Interactive Deep Reasoning

Pablo Donato

Partout team — LIX

May 30, 2022

Institut Polytechnique de Paris

Introduction

Context

Goal: Make proof assistants *easier* to use

- **Intuitive** and **discoverable** for newcomers
- **Productive** and **customizable** for experts

For now, focus on common logical heart:

Intuitionistic FOL

“A demo is worth a thousand words...”

Actema

- Fully graphical: no textual proof language
- **Different modes** of reasoning with a **single "syntax"**:

$$\begin{array}{ccc} \text{Click} & \Longleftrightarrow & \text{introduction/elimination} \\ \text{DnD} & \Longleftrightarrow & \text{backward/forward} \end{array}$$

Reasoning through Drag-and-Drop

- Socrates example:

$$\text{Backward} \iff \texttt{apply H1}$$
$$\text{Forward} \iff \texttt{apply H1 in H2}$$

- $\underline{A} \land B \vdash B \land (\underline{A} \lor C) \land D$ is trickier...

$$\dfrac{\dfrac{}{A, B \vdash B} \quad \dfrac{\dfrac{\dfrac{}{A, B \vdash A}}{A, B \vdash A \lor C}\lor R_1 \quad A, B \vdash D}{A, B \vdash (A \lor C) \land D}\land R}{\dfrac{A, B \vdash B \land (A \lor C) \land D}{A \land B \vdash B \land (A \lor C) \land D}\land L}\land R$$

```
destruct H as [HA HB].
split.
* admit.
* split.
  - left. assumption.
  - admit.
```

Idea: instead of *destroying* connectives, switch them

$$\underline{A} \wedge B \vdash \boxed{B \wedge}(\underline{A} \vee C) \wedge D$$

switch:

$$\triangleright\ B \wedge (\underline{A} \wedge B \vdash (\underline{A} \vee C)\boxed{\wedge D})$$

$$\triangleright\ B \wedge (\underline{A} \wedge B \vdash \underline{A}\boxed{\vee C}) \wedge D$$

$$\triangleright\ B \wedge ((\underline{A}\boxed{\wedge B} \vdash \underline{A}) \vee C) \wedge D$$

$$\triangleright\ B \wedge ((B \Rightarrow (\boxed{\underline{A} \vdash \underline{A}})) \vee C) \wedge D$$

identity:

$$\triangleright\ B \wedge ((B\boxed{\Rightarrow \top}) \vee C) \wedge D$$

unit elimination:

$$\triangleright\ B \wedge (\boxed{\top \vee C}) \wedge D$$

$$\triangleright\ B \wedge \boxed{\top \wedge D}$$

$$\triangleright\ B \wedge D$$

1. Unify linked subformulas
2. **Instantiate** unified variables
3. **Switch** uninstantiated quantifiers

$$\exists y. \forall x. R(x,y) \vdash \forall x'. \exists y'. R(x',y')$$

$\triangleright$ $\forall y. (\forall x. R(x,y) \vdash \forall x'. \exists y'. R(x',y'))$ $\quad x \longmapsto x'$

$\triangleright$ $\forall y. \forall x'. (\forall x. R(x,y) \vdash \exists y'. R(x',y'))$

$\triangleright$ $\forall y. \forall x'. (\forall x. R(x,y) \vdash R(x',y))$ $\quad y \longleftarrow y'$

$\triangleright$ $\forall y. \forall x'. (R(x',y) \vdash R(x',y))$

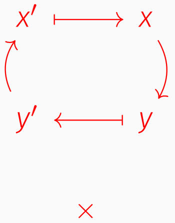
$\triangleright$ $\forall y. \forall x'. \top$ $\quad \checkmark$

$\triangleright^*$ $\top$

Linking under quantifiers — Donato et al. (2022)

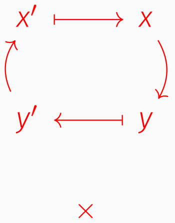
1. Unify linked subformulas
2. **Instantiate** unified variables
3. **Switch** uninstantiated quantifiers

$$\forall x'.\exists y'.\underline{R(x',y')} \vdash \exists y.\forall x.\underline{R(x,y)}$$



1. Unify linked subformulas
2. **Check** for ∀∃ dependency cycles
3. **Instantiate** unified variables
4. **Switch** uninstantiated quantifiers

$$\forall x'.\exists y'.\underline{R(x',y')} \vdash \exists y.\forall x.\underline{R(x,y)}$$



Add 4 rules $\Longrightarrow$ **rewrite** for free!

$$\underline{t} = u \vdash \underline{A} \quad \rhd \quad A\{t := u\} \qquad t = \underline{u} \vdash \underline{A} \quad \rhd \quad A\{u := t\}$$

$$\underline{t} = u * \underline{A} \quad \rhd \quad A\{t := u\} \qquad t = \underline{u} * \underline{A} \quad \rhd \quad A\{u := t\}$$

Compositional with semantics of **connectives**:

- **Quantifiers:** rewrite modulo *unification*
- **Implication:** *conditional* rewrite
- **Arbitrary** combinations are possible:

$$\forall x. x \neq 0 \Rightarrow \underline{f(x)} = g(x) \vdash \exists y. A(\underline{f(y)}) \lor B(y)$$

$$\rhd^* \quad \exists y. (y \neq 0 \land A(g(y))) \lor B(y)$$

Reasoning with bubbles

The chemical metaphor

Item	$\Longleftrightarrow$	Ion
Color	$\Longleftrightarrow$	Polarity
Logical connective	$\Longleftrightarrow$	Chemical bond
Click	$\Longleftrightarrow$	Heating
DnD	$\Longleftrightarrow$	Bimolecular reaction

The chemical metaphor

Item	$\Longleftrightarrow$	Ion
Color	$\Longleftrightarrow$	Polarity
Logical connective	$\Longleftrightarrow$	Chemical bond
Click	$\Longleftrightarrow$	Heating
DnD	$\Longleftrightarrow$	Bimolecular reaction

- Works well for $\Rightarrow$ and $\wedge$ only!

The chemical metaphor

Item	$\Longleftrightarrow$	Ion
Color	$\Longleftrightarrow$	Polarity
Logical connective	$\Longleftrightarrow$	Chemical bond
Click	$\Longleftrightarrow$	Heating
DnD	$\Longleftrightarrow$	Bimolecular reaction

- Works well for $\Rightarrow$ and $\wedge$ only!
- Problem: **context-scoping** through *premisses/tabs*

Adding bubbles

Idea: Internalize subgoals by nesting sequents

$$\sigma ::= \Gamma \; \textcircled{\sigma_1} \; \ldots \; \textcircled{\sigma_n} \; \Delta \qquad \Gamma ::= A_1 \; \ldots \; A_n \qquad \Delta ::= \varnothing \mid A$$

$$(A \vee B \Rightarrow C) \Rightarrow (A \Rightarrow C) \wedge (B \Rightarrow C)$$

Adding bubbles

Idea: Internalize subgoals by nesting sequents

$$\sigma ::= \Gamma \ \textcircled{\sigma_1} \ \ldots \ \textcircled{\sigma_n} \ \Delta \qquad \Gamma ::= A_1 \ \ldots \ A_n \qquad \Delta ::= \varnothing \mid A$$

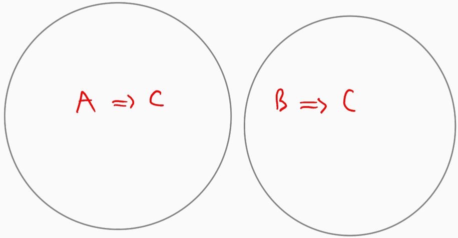
$$A \vee B \Rightarrow C \qquad (A \Rightarrow C) \wedge (B \Rightarrow C)$$

Adding bubbles

Idea: Internalize subgoals by nesting sequents

$$\sigma ::= \Gamma \ \textcircled{\sigma_1} \ \ldots \ \textcircled{\sigma_n} \ \Delta \qquad \Gamma ::= A_1 \ \ldots \ A_n \qquad \Delta ::= \varnothing \mid A$$

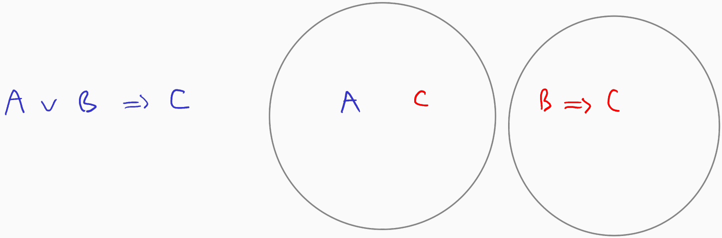
$A \vee B \Rightarrow C$



Adding bubbles

Idea: Internalize subgoals by nesting sequents

$$\sigma ::= \Gamma \ \textcircled{σ_1} \ \ldots \ \textcircled{σ_n} \ \Delta \qquad \Gamma ::= A_1 \ \ldots \ A_n \qquad \Delta ::= \varnothing \mid A$$

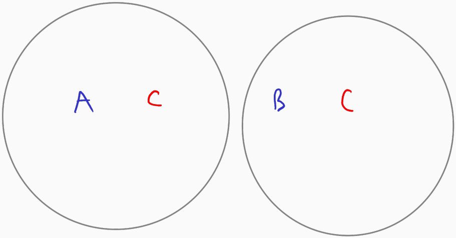


Adding bubbles

Idea: Internalize subgoals by nesting sequents

$$\sigma ::= \Gamma \; \textcircled{\sigma_1} \; \ldots \; \textcircled{\sigma_n} \; \Delta \qquad \Gamma ::= A_1 \; \ldots \; A_n \qquad \Delta ::= \varnothing \mid A$$

