The Structure of First-Order Causality

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The structure of logics

What is the **causality** induced by first-order connectives?

1. we introduce a game semantics
   (formula = game, proof = strategy)
2. we define a presentation of the category of games

Unifying points of view

Logics
invariants
of computation

Game Semantics
dynamics
of computation

Algebra
structure
of computation

First-order propositional logic

- Formulas:

$$A \quad ::= \quad \exists x.A \quad | \quad \forall x.A \quad | \quad A \wedge A \quad | \quad A \vee A \quad | \quad \ldots$$

First-order propositional logic

- Formulas:

$$A \quad ::= \quad \exists x.A \quad | \quad \forall x.A \quad | \quad A \wedge A \quad | \quad A \vee A \quad | \quad \ldots$$

- Rules:

$$\frac{\Gamma \vdash P, \Delta}{\Gamma \vdash \forall x.P, \Delta}(\forall) \qquad \frac{\Gamma \vdash P[t/x], \Delta}{\Gamma \vdash \exists x.P, \Delta}(\exists)$$

$$(\text{with } x \notin \mathrm{FV}(\Gamma, \Delta))$$

$$\frac{\Gamma \vdash A, \Delta \qquad \Gamma \vdash B, \Delta}{\Gamma \vdash A \wedge B, \Delta}(\wedge) \qquad \frac{\Gamma \vdash A, B, \Delta}{\Gamma \vdash A \vee B, \Delta}(\vee)$$

$$\vdots$$

Causality in proofs

$$\cfrac{\cfrac{\cfrac{\pi}{\Gamma \vdash A, B, \Delta}}{\Gamma \vdash A, \forall y.B, \Delta}(\forall)}{\Gamma \vdash \forall x.A, \forall y.B, \Delta}(\forall)$$

Causality in proofs

$$\dfrac{\dfrac{\dfrac{\pi}{\Gamma \vdash A, B, \Delta}}{\Gamma \vdash A, \forall y.B, \Delta}(\forall)}{\Gamma \vdash \forall x.A, \forall y.B, \Delta}(\forall) \quad \rightsquigarrow \quad \dfrac{\dfrac{\dfrac{\pi}{\Gamma \vdash A, B, \Delta}}{\Gamma \vdash \forall x.A, B, \Delta}(\forall)}{\Gamma \vdash \forall x.A, \forall y.B, \Delta}(\forall)$$

Causality in proofs

$$\dfrac{\dfrac{\dfrac{\pi}{\Gamma \vdash A[t/x], B[t'/y], \Delta}}{\Gamma \vdash A[t/x], \exists y.B, \Delta}(\exists)}{\Gamma \vdash \exists x.A, \exists y.B, \Delta}(\exists) \quad \rightsquigarrow \quad \dfrac{\dfrac{\dfrac{\pi}{\Gamma \vdash A[t/x], B[t'/y], \Delta}}{\Gamma \vdash \exists x.A, B[t'/y], \Delta}(\exists)}{\Gamma \vdash \exists x.A, \exists y.B, \Delta}(\exists)$$

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Causality in proofs

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\dfrac{\dfrac{\dfrac{\pi}{\Gamma \vdash A, B[\textcolor{red}{t}/y], \Delta}}{\Gamma \vdash A, \exists y.B, \Delta}(\exists)}{\Gamma \vdash \forall x.A, \exists y.B, \Delta}(\forall)
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$$

If $\textcolor{red}{x \notin \mathrm{FV}(t)}$!

Causality in proofs

Dependencies induced by proofs are of the form

$$\forall x \xrightarrow{\hspace{3cm}} \exists y$$

where the witness t given for y has x as free variable.

Games

Formulas

$$A \quad = \quad \exists x.A \quad | \quad \forall x.A \quad | \quad A \wedge A \quad | \quad A \vee A \quad | \quad \dots$$

will be interpreted as games $(M, \lambda, \leq)$:

- a set M of *moves*,
- a partial order $\leq$ on M called *causality*,
- a function $\lambda : M \to \{\forall, \exists\}$ indicating *polarity* ($\forall$: Opponent, $\exists$: Player)

Games

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$$\forall x.\forall y.(\forall z.P \vee \exists z'.Q)$$

