Algebraic Tools in Game Semantics

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Game Semantics

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$\Leftrightarrow$

there exists a *winning strategy*

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$$\forall x. \quad (x \neq 0 \quad \Rightarrow \quad (\exists y. \quad x = y + 1))$$

Proofs, programs and strategies

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should rather be: *what are the proofs of the formula?*

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So, the question: *is the following formula provable?*

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should rather be: *what are the proofs of the formula?*

What are the winning strategies?

Denotational Semantics

Proofs (or programs) are naturally organized in **categories**

- objects: formulas $A, B, \ldots$
- morphisms: $\pi : A \to B$ are proofs $\pi : A \Rightarrow B$
- composition $\rho \circ \pi : A \to C$ of $\pi : A \to B$ and $\rho : B \to C$:

$$\dfrac{\dfrac{\pi}{A \vdash B} \qquad \dfrac{\rho}{B \vdash C}}{A \vdash C}\,(\mathsf{Cut})$$

- identities $\mathrm{id}_A : A \to A$:

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Similarly, one can often build categories whose objects are game and morphisms are strategies.

Denotational Semantics

A *game semantics* is given by a functor

$$F \quad : \quad \textbf{Proofs} \quad \to \quad \textbf{Games}$$

i.e.

- a formula A is interpreted by a game $F(A)$,
- a proof $\pi : A \to B$ is interpreted as a strategy $F(\pi) : F(A) \to F(B)$

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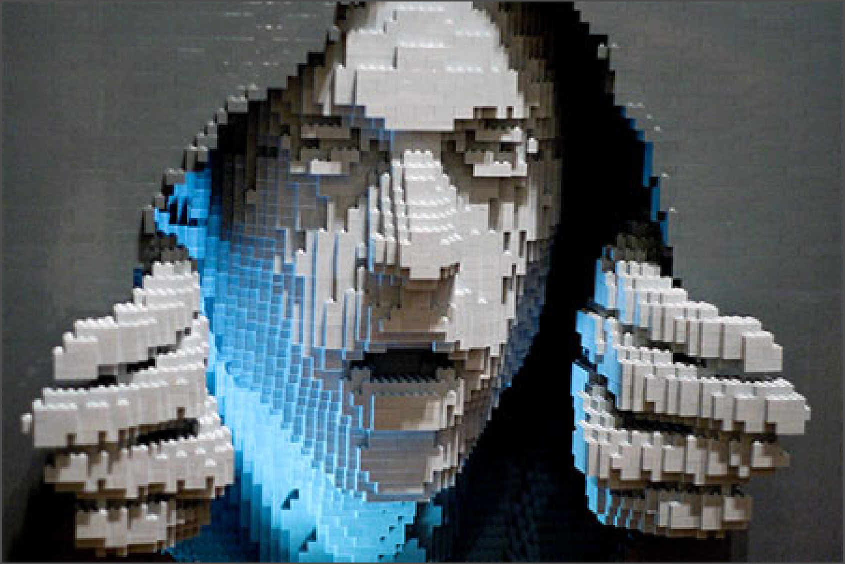
A semantics is *fully complete* when the functor is surjective.

The structure of logics

What is the **causality** induced by first-order connectives?

1. we introduce a game semantics
   (formula = game, proof = strategy)
2. we define a presentation of the category of games



Unifying points of view

Logics
invariants of computation

Game Semantics
dynamics of computation

Algebra
structure of computation

First-order propositional logic

- Formulas:

$$A \quad ::= \quad \exists x.A \quad | \quad \forall x.A \quad | \quad A \wedge A \quad | \quad A \vee A \quad | \quad \ldots$$

First-order propositional logic

- Formulas:

$$A \quad ::= \quad \exists x.A \quad | \quad \forall x.A \quad | \quad A \wedge A \quad | \quad A \vee A \quad | \quad \dots$$