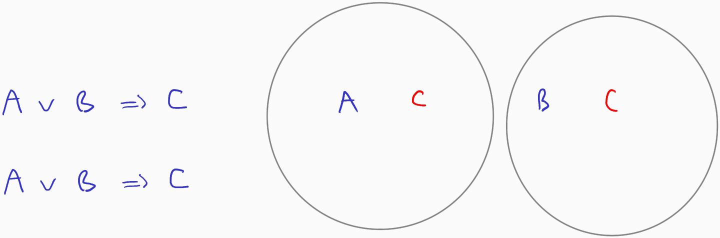
$A \vee B \Rightarrow C$



Adding bubbles

Idea: Internalize subgoals by nesting sequents

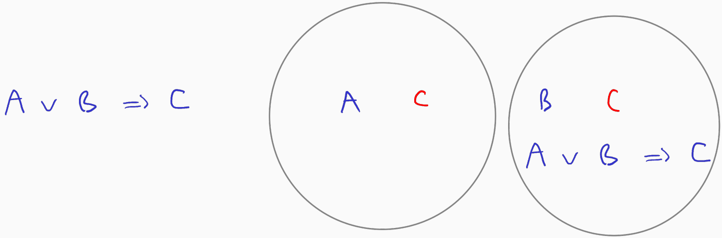
$$\sigma ::= \Gamma \ \textcircled{σ_1} \ \ldots \ \textcircled{σ_n} \ \Delta \qquad \Gamma ::= A_1 \ \ldots \ A_n \qquad \Delta ::= \varnothing \mid A$$



Adding bubbles

Idea: Internalize subgoals by nesting sequents

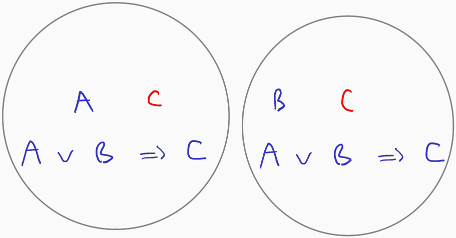
$\sigma ::= \Gamma \; \textcircled{\sigma_1} \; \ldots \; \textcircled{\sigma_n} \; \Delta \qquad \Gamma ::= A_1 \; \ldots \; A_n \qquad \Delta ::= \varnothing \mid A$



Adding bubbles

Idea: Internalize subgoals by nesting sequents

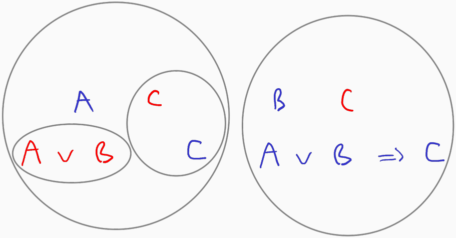
$\sigma ::= \Gamma \ \textcircled{σ_1} \ \ldots \ \textcircled{σ_n} \ \Delta \qquad \Gamma ::= A_1 \ \ldots \ A_n \qquad \Delta ::= \varnothing \mid A$



Adding bubbles

Idea: Internalize subgoals by nesting sequents

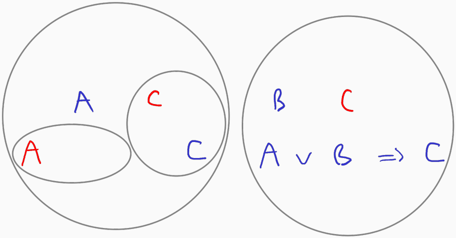
$\sigma ::= \Gamma \; \bigcirc\!\!\!\!\sigma_1 \; \ldots \; \bigcirc\!\!\!\!\sigma_n \; \Delta$ $\quad\quad \Gamma ::= A_1 \; \ldots \; A_n$ $\quad\quad \Delta ::= \varnothing \mid A$



Adding bubbles

Idea: Internalize subgoals by nesting sequents

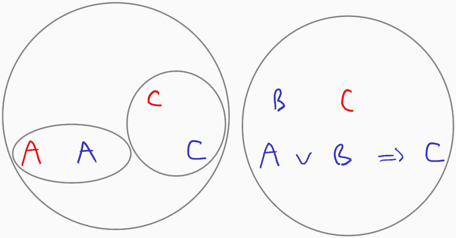
$$\sigma ::= \Gamma \ \textcircled{\sigma_1} \ \ldots \ \textcircled{\sigma_n} \ \Delta \qquad \Gamma ::= A_1 \ \ldots \ A_n \qquad \Delta ::= \varnothing \mid A$$



Adding bubbles

Idea: Internalize subgoals by nesting sequents

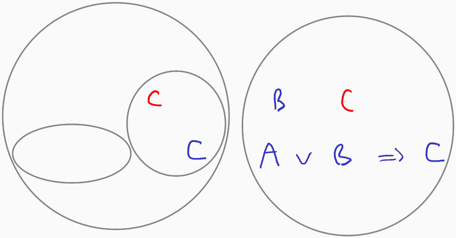
$\sigma ::= \Gamma \ \textcircled{\sigma_1} \ \ldots \ \textcircled{\sigma_n} \ \Delta \qquad \Gamma ::= A_1 \ \ldots \ A_n \qquad \Delta ::= \varnothing \mid A$



Adding bubbles

Idea: Internalize subgoals by nesting sequents

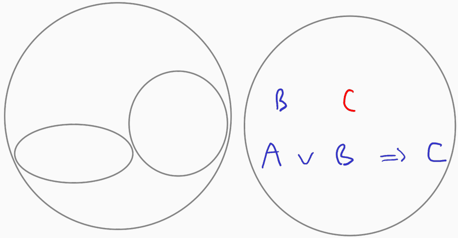
$\sigma ::= \Gamma \; \textcircled{\sigma_1} \; \ldots \; \textcircled{\sigma_n} \; \Delta \qquad \Gamma ::= A_1 \; \ldots \; A_n \qquad \Delta ::= \varnothing \mid A$



Adding bubbles

Idea: Internalize subgoals by nesting sequents

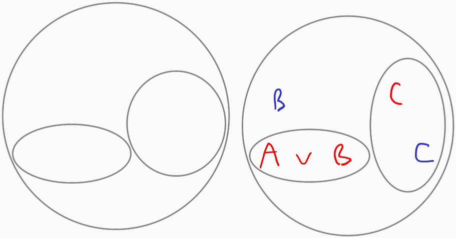
$\sigma ::= \Gamma \; \textcircled{\sigma_1} \; \ldots \; \textcircled{\sigma_n} \; \Delta \qquad \Gamma ::= A_1 \; \ldots \; A_n \qquad \Delta ::= \varnothing \mid A$



Adding bubbles

Idea: Internalize subgoals by nesting sequents

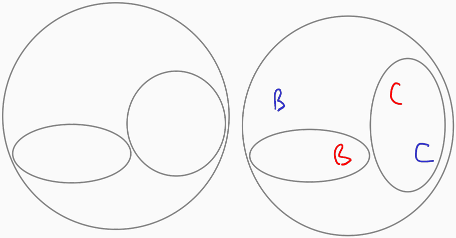
$\sigma ::= \Gamma \; \textcircled{\sigma_1} \; \ldots \; \textcircled{\sigma_n} \; \Delta \qquad \Gamma ::= A_1 \; \ldots \; A_n \qquad \Delta ::= \varnothing \mid A$



Adding bubbles

Idea: Internalize subgoals by nesting sequents

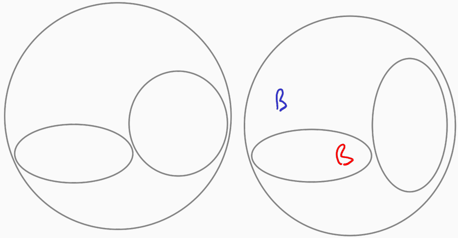
$\sigma ::= \Gamma \; \textcircled{\sigma_1} \ldots \textcircled{\sigma_n} \; \Delta \qquad \Gamma ::= A_1 \ldots A_n \qquad \Delta ::= \varnothing \mid A$



Adding bubbles

Idea: Internalize subgoals by nesting sequents

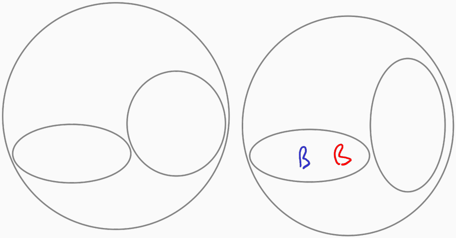
$\sigma ::= \Gamma \ \textcircled{\sigma_1} \ \ldots \ \textcircled{\sigma_n} \ \Delta$ $\qquad \Gamma ::= A_1 \ \ldots \ A_n$ $\qquad \Delta ::= \varnothing \mid A$



Adding bubbles

Idea: Internalize subgoals by nesting sequents

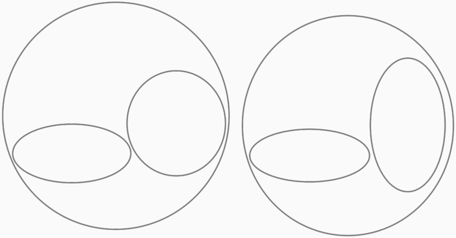
$\sigma ::= \Gamma \; \textcircled{σ_1} \; \ldots \; \textcircled{σ_n} \; \Delta \qquad \Gamma ::= A_1 \; \ldots \; A_n \qquad \Delta ::= \varnothing \mid A$



Adding bubbles

Idea: Internalize subgoals by nesting sequents

$\sigma ::= \Gamma \; \circled{\sigma_1} \; \dots \; \circled{\sigma_n} \; \Delta \qquad \Gamma ::= A_1 \; \dots \; A_n \qquad \Delta ::= \varnothing \mid A$



Adding bubbles

Idea: Internalize subgoals by nesting sequents

$$\sigma ::= \Gamma \ \textcircled{\sigma_1} \ \ldots \ \textcircled{\sigma_n} \ \Delta \qquad \Gamma ::= A_1 \ \ldots \ A_n \qquad \Delta ::= \varnothing \mid A$$

Reducing non-determinism

Moto: Non-reversibility reduces *freedom*

$$A \vee B \quad \rhd \quad A$$

$$A \vee B \quad \rhd \quad B$$

$$A \Rightarrow B \quad C \quad \rhd \quad \textcircled{A} \quad \textcircled{B \;\; C}$$