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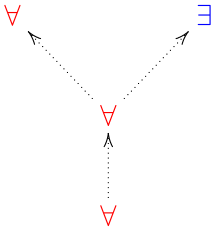
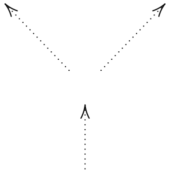


Strategies

$$\boxed{\text{strategy} \quad = \quad \text{dependency relation on the moves of the game}}$$

$$\vdash \forall x.\forall y.(\forall z.P \vee \exists z'.Q)$$

$\rightsquigarrow$



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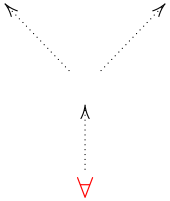

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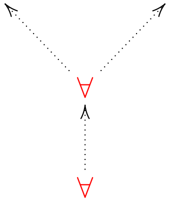

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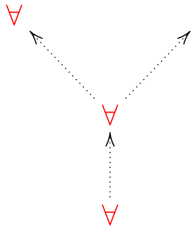

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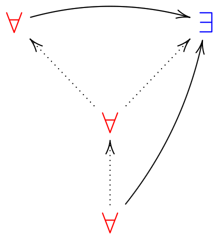

Free variables of t: $\{x, z\}$

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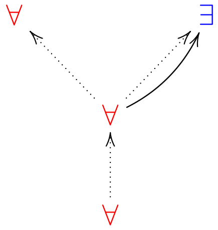

Free variables of t: $\{y\}$

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Free variables of t: $\emptyset$

Strategies

$$\begin{array}{rcl} \text{game } A & = & \text{partial order on the moves} \\ \text{strategy } \sigma & = & \text{relation on the moves} \end{array}$$

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A strategy $\sigma : A$ should moreover satisfy the following properties

1. Polarity: if $m \ \sigma \ n$ then m opponent and n player move
2. Acyclicity: the relation $\leq_A \ \cup \ \sigma$ is **acyclic**

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Forbids:

$$\begin{tikzcd} \forall \\ \exists \arrow[u, dotted] \arrow[u, bend right] \end{tikzcd}$$

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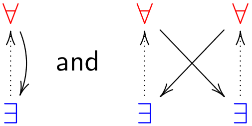

(similar to the correctness criterion of LL)

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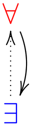
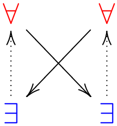
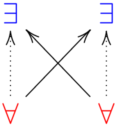


and


but not


(similar to the correctness criterion of LL)

A first step

We handle the case where connectives in formulas occur in leaves:

$$\forall x_1.\forall x_2.\exists x_3.\forall x_4.\forall x_5.\ \dots\ P(x_{i_1}, \dots, x_{i_k})$$

so games will be filiform (= total orders)

Interpreting proofs

A formula

$$A$$

is interpreted by a game

$$[\![A]\!]$$

Example

The formula

$$\forall x.\forall y.P$$

is interpreted by the game

$$\begin{tikzcd} \forall \\ \forall \arrow[u, dotted] \end{tikzcd}$$

Interpreting proofs

A sequent

$$A \vdash B$$

is interpreted by a game

$$[\![A]\!]^* \parr [\![B]\!]$$

Example

The sequent

$$\forall x.\forall y.P \vdash \forall z.P$$

is interpreted by the game

$$\begin{tikzcd} \exists & \\ \exists \arrow[u, dotted] & \forall \end{tikzcd}$$

Interpreting proofs

A proof

$$\dfrac{\vdots}{A \vdash B}$$

is interpreted by a strategy σ on the game

$$[\![A]\!]^* \parr [\![B]\!]$$

Example

The proof *is interpreted by the strategy*

$$\dfrac{\dfrac{\dfrac{\dfrac{}{z = z \vdash z = z}}{\forall y.z = y \vdash z = z}}{\forall x.\forall y.x = y \vdash z = z}}{\forall x.\forall y.x = y \vdash \forall z.z = z}$$

$$\begin{tikzcd}
\exists & \\
\exists \arrow[u, dotted] & \forall \arrow[l] \arrow[ul]
\end{tikzcd}$$

A monoidal category of games

We thus build a monoidal category **Games** whose

- objects A are filiform games
- morphisms $\sigma : A \to B$ are strategies on $A^* \parr B$

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Remark

It is not obvious that the acyclicity condition of strategies is preserved by composition.

So what?

This semantics is nice but

- why do strategies compose?
- what does it tell us about the structure of dependencies?
- are all the strategies definable (i.e. come from proofs)?