- Rules:

$$\frac{\Gamma \vdash P, \Delta}{\Gamma \vdash \forall x.P, \Delta}(\forall) \qquad \frac{\Gamma \vdash P[t/x], \Delta}{\Gamma \vdash \exists x.P, \Delta}(\exists)$$

(with $x \notin \mathrm{FV}(\Gamma, \Delta)$)

$$\frac{\Gamma \vdash A, \Delta \qquad \Gamma \vdash B, \Delta}{\Gamma \vdash A \wedge B, \Delta}(\wedge) \qquad \frac{\Gamma \vdash A, B, \Delta}{\Gamma \vdash A \vee B, \Delta}(\vee)$$

$$\vdots$$

Causality in proofs

$$\dfrac{\dfrac{\dfrac{\pi}{\Gamma \vdash A, B, \Delta}}{\Gamma \vdash A, \forall y.B, \Delta}(\forall)}{\Gamma \vdash \forall x.A, \forall y.B, \Delta}(\forall)$$

Causality in proofs

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If $x \notin \mathrm{FV}(t)$!

Causality in proofs

Dependencies induced by proofs are of the form

$$\forall x \longrightarrow \exists y$$

where the witness t given for y has x as free variable.

Formulas

$$A \quad = \quad \exists x.A \quad | \quad \forall x.A \quad | \quad A \wedge A \quad | \quad A \vee A \quad | \quad \dots$$

will be interpreted as games $(M, \lambda, \leq)$:

- a set M of *moves*,
- a partial order $\leq$ on M called *causality*,
- a function $\lambda : M \to \{\forall, \exists\}$ indicating *polarity* ($\forall$: Opponent, $\exists$: Player)

Formulas

$$A \quad = \quad \exists x.A \quad | \quad \forall x.A \quad | \quad A \wedge A \quad | \quad A \vee A \quad | \quad \ldots$$

$$\forall x.\forall y.(\forall z.P \vee \exists z'.Q)$$

Games

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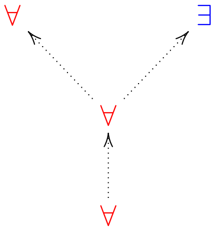
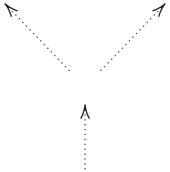


Strategies

strategy	$=$	dependency relation on the moves of the game

$$\vdash \forall x.\forall y.(\forall z.P \vee \exists z'.Q)$$

$\rightsquigarrow$



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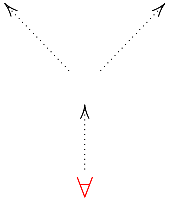

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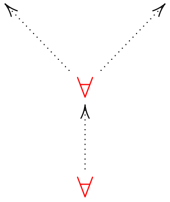

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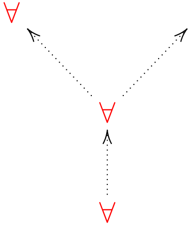

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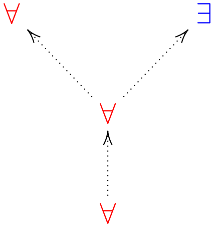

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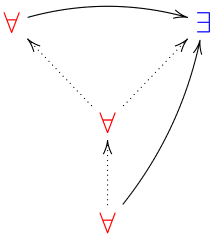

Free variables of t: $\{x, z\}$

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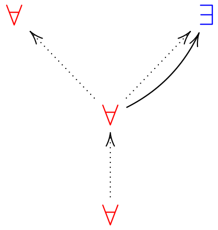

Free variables of t: $\{y\}$

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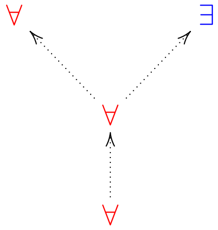

Free variables of t: $\emptyset$

Strategies

$$\begin{aligned} \text{game } A &= \text{partial order on the moves} \\ \text{strategy } \sigma &= \text{relation on the moves} \end{aligned}$$