- Hack: use only DnD
- New objective: **full formula decomposition** property
- (Guenot, 2013): only classical $\{\wedge, \vee\}$ and intuitionistic $\{\Rightarrow\}$

$$\Delta ::= \iota_1 \; \ldots \; \iota_n \qquad \iota ::= A \mid \sigma$$

$$A \vee B \quad \triangleright \quad A \;\; B \qquad\qquad A \Rightarrow B \quad \triangleright \quad \bigcirc\!\!\!\!\!\!\!\!\!\quad A \;\; B$$

$$B \vee (A \Rightarrow C) \qquad\qquad A \Rightarrow (B \vee C)$$

$$\Delta ::= \iota_1 \;\ldots\; \iota_n \qquad\qquad \iota ::= A \mid \sigma$$

$$A \vee B \quad \triangleright \quad A \;\; B \qquad\qquad A \Rightarrow B \quad \triangleright \quad (A \;\; B)$$

$$B \vee (A \Rightarrow C) \qquad (A \qquad B \vee C)$$

$\Delta ::= \iota_1 \ \ldots \ \iota_n \qquad \iota ::= A \mid \sigma$

$A \vee B \quad \triangleright \quad A \ \ B \qquad A \Rightarrow B \quad \triangleright \quad A \ \ B$

$B \vee (A \Rightarrow C) \qquad A \quad B \quad C$

Coloring bubbles — Grishin (b)

$$\Delta ::= \iota_1 \ \ldots \ \iota_n \qquad\qquad \iota ::= A \mid \sigma$$

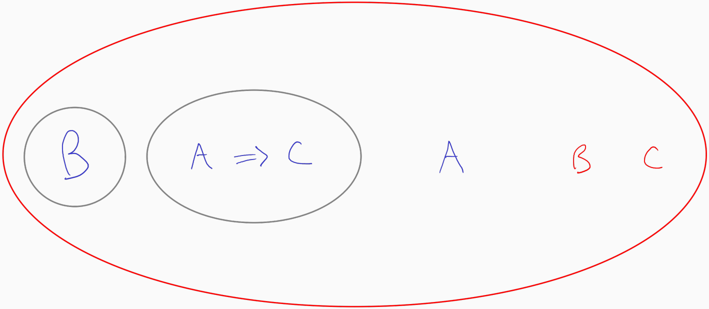
$$A \vee B \quad \triangleright \quad A \ \ B \qquad\qquad A \Rightarrow B \quad \triangleright \quad \bigcirc(A \ \ B)$$

$$B \vee (A \Rightarrow C) \qquad A \qquad B \quad C$$

Coloring bubbles

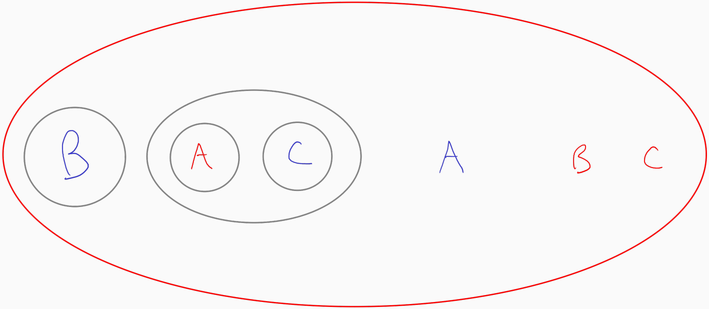
$$\Delta ::= \iota_1 \ \ldots \ \iota_n \qquad\qquad \iota ::= A \mid \sigma$$

$$A \vee B \quad \rhd \quad A \ \ B \qquad\qquad A \Rightarrow B \quad \rhd \quad (A \ \ B)$$



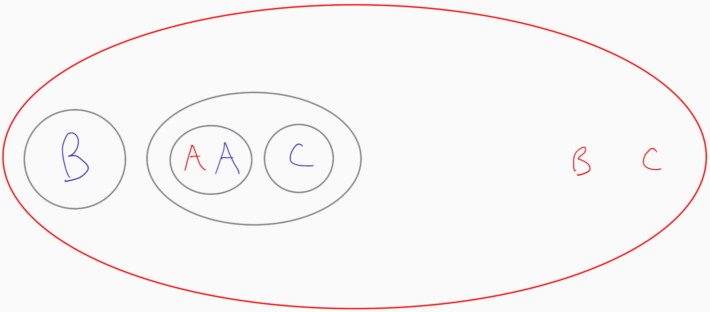
$\Delta ::= \iota_1 \ \ldots \ \iota_n$ $\qquad \iota ::= A \mid \sigma$

$A \vee B \quad \triangleright \quad A \ \ B$ $\qquad A \Rightarrow B \quad \triangleright \quad (A \ \ B)$



$$\Delta ::= \iota_1 \ \ldots \ \iota_n \qquad \iota ::= A \mid \sigma$$

$$A \vee B \quad \triangleright \quad A \;\; B \qquad\qquad A \Rightarrow B \quad \triangleright \quad \textcircled{A \;\; B}$$

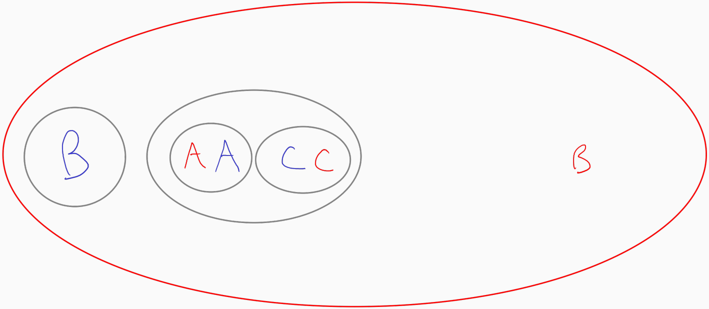


Coloring bubbles

Grishin (b)

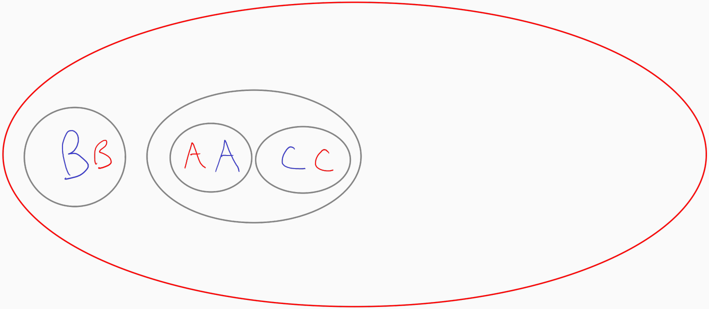
$$\Delta ::= \iota_1 \;\ldots\; \iota_n \qquad\qquad \iota ::= A \mid \sigma$$

$$A \vee B \quad \triangleright \quad A \;\; B \qquad\qquad A \Rightarrow B \quad \triangleright \quad \bigcirc\!\!\!\!\!\!\!\;\;A\;B$$



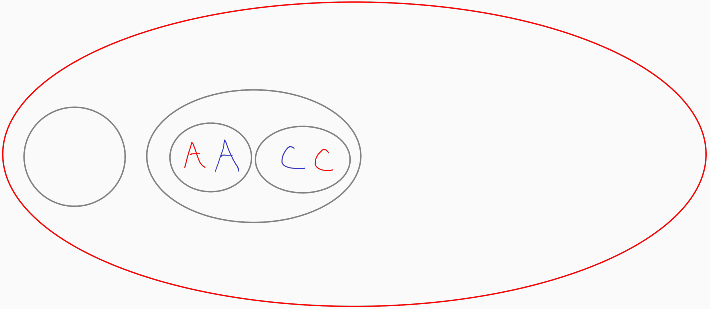
$$\Delta ::= \iota_1 \ \ldots \ \iota_n \qquad \iota ::= A \mid \sigma$$

$$A \vee B \quad \triangleright \quad A \ \ B \qquad A \Rightarrow B \quad \triangleright \quad \bigcirc(A \ \ B)$$



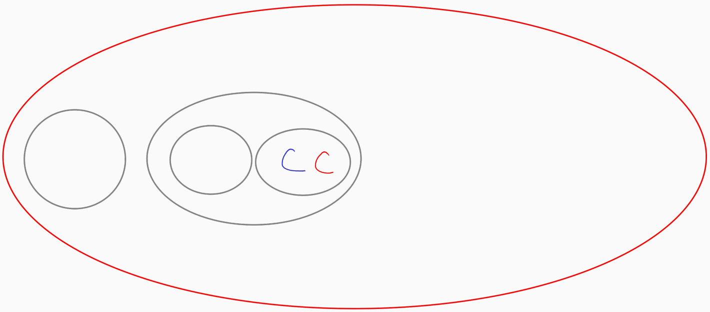
$$\Delta ::= \iota_1 \;\ldots\; \iota_n \qquad\qquad \iota ::= A \mid \sigma$$

$$A \vee B \quad \triangleright \quad A \;\; B \qquad\qquad A \Rightarrow B \quad \triangleright \quad (A \;\; B)$$



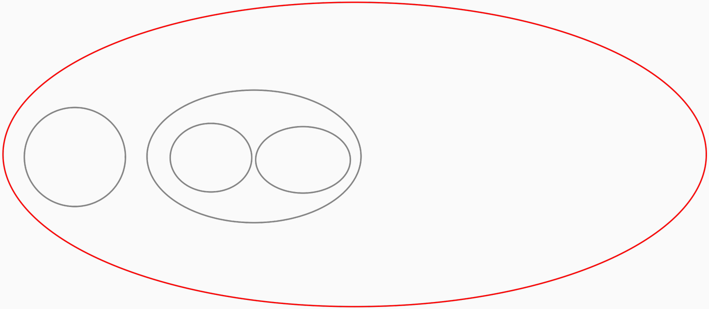
$$\Delta ::= \iota_1 \ \ldots \ \iota_n \qquad\qquad \iota ::= A \mid \sigma$$

$$A \vee B \quad \triangleright \quad A \ \ B \qquad\qquad A \Rightarrow B \quad \triangleright \quad \textcircled{A \ \ B}$$



$$\Delta ::= \iota_1 \ \ldots \ \iota_n \qquad \iota ::= A \mid \sigma$$

$$A \vee B \quad \triangleright \quad A \ \ B \qquad A \Rightarrow B \quad \triangleright \quad \bigcirc\!\!\!\!\!\!\!\!\; A \ \ B$$



$$\Delta ::= \iota_1 \;\ldots\; \iota_n \qquad \iota ::= A \mid \sigma$$

$$A \vee B \quad \triangleright \quad A \;\; B \qquad\qquad A \Rightarrow B \quad \triangleright \quad \textcircled{A \;\; B}$$

