{ (SEQ)} \\
\\
\frac{\langle c_1 \mid \sigma \rangle \longrightarrow \sigma'}{\langle \text{if } b \text{ then } c_1 \text{ else } c_2 \mid \sigma \rangle \longrightarrow \sigma'} \text{ (IF}_{\top}) \quad \text{si } \llbracket b \rrbracket_{\sigma}^{\mathbf{Bexp}} = \top \\
\\
\frac{\langle c_2 \mid \sigma \rangle \longrightarrow \sigma'}{\langle \text{if } b \text{ then } c_1 \text{ else } c_2 \mid \sigma \rangle \longrightarrow \sigma'} \text{ (IF}_{\perp}) \quad \text{si } \llbracket b \rrbracket_{\sigma}^{\mathbf{Bexp}} = \perp \\
\\
\frac{\langle c \mid \sigma \rangle \longrightarrow \sigma' \quad \langle \text{while } b \text{ do } c \mid \sigma' \rangle \longrightarrow \sigma''}{\langle \text{while } b \text{ do } c \mid \sigma \rangle \longrightarrow \sigma''} \text{ (WHILE}_{\top}) \quad \text{si } \llbracket b \rrbracket_{\sigma}^{\mathbf{Bexp}} = \top \\
\\
\overline{\langle \text{while } b \text{ do } c \mid \sigma \rangle \longrightarrow \sigma} \text{ (WHILE}_{\perp}) \quad \text{si } \llbracket b \rrbracket_{\sigma}^{\mathbf{Bexp}} = \perp
\end{array}$$

Figure 1: Règles de l'évaluation des commandes.

$$\begin{array}{c}
\overline{\langle x \mid \sigma \rangle \longrightarrow \langle \underline{\sigma(x)} \mid \sigma \rangle} \\
\\
\frac{\langle a_1 \mid \sigma \rangle \longrightarrow \langle a'_1 \mid \sigma' \rangle}{\langle a_1 + a_2 \mid \sigma \rangle \longrightarrow \langle a'_1 + a_2 \mid \sigma' \rangle} \quad \frac{\langle a_1 \mid \sigma \rangle \longrightarrow \langle a'_1 \mid \sigma' \rangle}{\langle a_1 * a_2 \mid \sigma \rangle \longrightarrow \langle a'_1 * a_2 \mid \sigma' \rangle} \\
\\
\frac{\langle a \mid \sigma \rangle \longrightarrow \langle a' \mid \sigma' \rangle}{\langle \underline{n} + a \mid \sigma \rangle \longrightarrow \langle \underline{n} + a' \mid \sigma' \rangle} \quad \frac{\langle a \mid \sigma \rangle \longrightarrow \langle a' \mid \sigma' \rangle}{\langle \underline{n} * a \mid \sigma \rangle \longrightarrow \langle \underline{n} * a' \mid \sigma' \rangle} \\
\\
\overline{\langle \underline{m} + \underline{n} \mid \sigma \rangle \longrightarrow \langle \underline{m+n} \mid \sigma \rangle} \quad \overline{\langle \underline{m} * \underline{n} \mid \sigma \rangle \longrightarrow \langle \underline{m \times n} \mid \sigma \rangle}
\end{array}$$

Figure 2: Règles de réduction des expressions arithmétiques.

$$\begin{array}{c}
\frac{\langle a_1 \mid \sigma \rangle \longrightarrow \langle a'_1 \mid \sigma' \rangle}{\langle a_1 < a_2 \mid \sigma \rangle \longrightarrow \langle a'_1 < a_2 \mid \sigma' \rangle} \\
\\
\frac{\langle a \mid \sigma \rangle \longrightarrow \langle a' \mid \sigma' \rangle}{\langle \underline{n} < a \mid \sigma \rangle \longrightarrow \langle \underline{n} < a' \mid \sigma' \rangle} \qquad \frac{\langle b \mid \sigma \rangle \longrightarrow \langle b' \mid \sigma' \rangle}{\langle \text{not } b \mid \sigma \rangle \longrightarrow \langle \text{not } b' \mid \sigma' \rangle} \\
\\
\frac{}{\langle \underline{m} < \underline{n} \mid \sigma \rangle \longrightarrow \langle \text{true} \mid \sigma \rangle} \quad \text{si } m < n \qquad \frac{}{\langle \text{not false} \mid \sigma \rangle \longrightarrow \langle \text{true} \mid \sigma \rangle} \\
\\
\frac{}{\langle \underline{m} < \underline{n} \mid \sigma \rangle \longrightarrow \langle \text{false} \mid \sigma \rangle} \quad \text{si } m \geq n \qquad \frac{}{\langle \text{not true} \mid \sigma \rangle \longrightarrow \langle \text{false} \mid \sigma \rangle}
\end{array}$$