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A strategy $\sigma : A$ should moreover satisfy the following properties

1. Polarity: if $m \ \sigma \ n$ then m opponent and n player move
2. Acyclicity: the relation $\leq_A \cup \sigma$ is **acyclic**

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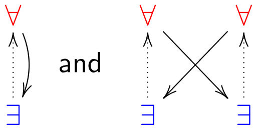

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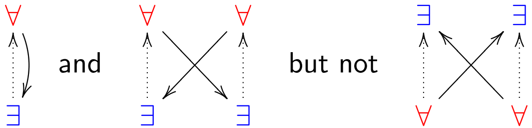
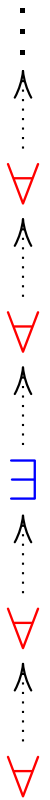


A first step

We handle the case where connectives in formulas occur in leaves:

$$\forall x_1.\forall x_2.\exists x_3.\forall x_4.\forall x_5.\ \ldots\ P(x_{i_1},\ldots,x_{i_k})$$

so games will be filiform (= total orders)



Interpreting proofs

A formula

$$A$$

is interpreted by a game

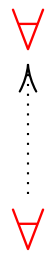
$$[\![A]\!]$$

Example

The formula

$$\forall x.\forall y.P$$

is interpreted by the game



Interpreting proofs

A sequent

$$A \vdash B$$

is interpreted by a game

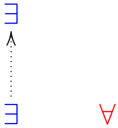
$$\llbracket A \rrbracket^* \parr \llbracket B \rrbracket$$

Example

The sequent

$$\forall x.\forall y.P \vdash \forall z.P$$

is interpreted by the game



Interpreting proofs

A proof

$$\dfrac{\vdots}{A \vdash B}$$

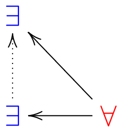
is interpreted by a strategy σ on the game

$$[\![A]\!]^* \parr [\![B]\!]$$

Example

The proof *is interpreted by the strategy*

$$\dfrac{\dfrac{\dfrac{\dfrac{}{z = z \vdash z = z}}{\forall y.z = y \vdash z = z}}{\forall x.\forall y.x = y \vdash z = z}}{\forall x.\forall y.x = y \vdash \forall z.z = z}$$



A monoidal category of games

We thus build a monoidal category **Games** whose

- objects A are filiform games
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Remark

It is not obvious that the acyclicity condition of strategies is preserved by composition.

So what?

This semantics is nice but

- why do strategies compose?
- what does it tell us about the structure of dependencies?
- are all the strategies definable (i.e. come from proofs)?

We need algebraic tools!

Presenting monoids

A finite description of a monoid can be given using a *presentation*:

$$M \quad \cong \quad \langle G \quad | \quad R \rangle$$

with

- G: *generators*
- $R \subseteq G^* \times G^*$: *relations*

meaning that

$$M \quad \cong \quad G^* / \equiv$$

Example

$$\mathbb{N} \times \mathbb{N} \quad \cong \quad \langle a, b \quad | \quad ba = ab \rangle$$

Presenting monoidal categories

Similarly, we can give presentations of monoidal categories using **polygraphs** [Street76, Power90, Burroni93].

We construct a polygraph presenting the category **Games**.

The simplicial category

The simplicial category Δ is the category whose

- objects are sets $[n] = \{0, 1, \ldots, n-1\}$ with $n \in \mathbb{N}$,
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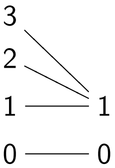

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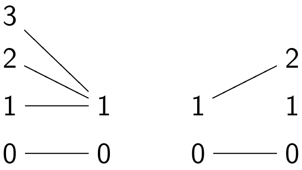

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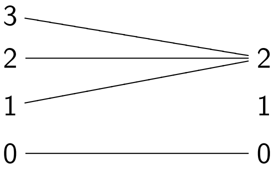

