Asynchronous innocence

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Game theoretical semantics

- We study logic from a dynamic point of view: **interaction**
- We try to **recover the syntax from the semantics**
- We want our model to be
  - **general**
  - **concurrent**
  - **natural** / **elegant**
- *Disclaimer: this is a work in progress*

Main features

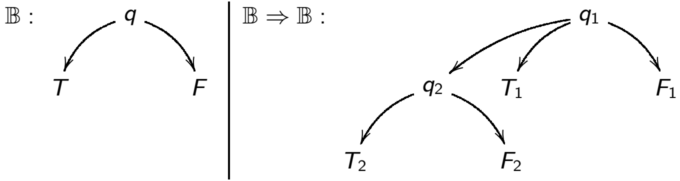
- Games played on graphs (Mazurkiewicz traces)
- A diagrammatic characterization of innocence
- A *positional* characterization of innocent strategies
- A general model of interaction

Road map

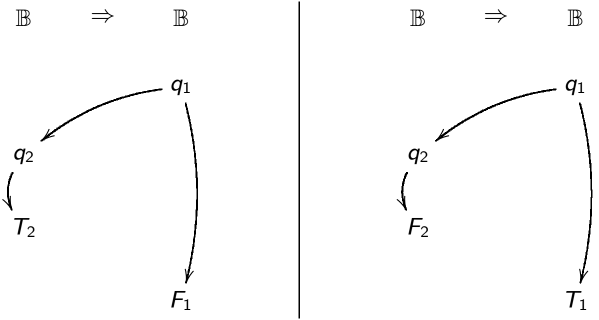
1. Pointer games
2. Asynchronous games + sequential strategies
3. Asynchronous games + concurrent strategies

Usual (pointer) games

- A *game* is played on an arena

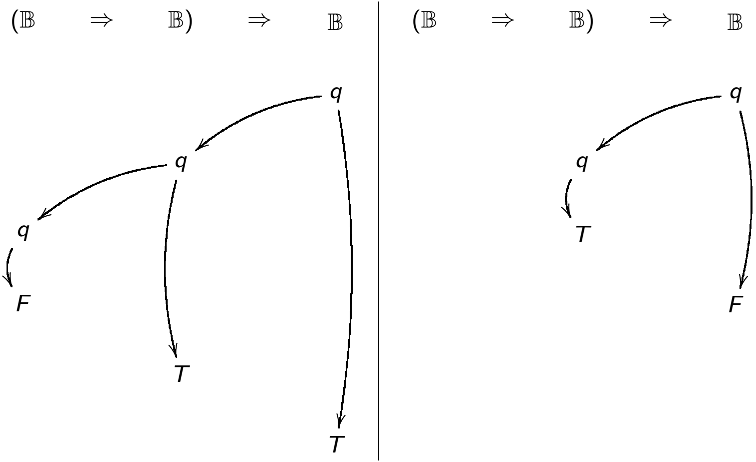


- A *strategy* is a set of plays



Innocence

- Playing with the rear-view mirror.



Towards a more general framework
Games on event structures

- Games are now played event structures: $(M, \preceq)$

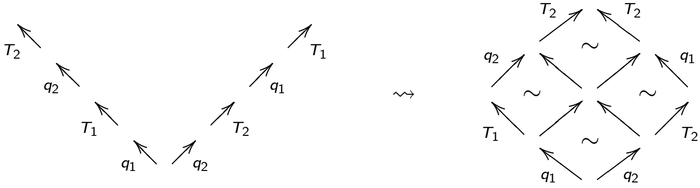
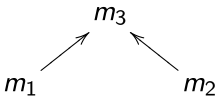


Figure: $\mathbb{B} \otimes \mathbb{B}$: trees vs dags

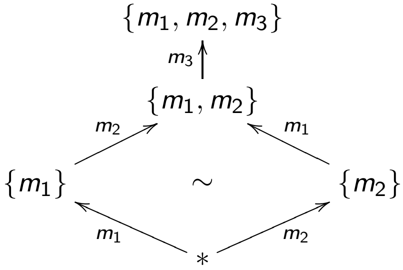
- $+$ a polarization function: $\lambda : M \to \{-1, +1\}$

Towards a more general framework
Positions

- Position: finite compatible downward closed subset of M
- Positional graph $\mathcal{G}(M)$:
  - positions: x, y, ... of M
  - arrows: $x \xrightarrow{m} x + \{m\}$