$\boxed{\text{We need algebraic tools!}}$

Presenting monoids

A finite description of a monoid can be given using a *presentation*:

$$M \quad \cong \quad \langle G \quad | \quad R \rangle$$

with

- G: *generators*
- $R \subseteq G^* \times G^*$: *relations*

meaning that

$$M \quad \cong \quad G^* / \equiv$$

Example

$$\mathbb{N} \times \mathbb{N} \quad \cong \quad \langle a, b \quad | \quad ba = ab \rangle$$

Presenting monoidal categories

Similarly, we can give presentations of monoidal categories using **polygraphs** [Street76, Power90, Burroni93].

We construct a polygraph presenting the category **Games**.

The simplicial category

The simplicial category Δ is the category whose

- objects are sets $[n] = \{0, 1, \ldots, n-1\}$ with $n \in \mathbb{N}$,
- morphisms are increasing functions

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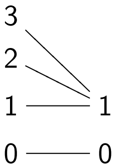

$: [4] \to [2]$

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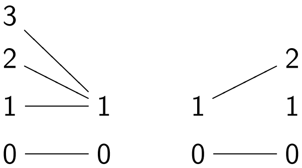
It is a category: horizontal composition ($\circ$)



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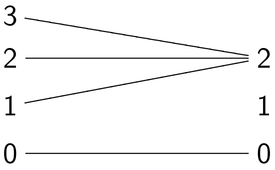

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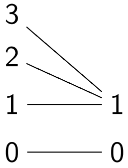
This category is monoidal: vertical composition ($\otimes$)



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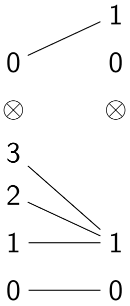

$: [1] \to [2]$

$: [4] \to [2]$

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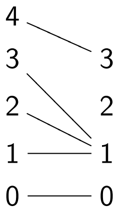
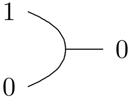


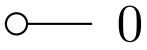
$: [5] \to [4]$

A theory of monoids

The category Δ contains two generating morphisms:

$$\mu : [2] \to [1] \qquad \text{and} \qquad \eta : [0] \to [1]$$





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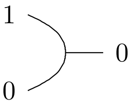
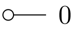
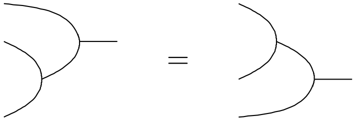
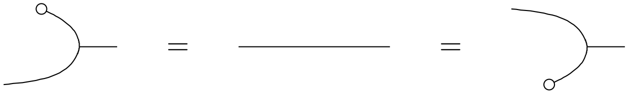




satisfying



and



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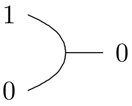

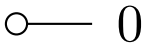

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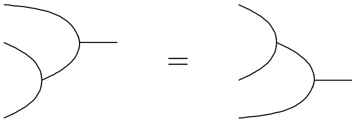


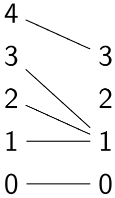
$$\mu \circ (\mathrm{id}_{[1]} \otimes \mu) \quad = \quad \mu \circ (\mu \otimes \mathrm{id}_{[1]})$$

and

$$\mu \circ (\eta \otimes \mathrm{id}_{[1]}) \quad = \quad \mathrm{id}_{[1]} \quad = \quad \mu \circ (\mathrm{id}_{[1]} \otimes \eta)$$

A theory of monoids

$\boxed{\mu \text{ and } \eta \text{ generate } \Delta}$



A theory of monoids

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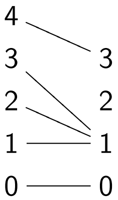
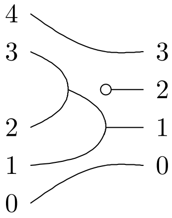