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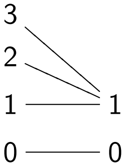
This category is monoidal: vertical composition ($\otimes$)



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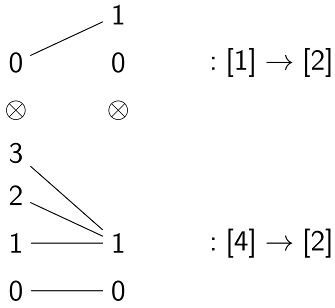

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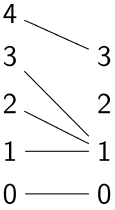
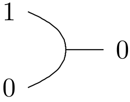


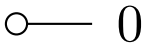
$: [5] \to [4]$

A theory of monoids

The category Δ contains two generating morphisms:

$$\mu : [2] \to [1] \qquad \text{and} \qquad \eta : [0] \to [1]$$





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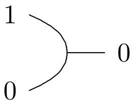
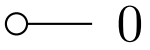
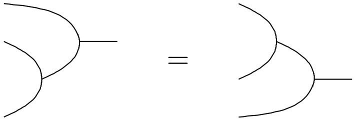
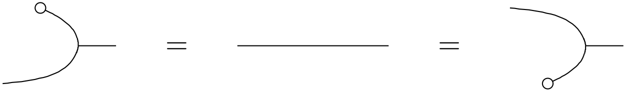




satisfying



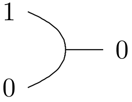
and

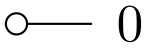


A theory of monoids

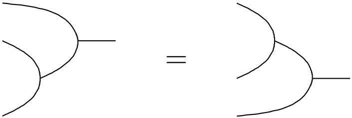
The category Δ contains two generating morphisms:

$$\mu : [2] \to [1] \qquad \text{and} \qquad \eta : [0] \to [1]$$



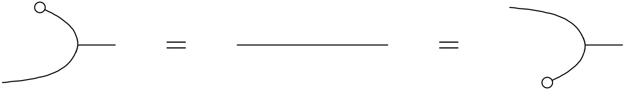


satisfying



$$\mu \circ (\mathrm{id}_{[1]} \otimes \mu) \quad = \quad \mu \circ (\mu \otimes \mathrm{id}_{[1]})$$

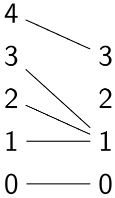
and



$$\mu \circ (\eta \otimes \mathrm{id}_{[1]}) \quad = \quad \mathrm{id}_{[1]} \quad = \quad \mu \circ (\mathrm{id}_{[1]} \otimes \eta)$$

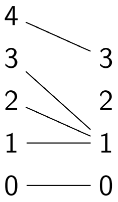
A theory of monoids

μ and η generate Δ

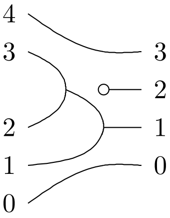


A theory of monoids

μ and η generate Δ



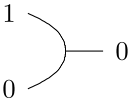
$\rightsquigarrow$

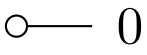


A presentation of the category Δ

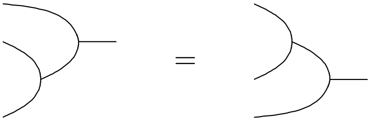
The category Δ is monoidally isomorphic to the free monoidal category on the two generators

$$\mu : [2] \to [1] \qquad \text{and} \qquad \eta : [0] \to [1]$$

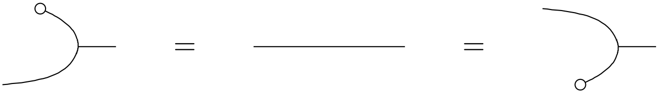




quotiented by the relations



and



The game theory

strict monoidal functor $\Delta \to \mathcal{C}$
$$=$$
monoid in $\mathcal{C}$

$$\mathbf{Mon}(\mathcal{C}) \quad \cong \quad \mathbf{StrMonCat}(\Delta, \mathcal{C})$$

The game theory

strict monoidal functor **Games** $\to \mathcal{C}$

$=$

?????