$A \Rightarrow (B \vee C)$ $\qquad B \vee (A \Rightarrow C)$

$\Delta ::= \iota_1 \ \ldots \ \iota_n$ $\qquad \iota ::= A \mid \sigma$

$A \vee B \quad \triangleright \quad A \ \ B \qquad A \Rightarrow B \quad \triangleright \quad (A \ \ B)$

$A \Rightarrow (B \vee C) \qquad B \quad A \Rightarrow C$

Red bubbles

Grishin (a)

$\Delta ::= \iota_1 \;\ldots\; \iota_n$ $\qquad$ $\iota ::= A \mid \sigma$

$A \vee B \quad \triangleright \quad A \;\; B$ $\qquad$ $A \Rightarrow B \quad \triangleright \quad (A \;\; B)$

$A \Rightarrow (B \vee C)$ $\qquad$ $B \quad (A \quad C)$

Red bubbles

$$\Delta ::= \iota_1 \;\ldots\; \iota_n \qquad\qquad \iota ::= A \mid \sigma$$

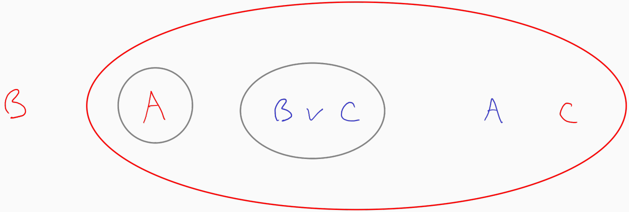
$$A \vee B \quad \triangleright \quad A \;\; B \qquad\qquad A \Rightarrow B \quad \triangleright \quad (A \;\; B)$$

$$B \qquad A \Rightarrow (B \vee C) \quad A \quad C$$

Red bubbles

$$\Delta ::= \iota_1 \ \ldots \ \iota_n \qquad\qquad \iota ::= A \mid \sigma$$

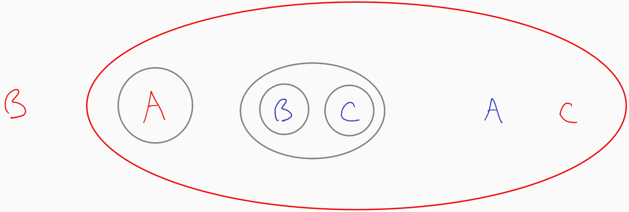
$$A \vee B \quad \triangleright \quad A \ \ B \qquad\qquad A \Rightarrow B \quad \triangleright \quad \bigcirc\!\!\!\!\!\!\!\!\!\! A \ \ B$$



$\Delta ::= \iota_1 \; \ldots \; \iota_n \qquad \iota ::= A \mid \sigma$

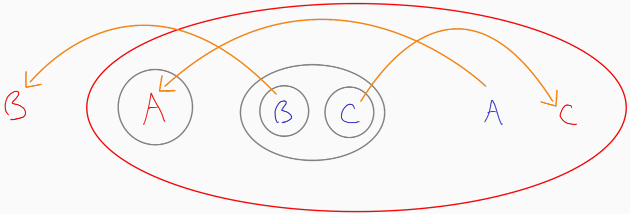
$A \vee B \quad \triangleright \quad A \;\; B$

$A \Rightarrow B \quad \triangleright \quad (A \;\; B)$



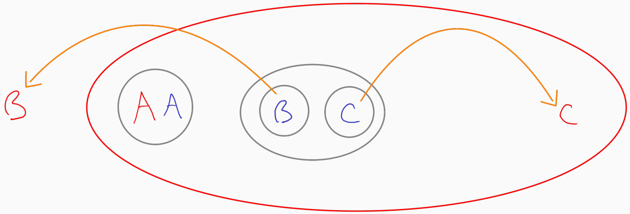
$$\Delta ::= \iota_1 \ \ldots \ \iota_n \qquad \iota ::= A \mid \sigma$$

$$A \vee B \quad \rhd \quad A \ \ B \qquad\qquad A \Rightarrow B \quad \rhd \quad (A \ \ B)$$



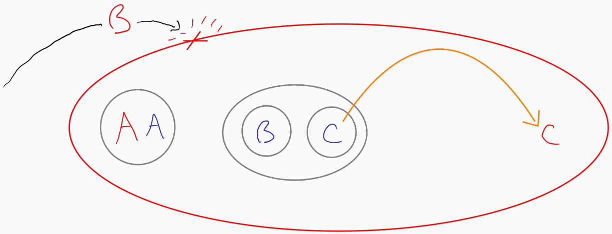
$$\Delta ::= \iota_1 \ \ldots \ \iota_n \qquad \iota ::= A \mid \sigma$$

$$A \vee B \quad \triangleright \quad A \ \ B \qquad A \Rightarrow B \quad \triangleright \quad \textcircled{A \ \ B}$$



$$\Delta ::= \iota_1 \ \ldots \ \iota_n \qquad \iota ::= A \mid \sigma$$

$$A \vee B \quad \triangleright \quad A \ \ B \qquad\qquad A \Rightarrow B \quad \triangleright \quad (A \ \ B)$$



Meaning

$$\left[\!\left[\Gamma \; (\Gamma_1 \; \mathcal{S}_1 \; \Delta_1) \; \ldots \; (\Gamma_n \; \mathcal{S}_n \; \Delta_n) \; \Delta \right]\!\right]$$

$$=$$

$$\bigwedge_i [\![\Gamma \; \Gamma_i \; \mathcal{S}_i \; \Delta_i \; \Delta]\!]$$

Need for a (non-trivializing) base case:

$$\sigma ::= \underbrace{\Gamma \vdash \Delta}_{\text{subgoal}} \mid \underbrace{\Gamma \; \mathcal{S} \; \Delta}_{\text{branching}}$$

Blue bubbles

- Allow nesting in hypotheses $\Longrightarrow$ dual-intuitionistic logic

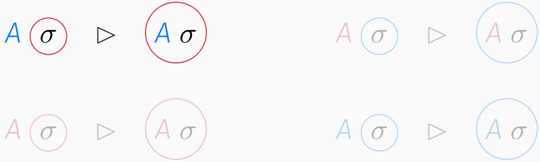
$$\Gamma ::= \iota_1 \;\ldots\; \iota_n$$

- Rules for **subtraction** $-$ dual to $\Rightarrow$:

$$A - B \quad \triangleright \quad \textcircled{A \;\; B} \qquad\qquad A - B \quad \triangleright \quad \textcircled{A} \;\; \textcircled{B}$$

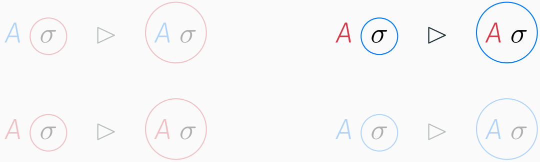
- Blue bubbles **hermetic** to blue items

A new view on classical VS intuitionistic



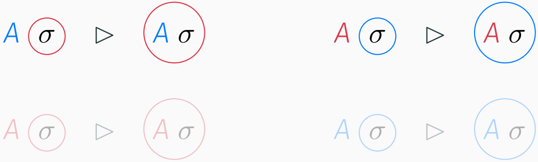
Intuitionistic logic

A new view on classical VS intuitionistic



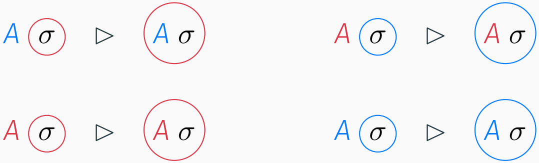
Dual-intuitionistic logic

A new view on classical VS intuitionistic



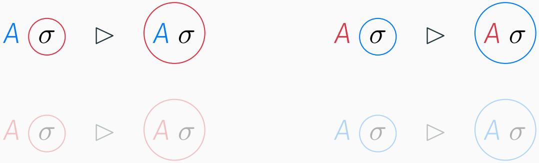
Bi-intuitionistic logic

A new view on classical VS intuitionistic



Classical logic

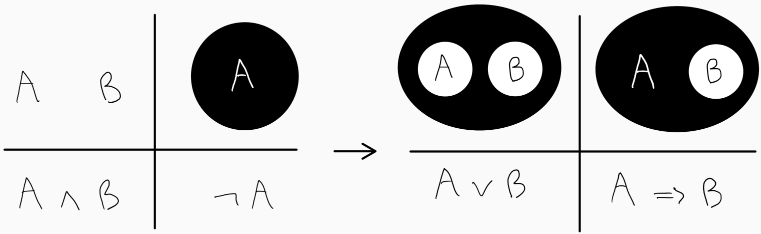
A new view on classical VS intuitionistic



*Intuitionism = same polarities **repel** eachother*

Reasoning without formulas

- ∨ solved, but ⇒ still irreversible!
- Key idea: space is **polarized**, *not items*
- In classical logic:



Illative transformations (Peirce, 1882)

Only 3 edition principles!

