# Exam 2011-2012
# **Algorithms and Complexity of Constraint Satisfaction Problems**

MPRI 2.31.1

29 November 2011

**Remark:** I will focus on (1) correctness and (2) clarity in your exam document.

## Exercise 1

Let $R$ be an arbitrary Boolean relation of arity $k \geq 3$ with the following properties:

- $R$ contains all tuples with exactly one entry 1 and all other entries 0,
- $R$ does not contain the tuples $(0, \dots, 0)$ and $(1, \dots, 1)$.
- When $R$ contains a tuple $(t_1, \dots, t_k)$, then it also contains all tuples $(s_1, \dots, s_k) \neq (0, \dots, 0)$ such that $s_i \leq t_i$ for all $i \leq k$.

Determine the computational complexity of $\textsc{csp}(R)$.

## Exercise 2

Can the relation (constraint) $R(x, y, z) = [x \to (y \to z)]$ be implemented by the relations (constraints) $K(x, y) = [(x \to y) \to x]$ and $T(x, y, z) = [(x \to z) \to (y \to z)]$ ? Justify your answer.

## Exercise 3

**Problem:** YET ANOTHER SAT $(S)$
*Input:* An $S$-formula $\varphi$ and three models $m, m', m''$ of $\varphi$.
*Question:* Is there another model $m_*$ of $\varphi$, different from $m$, $m'$, and $m''$?

Show that when all relations in $S$ are Horn, then YET ANOTHER SAT $(S)$ can be solved in polynomial time.