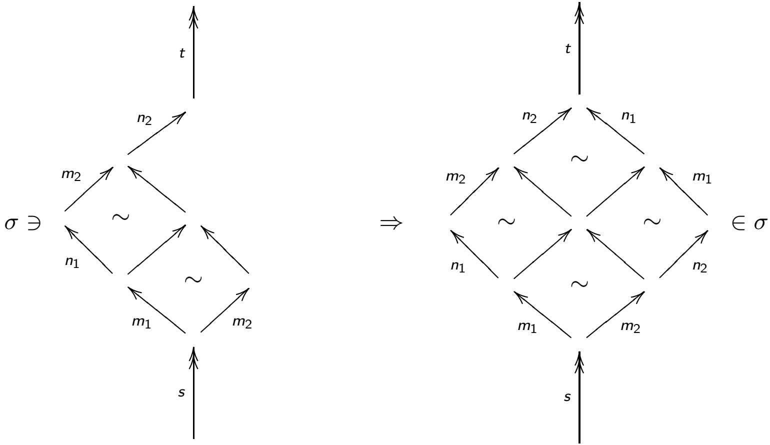


$(M, \preceq)$

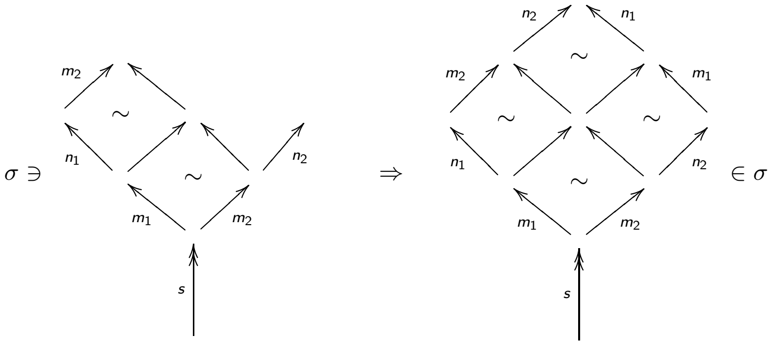


$\mathcal{G}(M)$

Diagrammatic innocence: backward consistency



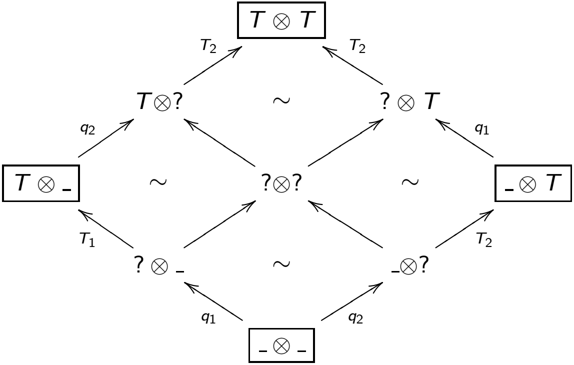
Diagrammatic innocence: forward consistency



Positionality

A strategy is positional when for every two plays $s_1, s_2 : * \twoheadrightarrow x$

$$s_1 \in \sigma \text{ and } s_2 \in \sigma \text{ and } s_1 \cdot t \in \sigma \quad \Rightarrow \quad s_2 \cdot t \in \sigma$$



Innocent strategies are positional

- A strategy σ is characterized by σ°
- Composition can be seen as a relational composition on positions:

$$(\sigma;\tau)^\circ = \sigma^\circ;\tau^\circ$$

  ($\rightarrow$ monoïdal functor to *Rel*)

Our work

- We now play on asynchronous graphs (instead of event structures).
- No more O/P alternation.
- We seek connections with existing models.

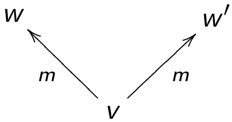
Asynchronous graphs

Let M be a set of *moves*.

Definition

An *asynchronous graph* is a graph whose edges are labeled by moves and which satisfies:

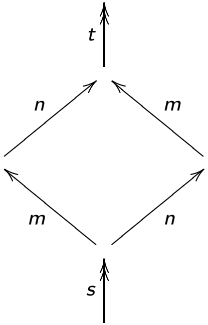
- *linearity*: at most one occurrence of a move in a path
- *determinism*:



$\Rightarrow w = w'$

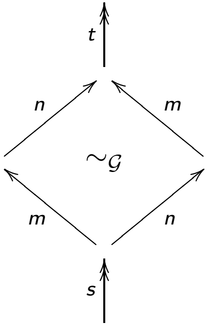
(and the dual)

Towards true concurrency: homotopy



Towards true concurrency: homotopy

What is really meaningful is not the precise order of the moves but what moves were played in the path.



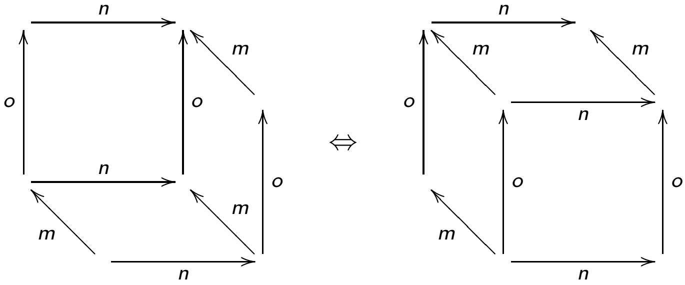
- We will work with $[\mathcal{G}]$ which is the free category generated by $\mathcal{G}$ whose arrows are quotiented by $\sim_{\mathcal{G}}$.