- **(De-)Iteration** (*copy/cut-paste*):

$$G \;\; H\{\} \quad \equiv \quad G \;\; H\{G\}$$

- **Insertion**:

$$\triangleright \quad G$$

- **Deletion**:

$$G \quad \triangleright$$

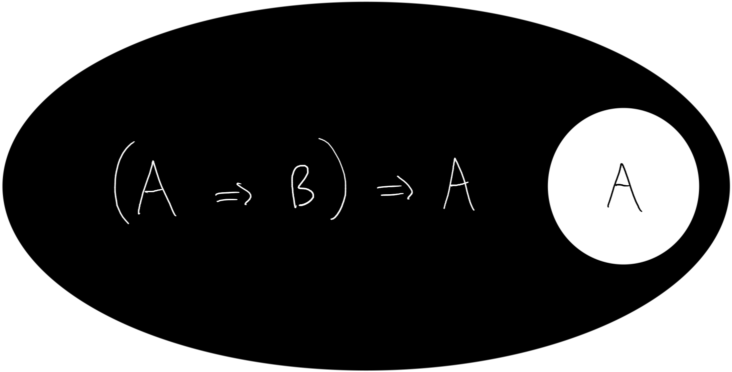
And a space principle, the **double-cut** law:

$$\equiv \quad G$$

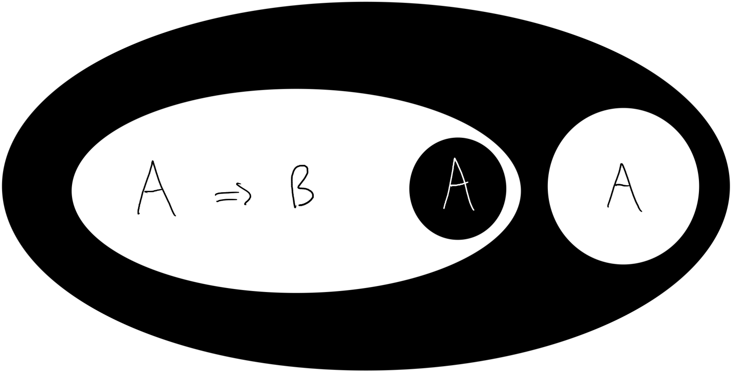
Example: Peirce's law

$$((A \Rightarrow B) \Rightarrow A) \Rightarrow A$$

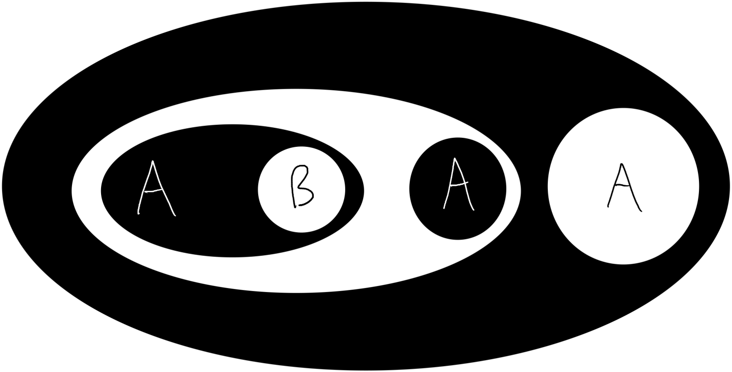
Example: Peirce's law



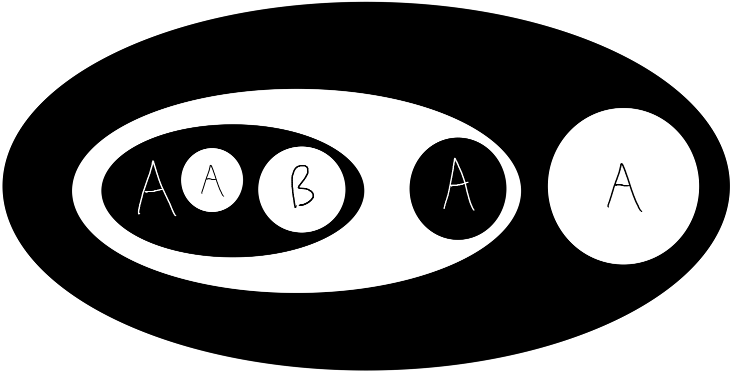
Example: Peirce's law



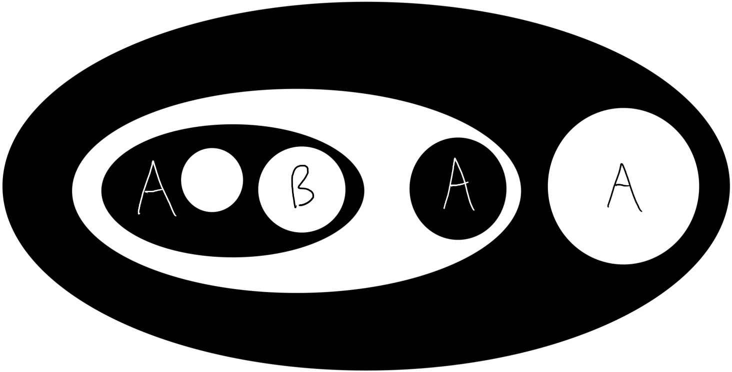
Example: Peirce's law



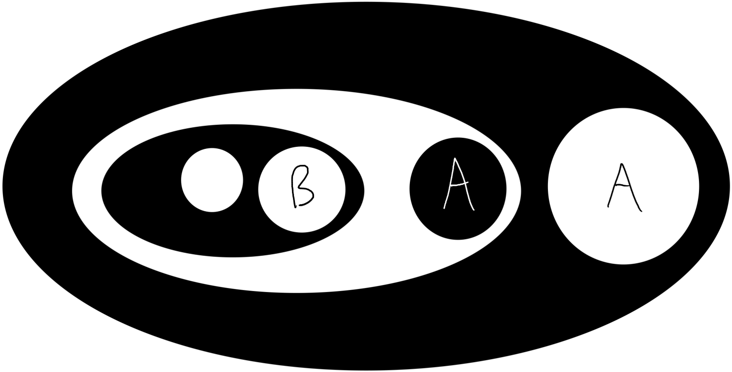
Example: Peirce's law



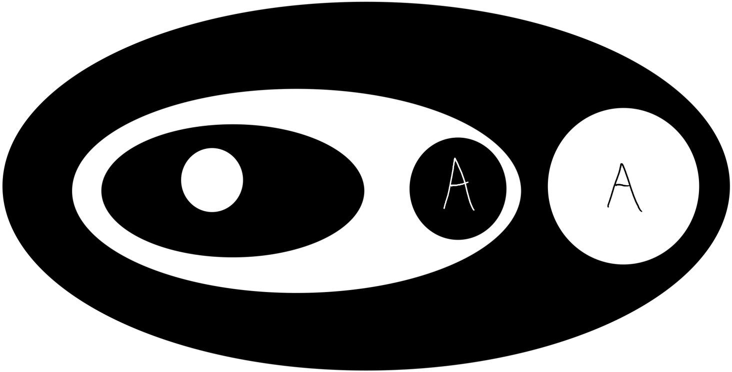
Example: Peirce's law



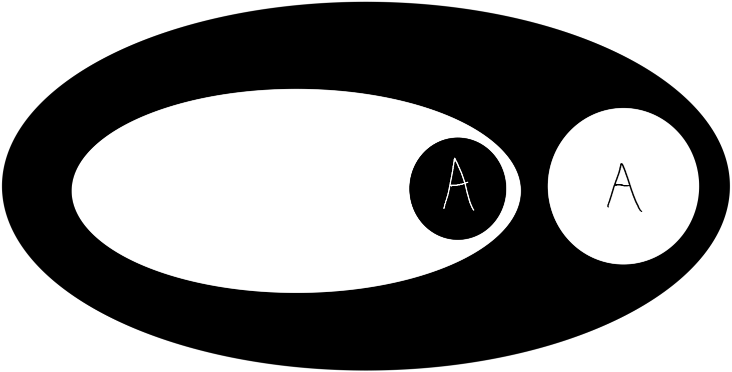
Example: Peirce's law



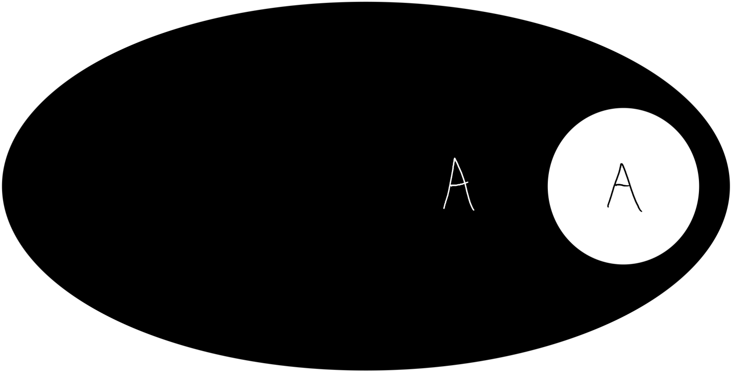
Example: Peirce's law



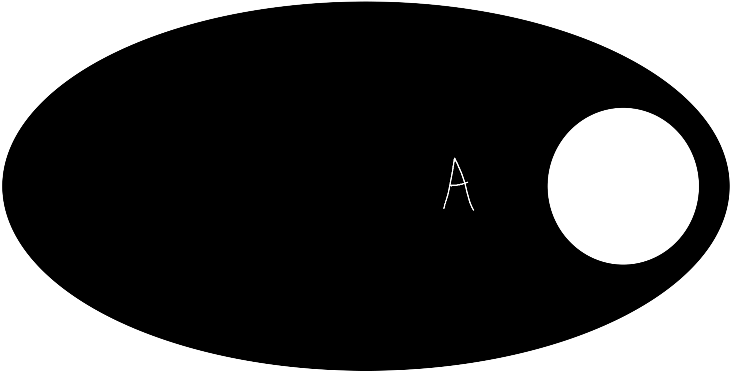
Example: Peirce's law



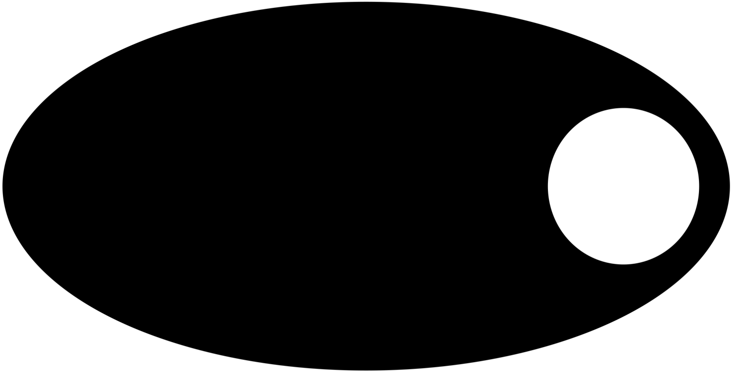
Example: Peirce's law



Example: Peirce's law

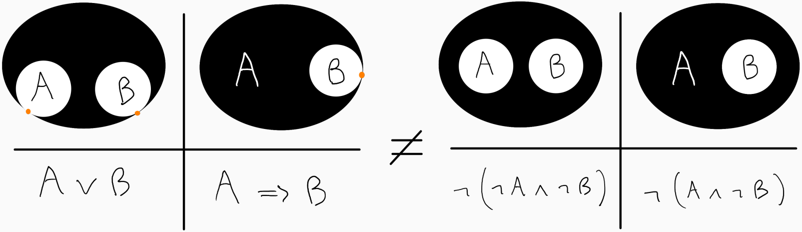


Example: Peirce's law



Example: Peirce's law

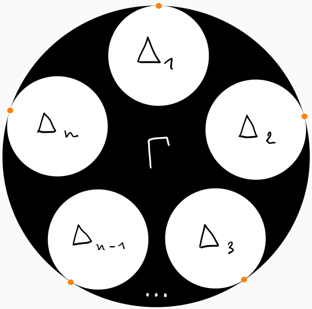
Officialize Peirce's scroll

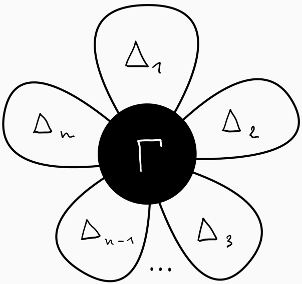


Flowers

Turn *inloops* into petals

$$\Gamma \vdash \Delta_1 ; \ldots ; \Delta_n$$

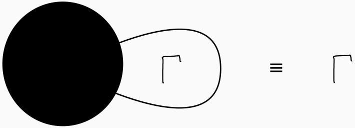




(De-)iteration is now called pollination.