The game theory

strict monoidal functor **Games** $\to \mathcal{C}$

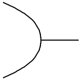
$=$

?????

The corresponding theory is a polarized variant of *bicommutative bialgebras*

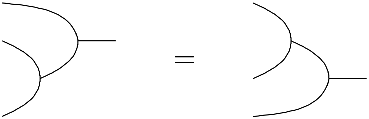
The simplicial category Δ: increasing functions.

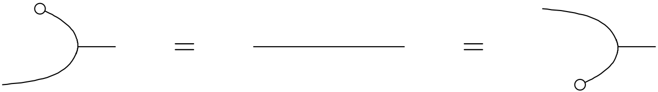
- Generators:



and

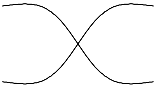
- Relations:



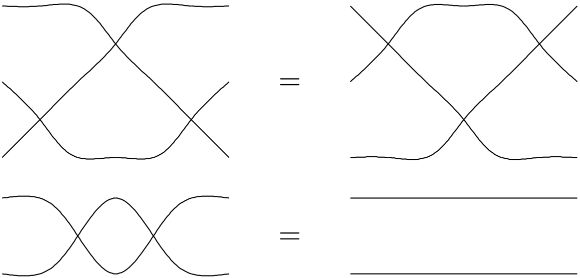


The category **Bij**: bijections.

- Generators:

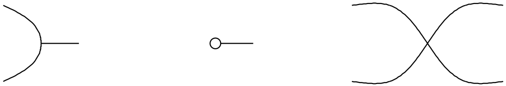


- Relations:

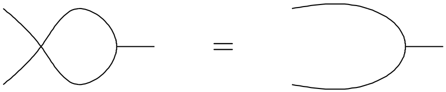


The category **F**: functions.

- Generators:



- Relations: monoid + symmetry +



...

The category $\mathbf{F}^{\mathrm{op}}$: "cofunctions".

- Generators:

- Relations:

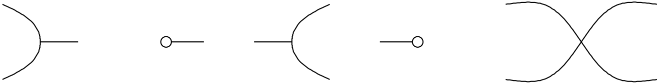
$\cdots$

Presentations

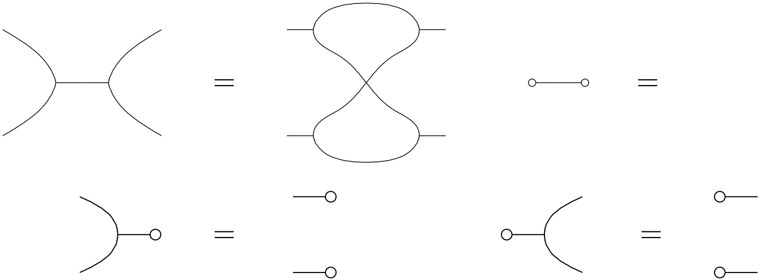
The theory of bicommutative bialgebras

The category **Mat**($\mathbb{N}$): $\mathbb{N}$-valued matrices.

- Generators:

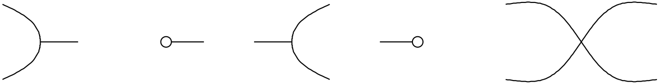


- Relations: commutative monoid + commutative comonoid +



The category **FRel**: relations

- Generators:



- Relations: bicommutative bialgebra which is *qualitative*:

The category **Games**

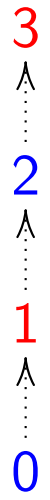
The category **Games** is the category whose