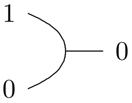
$\rightsquigarrow$

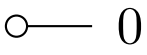


A presentation of the category Δ

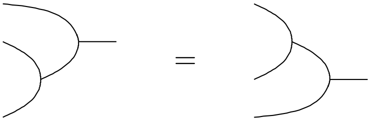
The category Δ is monoidally isomorphic to the free monoidal category on the two generators

$$\mu : [2] \to [1] \qquad \text{and} \qquad \eta : [0] \to [1]$$

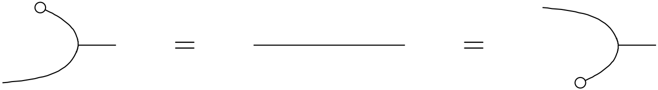




quotiented by the relations



and



The game theory

strict monoidal functor $\Delta \to \mathcal{C}$
$$=$$
monoid in $\mathcal{C}$

$$\mathbf{Mon}(\mathcal{C}) \quad \cong \quad \mathbf{StrMonCat}(\Delta, \mathcal{C})$$

The game theory

strict monoidal functor **Games** $\rightarrow \mathcal{C}$

$=$

?????

The game theory

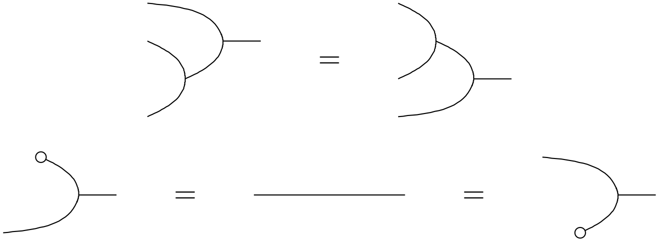
$$\boxed{\begin{array}{c}\text{strict monoidal functor } \mathbf{Games} \to \mathcal{C} \\ = \\ ?????\end{array}}$$

The corresponding theory is a polarized variant of *bicommutative bialgebras*

The simplicial category Δ: increasing functions.

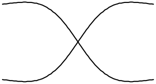
- Generators:

- Relations:

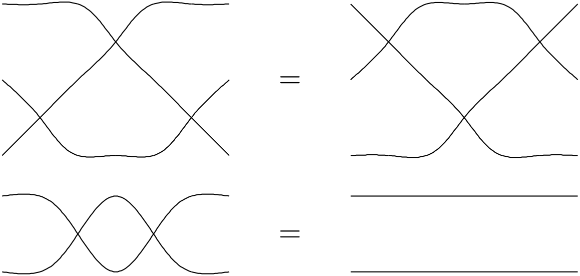


The category **Bij**: bijections.

- Generators:

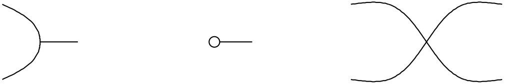


- Relations:

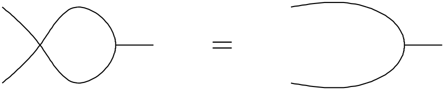


The category **F**: functions.

- Generators:



- Relations: monoid + symmetry +



...

The category $\mathbf{F}^{\mathrm{op}}$: "cofunctions".

- Generators:

- Relations:

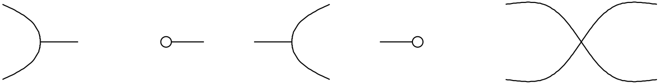
$$\ldots$$

Presentations

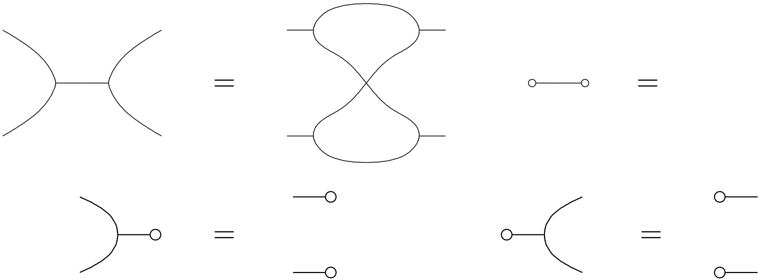
The theory of bicommutative bialgebras

The category **Mat**($\mathbb{N}$): $\mathbb{N}$-valued matrices.