Decomposition law:

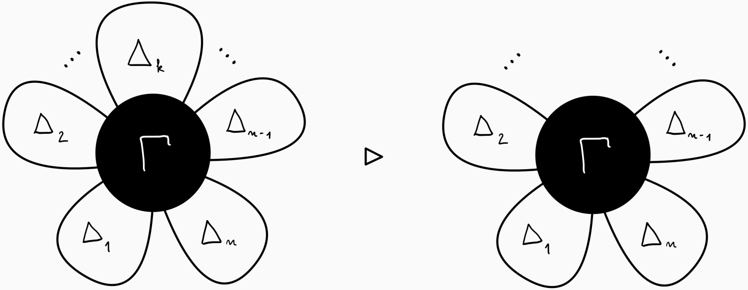
$\equiv G$ becomes



Disjunction and falsity

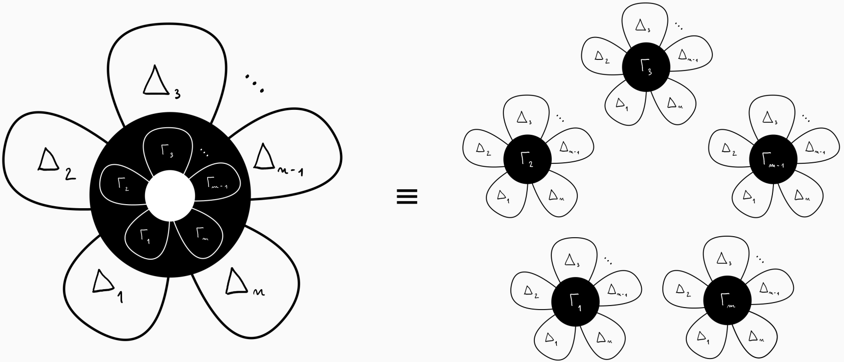
Deletion splits in two:

$F \quad \triangleright$



Disjunction and falsity

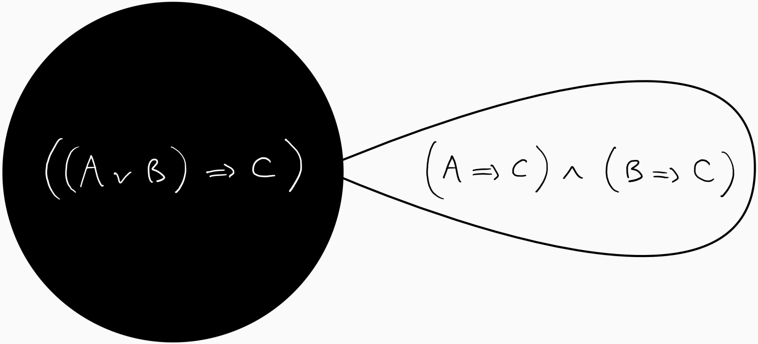
Reproduction rule for *case reasoning*:



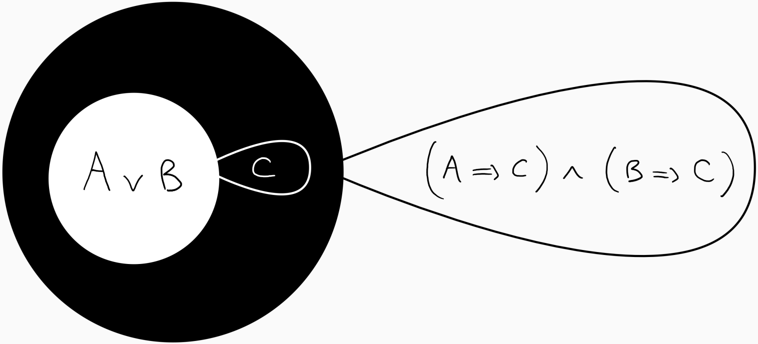
Example: disjunction elimination

$$((A \vee B) \Rightarrow C) \Rightarrow (A \Rightarrow C) \wedge (B \Rightarrow C)$$