A diagrammatical characterization of innocence

Definition

An asynchronous graph is called *innocent* when it is:

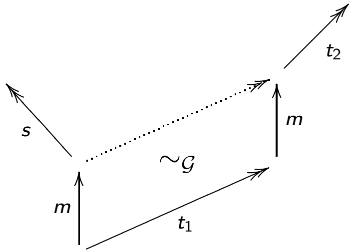
- *stable* / *costable* (cube property):



Residual techniques

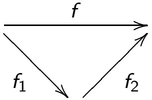
- Arrows in $[\mathcal{G}]$ are epi and mono

$$h_1 \cdot f \cdot h_2 \sim_{\mathcal{G}} h_1 \cdot g \cdot h_2 \Rightarrow f \sim_{\mathcal{G}} g$$

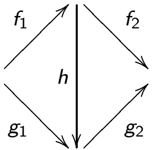


The lattice of factorizations of a path

- A factorization of a path:



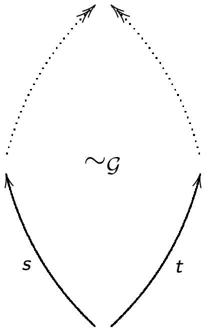
- The catergory of factorizations:



- Ordering factorizations: $(f_1, f_2) \preceq (g_1, g_2)$ iff f_1 is a prefix of g_1 modulo $\sim_{\mathcal{G}}$
- Property: $f \preceq g$ iff every move in f is also a move in g
- Property: the factorizations of a path is a distributive lattice

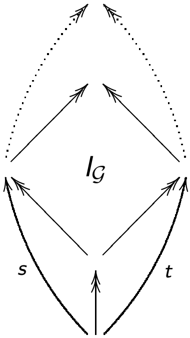
The lattice of factorizations of a path

- Intersection of two paths



The lattice of factorizations of a path

- Intersection of two paths



- $\Rightarrow$ structure of *distributive lattice* of the prefixes of a paths (ordered by inclusion of the set of moves)

Strategies

Take one distinguished vertex $*$ in a costable asynchronous graph.

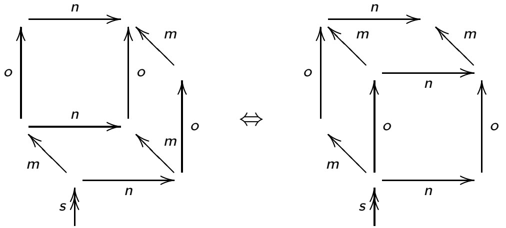
Definition

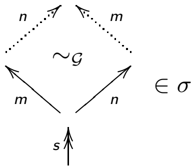
A *strategy* is a set of paths starting from $*$, closed under prefix.

Definition

An *innocent strategy* is a strategy which is:

- *deterministic*
- *stable / costable*
- *closed under local union*



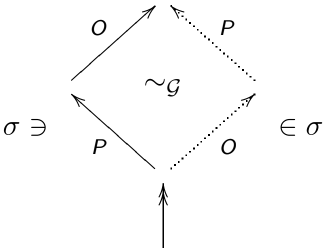


Positionality

- An innocent strategy σ is a subgraph of $\mathcal{G}$ which is innocent.

Polarizing games!

- Now, we polarize the moves: $\lambda : M \to \{-1, +1\}$
- A *game*: $A = (M_A, \mathcal{G}_A, \lambda_A)$
- Some more conditions are now required to hold for innocent strategies



$\approx$ every O-move points to the preceding move in a view

Complete positions

- A position is said to be *complete* when no more player move can be played.
- σ°: complete positions

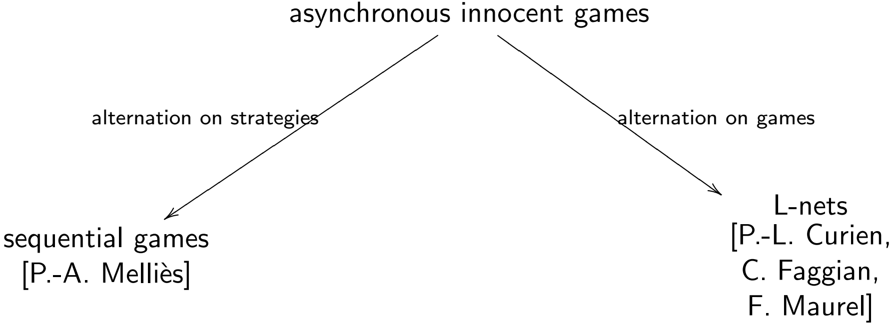
Positionality

- An innocent strategy σ is characterized by its complete positions σ°
- If we add a payoff condition on strategies, we then have

$$(\sigma; \tau)^\circ = \sigma^\circ; \tau^\circ$$

  ($\approx$ acyclicity criterion on nets?)
- And now, two innocent strategies compose

Recovering other game models



What's next

- Characterizing the usual game models in our framework: sequential games, L-nets, ...
- Full completeness
- Pinch holes in the homotopy to have models of concurrent languages: CCS, π-calculus, ...