Figure 3: Règles de réduction des expressions booléennes.

$$\begin{array}{c}
\frac{\langle a \mid \sigma \rangle \longrightarrow \langle a' \mid \sigma' \rangle}{\langle x := a \mid \sigma \rangle \longrightarrow \langle x := a' \mid \sigma' \rangle} \text{ (SET)} \\
\\
\frac{}{\langle x := \underline{n} \mid \sigma \rangle \longrightarrow \langle \text{skip} \mid \sigma[x \mapsto n] \rangle} \text{ (SET}_n\text{)} \\
\\
\frac{\langle c_1 \mid \sigma \rangle \longrightarrow \langle c'_1 \mid \sigma' \rangle}{\langle c_1; c_2 \mid \sigma \rangle \longrightarrow \langle c'_1; c_2 \mid \sigma' \rangle} \text{ (SEQ}_c\text{)} \qquad \frac{}{\langle \text{skip}; c \mid \sigma \rangle \longrightarrow \langle c \mid \sigma \rangle} \text{ (SEQ}_b\text{)} \\
\\
\frac{\langle b \mid \sigma \rangle \longrightarrow \langle b' \mid \sigma' \rangle}{\langle \text{if } b \text{ then } c_1 \text{ else } c_2 \mid \sigma \rangle \longrightarrow \langle \text{if } b' \text{ then } c_1 \text{ else } c_2 \mid \sigma' \rangle} \text{ (IF)} \\
\\
\frac{}{\langle \text{if true then } c_1 \text{ else } c_2 \mid \sigma \rangle \longrightarrow \langle c_1 \mid \sigma \rangle} \text{ (IF}_\top\text{)} \\
\\
\frac{}{\langle \text{if false then } c_1 \text{ else } c_2 \mid \sigma \rangle \longrightarrow \langle c_2 \mid \sigma \rangle} \text{ (IF}_\perp\text{)} \\
\\
\frac{}{\langle \text{while } b \text{ do } c \mid \sigma \rangle \longrightarrow \langle \text{if } b \text{ then } (c; \text{while } b \text{ do } c) \text{ else skip} \mid \sigma \rangle} \text{ (WHILE)}
\end{array}$$

Figure 4: Règles de réduction des commandes.

$$\begin{array}{c}
\frac{}{\Gamma \vdash \underline{n} : \mathbf{int}} \text{(INT)} \\
\frac{(x : A) \in \Gamma}{\Gamma \vdash x : A} \text{(VAR)} \\
\frac{\Gamma, x : A \vdash t : B}{\Gamma \vdash \mathbf{fun } x \rightarrow t : A \Rightarrow B} \text{(FUN)} \\
\frac{}{\Gamma \vdash \mathbf{fst} : A \times B \Rightarrow A} \text{(FST)} \\
\frac{\Gamma \vdash t : A \quad \Gamma \vdash u : B}{\Gamma \vdash (t, u) : A \times B} \text{(PAIR)}
\end{array}
\qquad
\begin{array}{c}
\frac{}{\Gamma \vdash \underline{b} : \mathbf{bool}} \text{(BOOL)} \\
\frac{}{\Gamma \vdash \mathbf{add} : \mathbf{int} \times \mathbf{int} \Rightarrow \mathbf{int}} \text{(ADD)} \\
\frac{\Gamma \vdash t : A \Rightarrow B \quad \Gamma \vdash u : A}{\Gamma \vdash t u : B} \text{(APP)} \\
\frac{}{\Gamma \vdash \mathbf{snd} : A \times B \Rightarrow B} \text{(SND)}
\end{array}$$