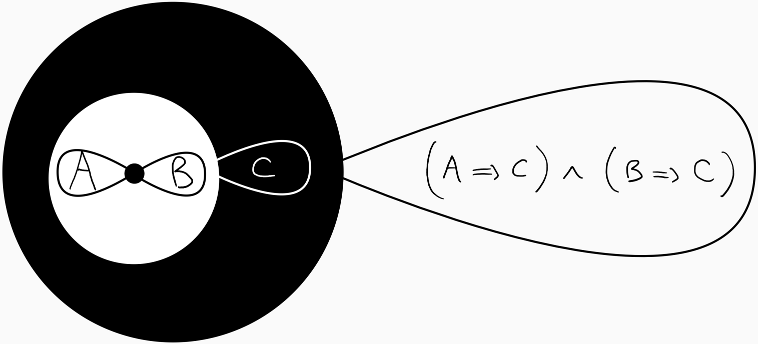
Example: disjunction elimination



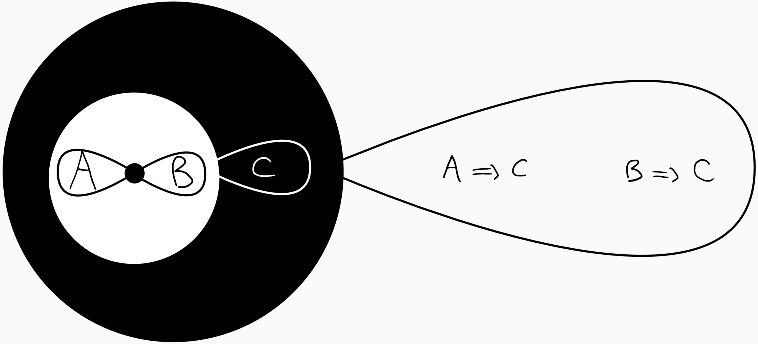
Example: disjunction elimination



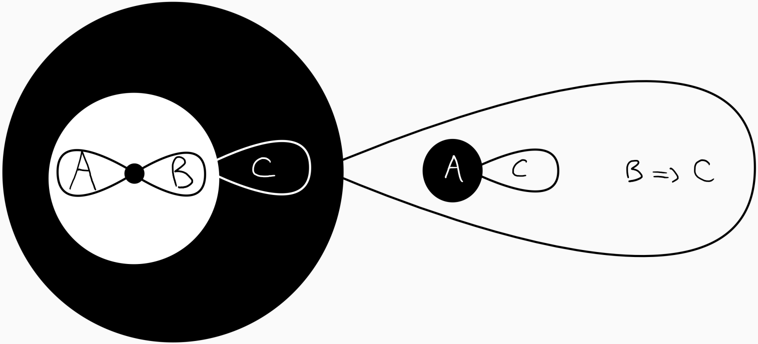
Example: disjunction elimination



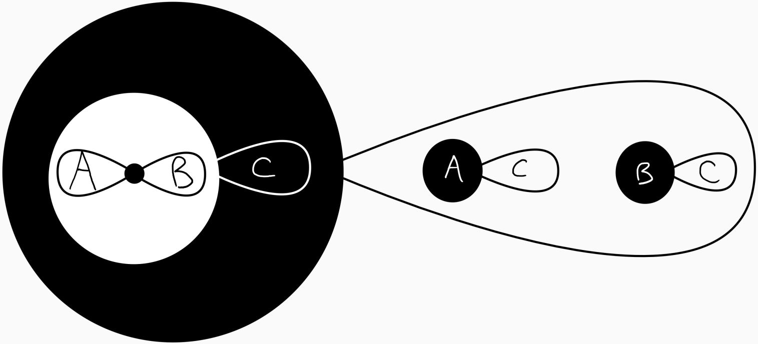
Example: disjunction elimination



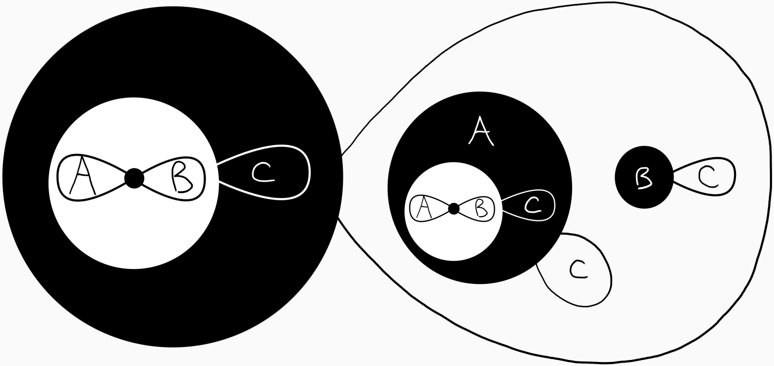
Example: disjunction elimination



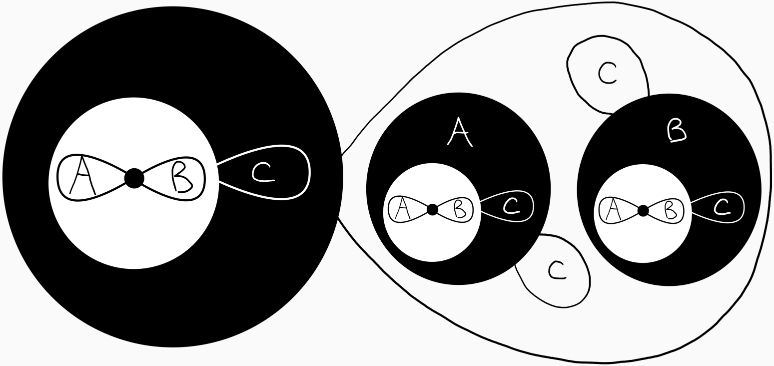
Example: disjunction elimination



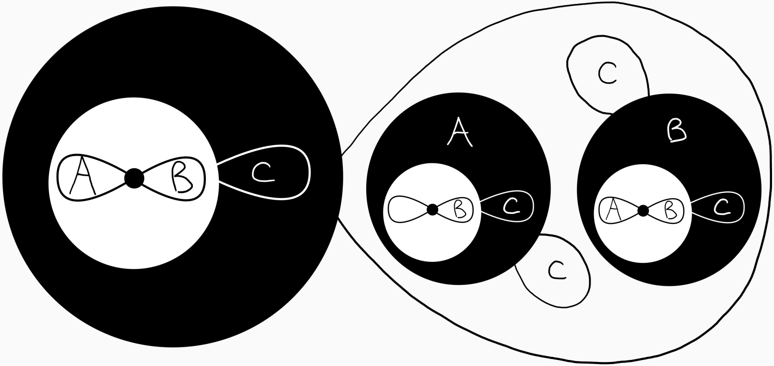
Example: disjunction elimination



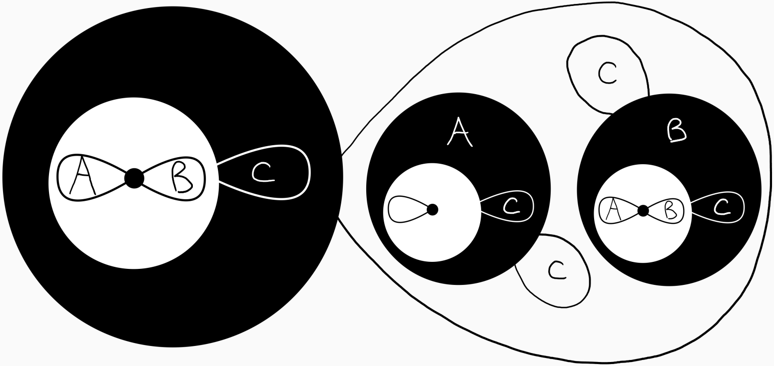
Example: disjunction elimination



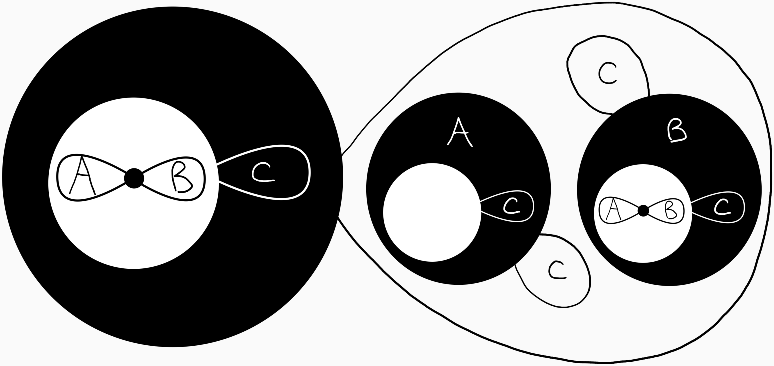
Example: disjunction elimination



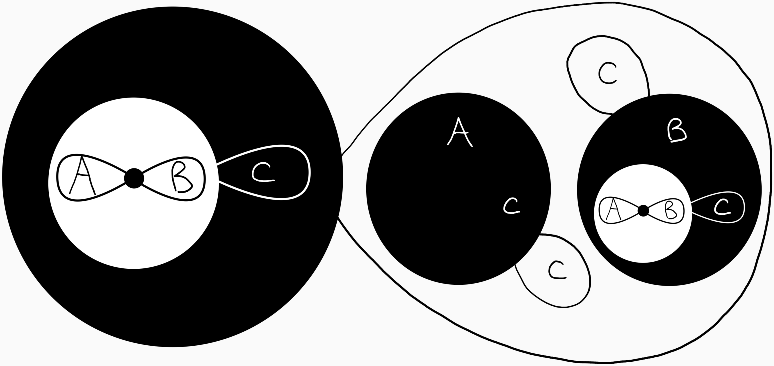
Example: disjunction elimination



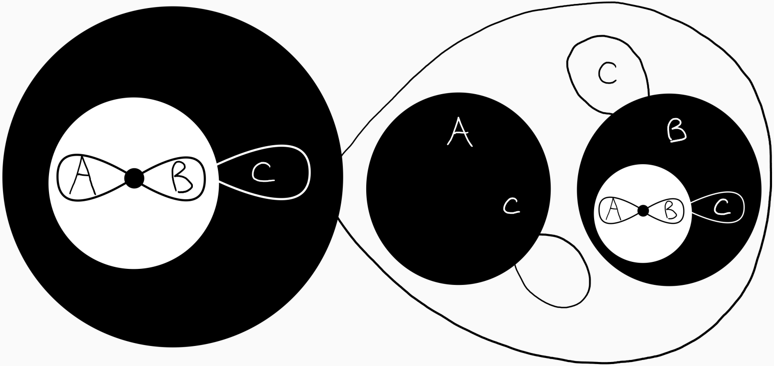
Example: disjunction elimination



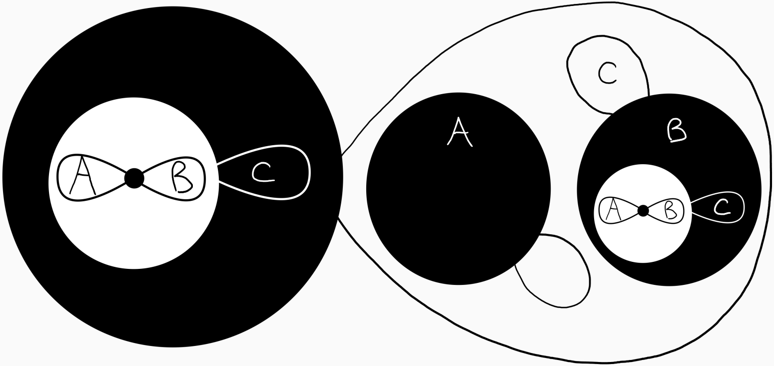
Example: disjunction elimination



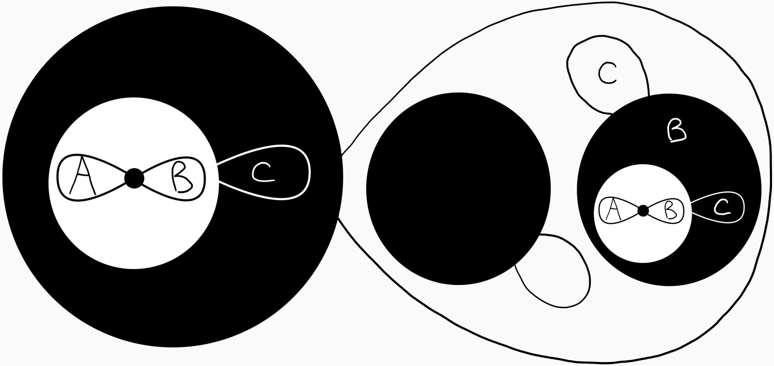
Example: disjunction elimination



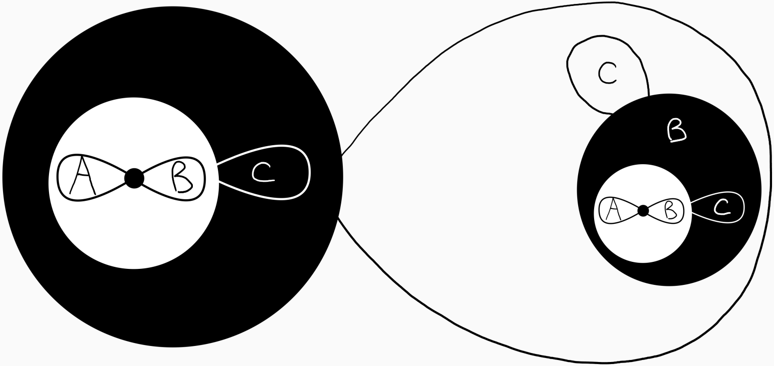
Example: disjunction elimination



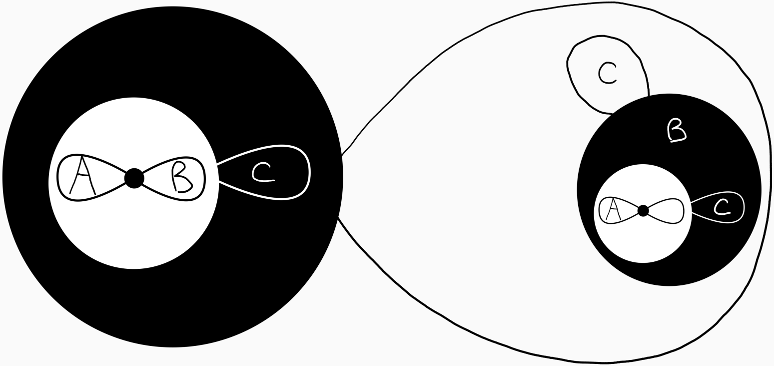
Example: disjunction elimination



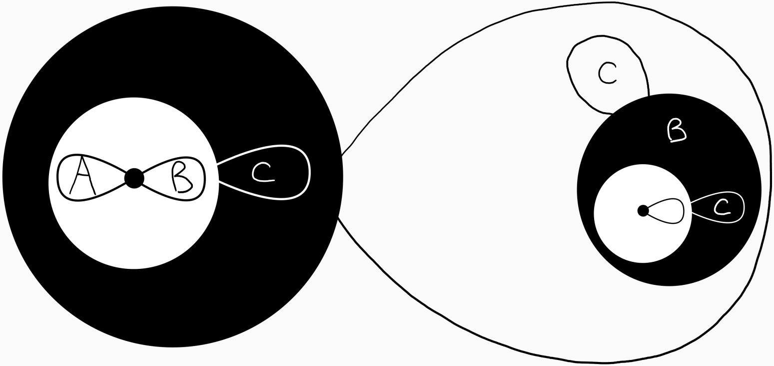
Example: disjunction elimination



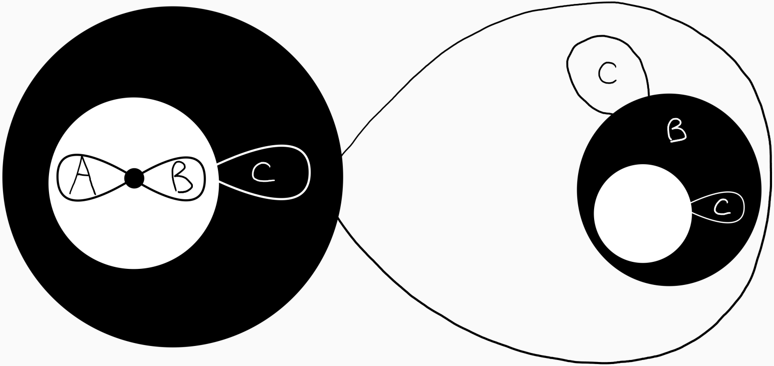
Example: disjunction elimination



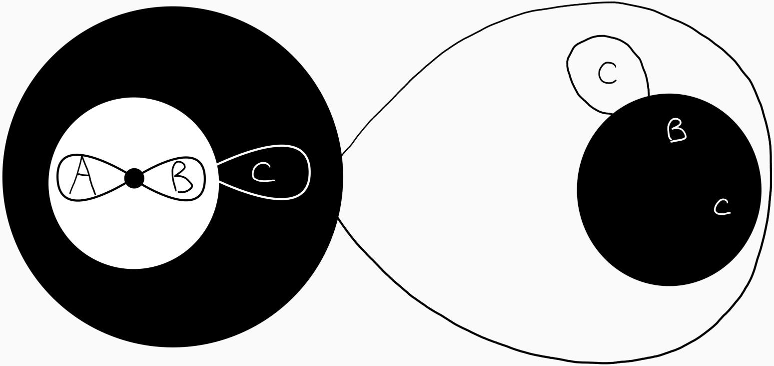
Example: disjunction elimination



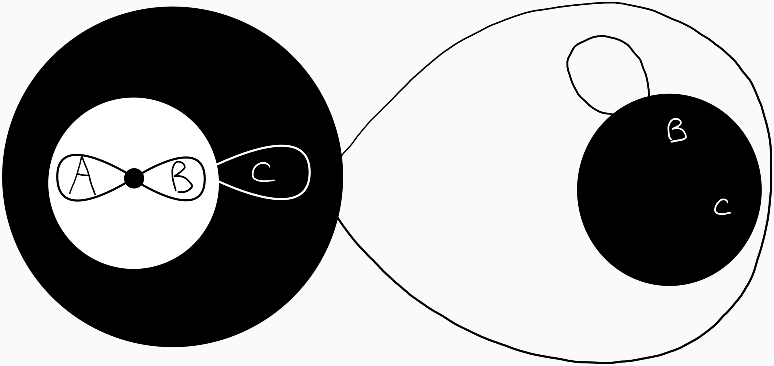
Example: disjunction elimination



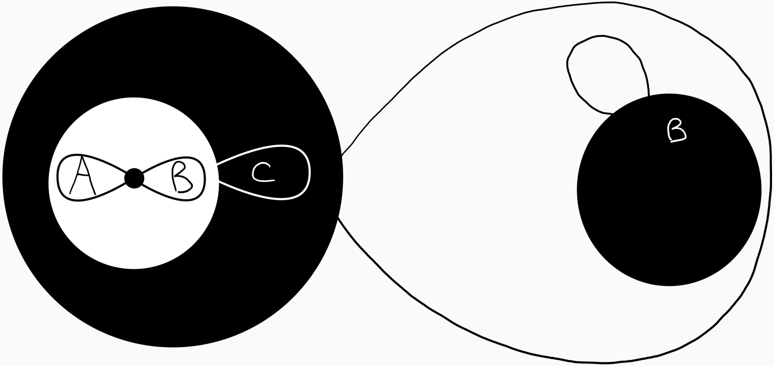
Example: disjunction elimination



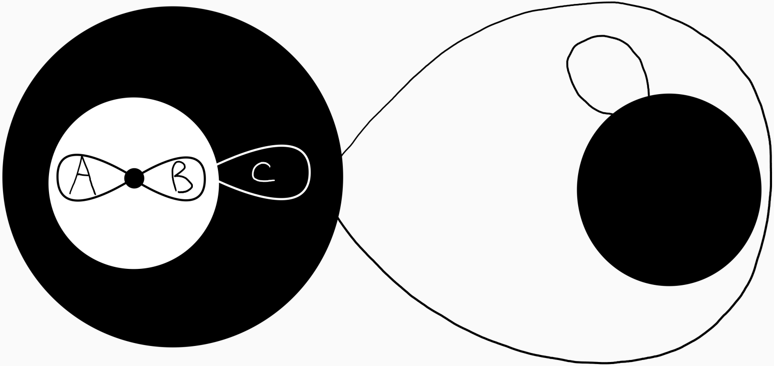
Example: disjunction elimination



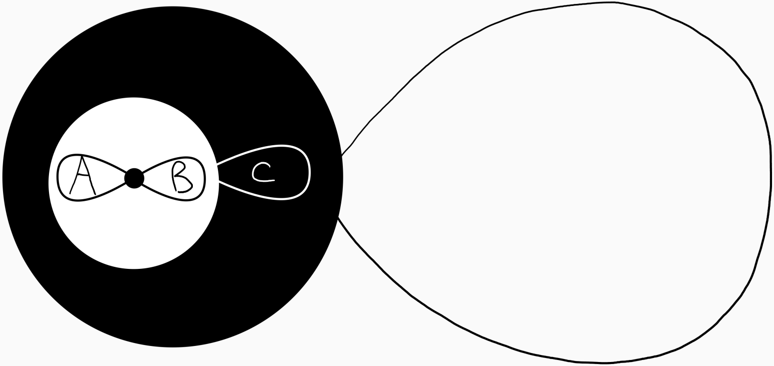
Example: disjunction elimination



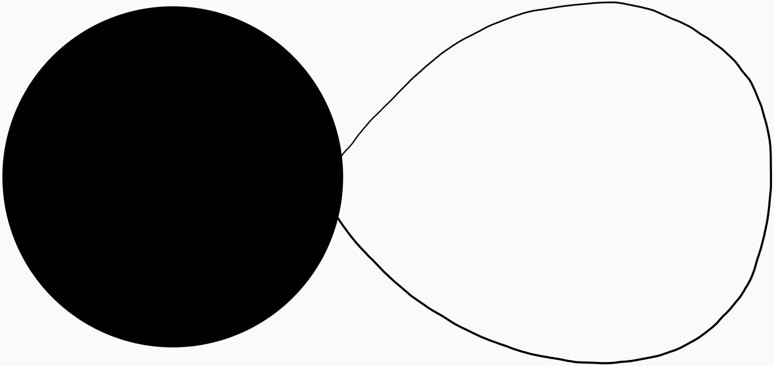
Example: disjunction elimination



Example: disjunction elimination



Example: disjunction elimination

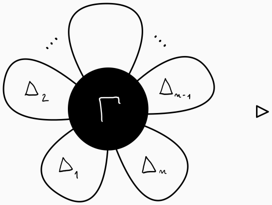


Example: disjunction elimination

Full reversibility

A *fully-reversible* variant conjectured complete:

- Pollination + Decomposition + Reproduction
- Petal removal all at once:



Basis for a connection method-like **proof-search procedure**.

Future works

Graphical interfaces

- Interface Actema with **Coq** (for real case studies)
- Custom (interactive?) **notations**
- **ZUI** for nested judgments (demo)

Proof theory

- **Extensions** and variants (classical, linear, higher-order)
- Computational content/**Curry-Howard** ((Guenot, 2013), (Elliott, 2007))
- Notion of **cut**, internal cut-elimination
- Completeness and **automated proof-search** in reversible fragment

SFL, bubbles and flowers
Bubbles and flowers
Flowers

Thank you!

References

Bertot, Y., Kahn, G., and Théry, L. (1994). Proof by pointing. In Hagiya, M. and Mitchell, J. C., editors, *Theoretical Aspects of Computer Software*, volume 789, pages 141–160. Springer Berlin Heidelberg. Series Title: Lecture Notes in Computer Science.

Chaudhuri, K. (2013). Subformula linking as an interaction method. In Blazy, S., Paulin-Mohring, C., and Pichardie, D., editors, *Interactive Theorem Proving*, volume 7998, pages 386–401. Springer Berlin Heidelberg. Series Title: Lecture Notes in Computer Science.

Clouston, R., Dawson, J., Goré, R., and Tiu, A. (2013). Annotation-Free Sequent Calculi for Full Intuitionistic Linear Logic. page 18 pages. Artwork Size: 18 pages Medium: application/pdf Publisher: Schloss Dagstuhl - Leibniz-Zentrum fuer Informatik GmbH, Wadern/Saarbruecken, Germany.

Donato, P., Strub, P.-Y., and Werner, B. (2022). A drag-and-drop proof tactic. In *Proceedings of the 11th ACM SIGPLAN International Conference on Certified Programs and Proofs*, CPP 2022, page 197–209, New York, NY, USA. Association for Computing Machinery.

Elliott, C. M. (2007). Tangible functional programming. In *Proceedings of the 12th ACM SIGPLAN International Conference on Functional Programming*, ICFP '07, page 59–70, New York, NY, USA. Association for Computing Machinery.

Guenot, N. (2013). *Nested Deduction in Logical Foundations for Computation*. phdthesis, Ecole Polytechnique X.

Oostra, A. (2010). Los gráficos alfa de peirce aplicados a la lógica intuicionista. In *Cuadernos de Sistemática Peirceana*, pages 25–60.