- Generators:

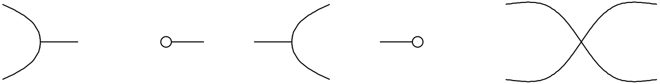


- Relations: commutative monoid + commutative comonoid +



The category **FRel**: relations

- Generators:



- Relations: bicommutative bialgebra which is *qualitative*:

The category **Games**

The category **Games** is the category whose

- objects are integers

$$[n] = \{0, 1, 2, \ldots, n-1\}$$

together with a polarization function

$$\lambda : [n] \longrightarrow \{\exists, \forall\}$$

$$\begin{tikzcd}
3 \\
2 \arrow[u, dotted] \\
1 \arrow[u, dotted] \\
0 \arrow[u, dotted]
\end{tikzcd}$$

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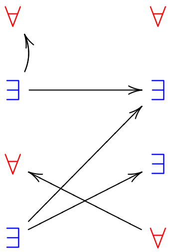
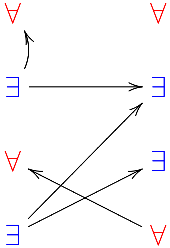
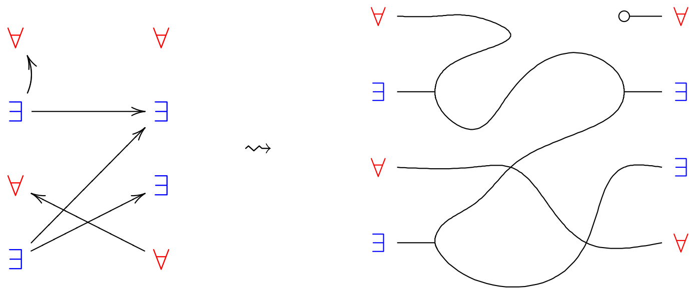
- morphisms are strategies.



The structure of wires



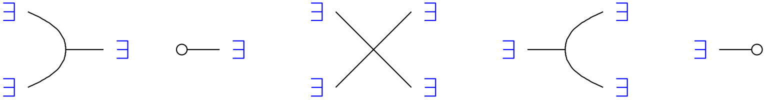
The structure of wires



The presentation of **Games**

Two objects $\exists$ and $\forall$ with

- five generators

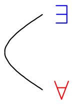


inducing a structure of *qualitative bicommutative bialgebra*,

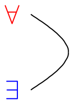
The presentation of **Games**

Two objects $\exists$ and $\forall$ with

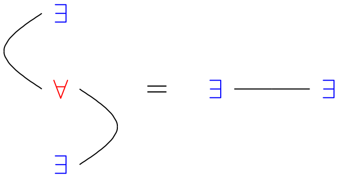
- five generators
- a duality $\exists \dashv \forall$:



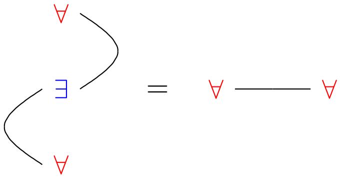
and



such that



and



(the axioms for adjunctions)

The theory **Games**

That's it!

strict monoidal functor **Games** $\to \mathcal{C}$

$$=$$

dual pair of bicommutative qualitative bialgebras

$$\mathbf{Games}(\mathcal{C}) \quad \cong \quad \mathbf{StrMonCat}(\mathbf{Games}, \mathcal{C})$$

Technical byproducts

From this presentation we deduce that

- strategies do **compose**
  (the acyclicity condition is preserved by composition)
- strategies are **definable**
  (i.e. are the interpretations of proofs)

Abstract methodology

We have replaced an *external* definition of the category **Games**:

by an *internal* definition:

Abstract methodology

We have replaced an *external* definition of the category **Games**:

- category of relations which satisfy conditions (polarity + acyclicity)

by an *internal* definition:

- presentation of the category

Abstract methodology

We have replaced an *external* definition of the category **Games**:

- category of relations which satisfy conditions (polarity + acyclicity)
- **restricting**

by an *internal* definition:

- presentation of the category
- **generating**

Abstract methodology

We have replaced an *external* definition of the category **Games**:

- category of relations which satisfy conditions
  (polarity + acyclicity)
- **restricting**
- global correctness

by an *internal* definition:

- presentation of the category
- **generating**
- local correctness

About proofs

To show these results, we have used a technique elaborated by Burroni and Lafont:

- working on terms modulo a congruence:
  rewriting to canonical forms

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About proofs

To show these results, we have used a technique elaborated by Burroni and Lafont:

- working on terms modulo a congruence:
  rewriting to canonical forms
- very systematic and involves considering lots of cases...
- ...which is good news: *mechanization*

Next steps

- extend to formulas with connectives
- internal formulation of the correctness criterion of linear logic?
- tools for computer assisted semantic analysis of programs
- . . .

Thanks!

Any question?