Figure 5: Règles de typage.

$$\begin{array}{c}
\frac{}{\underline{n} \longrightarrow \underline{n}} \text{(INT)} \\
\frac{}{\mathbf{add} \longrightarrow \mathbf{add}} \text{(ADD)} \\
\frac{}{(\mathbf{fun } x \rightarrow t) \longrightarrow (\mathbf{fun } x \rightarrow t)} \text{(FUN)} \\
\frac{t \longrightarrow t' \quad u \longrightarrow u'}{(t, u) \longrightarrow (t', u')} \text{(PAIR)} \\
\frac{}{\mathbf{fst} \longrightarrow \mathbf{fst}} \text{(FST)} \\
\frac{}{\mathbf{snd} \longrightarrow \mathbf{snd}} \text{(SND)}
\end{array}
\qquad
\begin{array}{c}
\frac{}{\underline{b} \longrightarrow \underline{b}} \text{(BOOL)} \\
\frac{t \longrightarrow \mathbf{add} \quad u \longrightarrow (m, n)}{t u \longrightarrow m + n} \text{(SUM)} \\
\frac{t \longrightarrow (\mathbf{fun } x \rightarrow t') \quad u \longrightarrow u' \quad t'[x \mapsto u'] \longrightarrow v}{t u \longrightarrow v} \text{(APP)} \\
\frac{t \longrightarrow \mathbf{fst} \quad u \longrightarrow (v_1, v_2)}{t u \longrightarrow v_1} \text{(FSTP)} \\
\frac{t \longrightarrow \mathbf{snd} \quad u \longrightarrow (v_1, v_2)}{t u \longrightarrow v_2} \text{(SNDP)}
\end{array}$$

Figure 6: Évaluation en mini-ML.

Pour v, v_1 et v_2 des valeurs :

$$\begin{array}{c}
\frac{t \longrightarrow t'}{t v \longrightarrow t' v} \text{ (APPL)} \qquad \frac{u \longrightarrow u'}{t u \longrightarrow t u'} \text{ (APPR)} \\
\\
\frac{}{(\mathbf{fun} \ x \ \rightarrow \ t) \ v \longrightarrow t[x \mapsto v]} \text{ (APP)} \qquad \frac{}{\mathbf{add} \ (\underline{m}, \underline{n}) \longrightarrow \underline{m + n}} \text{ (ADD)} \\
\\
\frac{t \longrightarrow t'}{(t, v) \longrightarrow (t', v)} \text{ (PAIRL)} \qquad \frac{u \longrightarrow u'}{(t, u) \longrightarrow (t, u')} \text{ (PAIRR)} \\
\\
\frac{}{\mathbf{fst} \ (v_1, v_2) \longrightarrow v_1} \text{ (FST)} \qquad \frac{}{\mathbf{snd} \ (v_1, v_2) \longrightarrow v_2} \text{ (SND)}
\end{array}$$

Figure 7: Réduction en mini-ML.

Typage:

$$\frac{}{\Gamma \vdash \mathbf{fix} : (A \Rightarrow A) \Rightarrow A} \text{ (FIX)}$$

Évaluation:

$$\frac{t \twoheadrightarrow \mathbf{fun} \ x \ \rightarrow \ t' \quad t'[x \mapsto \mathbf{fix} \ t'] \twoheadrightarrow v}{\mathbf{fix} \ t \twoheadrightarrow v}$$

Réduction:

$$\frac{t \longrightarrow t'}{\mathbf{fix} \ t \longrightarrow \mathbf{fix} \ t'} \qquad \frac{}{\mathbf{fix} \ (\mathbf{fun} \ x \ \rightarrow \ t) \longrightarrow t[x \mapsto \mathbf{fix} \ (\mathbf{fun} \ x \ \rightarrow \ t)]}$$

Figure 8: Point fixe en mini-ML.