- objects are integers

$$[n] = \{0, 1, 2, \ldots, n-1\}$$

  together with a polarization function

$$\lambda : [n] \to \{\exists, \forall\}$$



The category **Games**

The category **Games** is the category whose

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$$\lambda : [n] \to \{\exists, \forall\}$$

$\forall$

$\exists$

$\forall$

$\exists$

The category **Games** is the category whose

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  together with a polarization function

$$\lambda : [n] \to \{\exists, \forall\}$$

$\forall$	$\forall$
$\exists$	$\exists$
$\forall$	$\exists$
$\exists$	$\forall$

The category **Games**

The category **Games** is the category whose

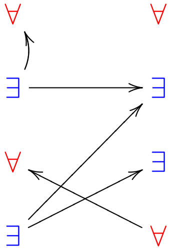
- objects are integers

$$[n] = \{0, 1, 2, \ldots, n-1\}$$

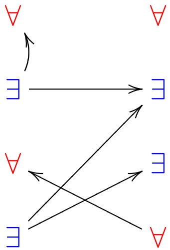
  together with a polarization function

$$\lambda : [n] \to \{\exists, \forall\}$$

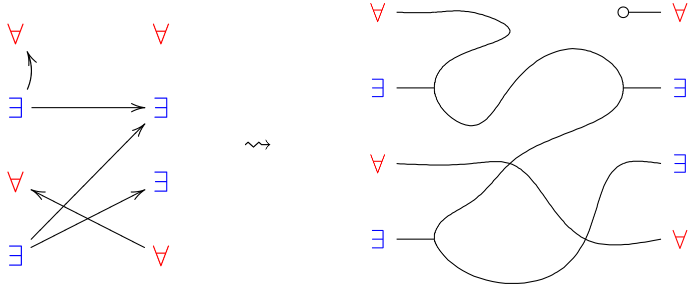
- morphisms are strategies.



The structure of wires



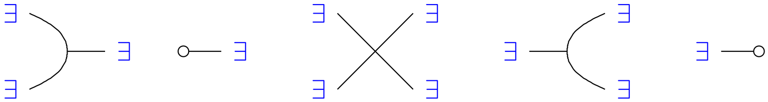
The structure of wires



The presentation of **Games**

Two objects $\exists$ and $\forall$ with

- five generators



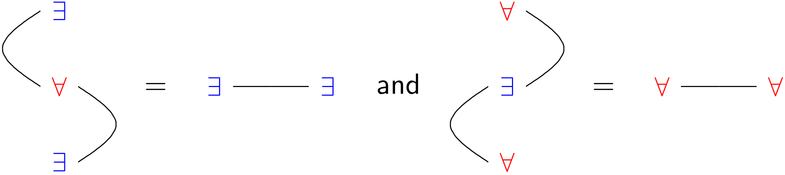
  inducing a structure of *qualitative bicommutative bialgebra*,

The presentation of **Games**

Two objects $\exists$ and $\forall$ with

- five generators
- a duality $\exists \dashv \forall$:

such that



(the axioms for adjunctions)

The theory **Games**

That's it!

strict monoidal functor **Games** $\to \mathcal{C}$

$=$

dual pair of bicommutative qualitative bialgebras

$$\mathbf{Games}(\mathcal{C}) \quad \cong \quad \mathbf{StrMonCat}(\mathbf{Games}, \mathcal{C})$$

Technical byproducts

From this presentation we deduce that

- strategies do **compose**
  (the acyclicity condition is preserved by composition)
- strategies are **definable**
  (i.e. are the interpretations of proofs)

Abstract methodology

We have replaced an *external* definition of the category **Games**:

- category of relations which satisfy conditions
  (polarity + acyclicity)
- **restricting**
- global correctness

by an *internal* definition:

- presentation of the category
- **generating**
- local correctness

Next steps

- extend to formulas with connectives
- links with synthesis of electric circuits
- tools for computer assisted semantic analysis of programs
- ...

Thanks!

Any question?